Theoretical interpretation of the experiments on Parametric X-ray radiation in case of backward diffraction.

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Abstract

The spectral-angular and angular distributions of parametric X-radiation for case of backward diffraction (particular case of Bragg geometry scheme) is discussed. It is shown that in case of Bragg geometry it is necessary to use dynamical approach for PXR consideration. The comparison of the theory and experiment is carried out.

Introduction

Since the theoretical prediction of Parametric X-radiation (PXR) in crystals [1]–[4] and its experimental observation in 1985 [5, 6] a great number of experiments dedicated to studying of PXR characteristics has been carried out. Most of these experiments were performed in schemes of Laue geometry (Fig.1a) and so called extremely asymmetric geometry, in which PXR photons were emitted from the crystal through the lateral surface of crystal plate at the right angle relative to the electron beam velocity $\vec{v}$ (see Fig.1b).

In this case for theoretical interpretation of experimental data it is sufficient to use simplified by the special way exact theory, developed in [7, 8]. Specified simplification of the theory is similar in some details to kinematical approximation used by Ter-Mikaelyan for description of resonance radiation of charged particle in medium with periodical dialectical permittivity [9]. Let us remind, that the typical property of resonance radiation (and its main difference from PXR) is dependence of emitted photons energy on the energy of charged particles, while the PXR frequency is constant and determines only by the crystal lattice period and direction of charged particle propagation relative to crystallographic planes. However, the attempts of using some simplified variants of the theory for explanation of Bragg geometry experiments (see Fig.1c) appeared to be unsuccessful. In this paper it is shown that

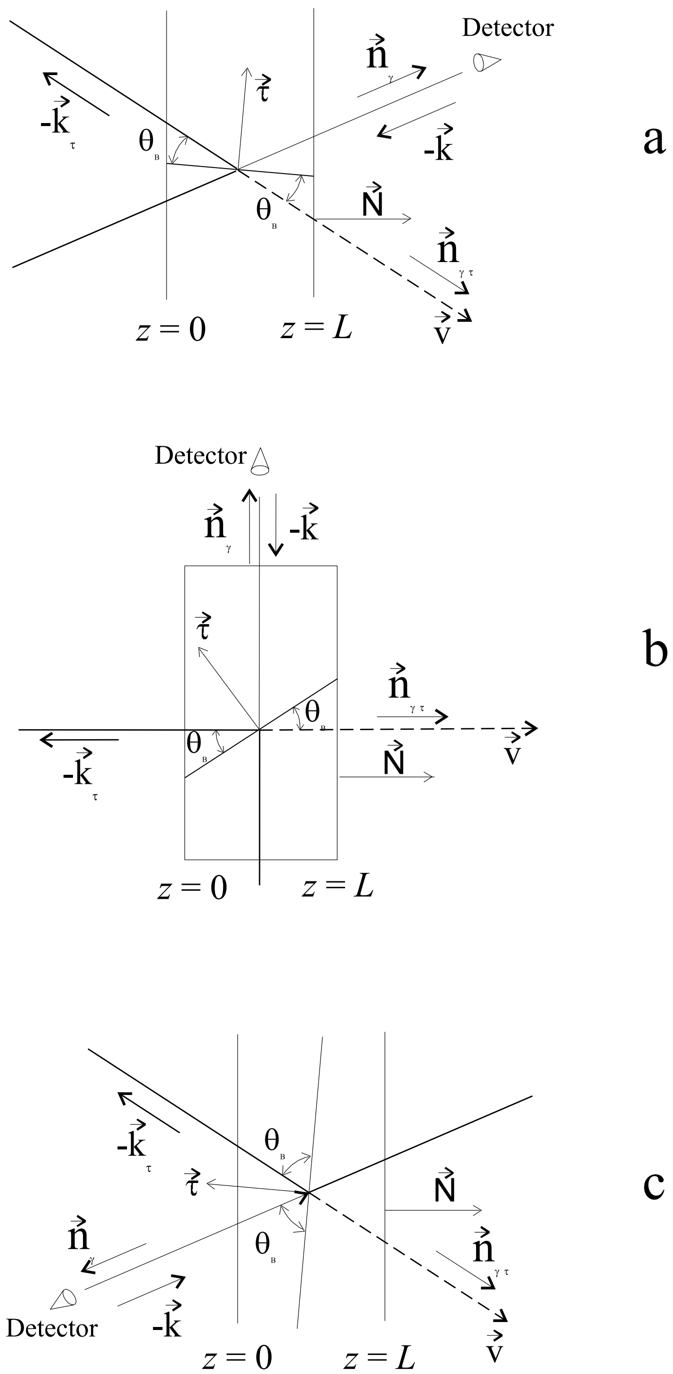

Figure 1: **The schemes of different geometries of PXR observation: a – Laue, b – Extremely asymmetrical, c – Bragg.**

for theoretical description of experiments in Bragg scheme it is necessary to use dynamical theory, developed in [10, 11].

In that way, there's a series of experiments on PXR measurement in scheme of Bragg geometry, which did not get any theoretical interpretation. In the this work we present and discuss the results of numerical calculations of spectral-angular and angular distributions of PXR in backward geometry, which is a particular case of Bragg geometry scheme, for the experimental parameters corresponding to the Mainz microtrone [12]. The experimental data were kindly given to us by professor H.Backe with colleagues, and as far as they are not published, we don't present them in this paper. In addition, we performed calculations of angular distributions for the experiment [13], which was also made in the backward geometry.

1 General expressions for PXR spectral-angular intensity in Bragg diffraction scheme

The spectral- angular distribution of radiation, generated by the charged particle at pass through the crystal plate into the maximum at the angle $2\theta_B$ (θ_B – angle between the particle velocity vector $\vec{v}$ and planes corresponding to vector $\vec{\tau}$) relative to the direction of its velocity in scheme of Bragg diffraction is given by the following expression [11, 14]:

$$\frac{d^2N_s}{d\omega d\vec{O}} = \frac{e^2Q^2\omega}{4\pi^2\hbar c^3}(\vec{e}_{\tau s}\vec{v})^2 \left| \sum_{\mu=1,2} \gamma^{\tau}_{\mu s} \left[\frac{1}{\omega - \vec{k}_{\tau}\vec{v}} - \frac{1}{\omega - \vec{k}_{\mu\tau s}\vec{v}} \right] \left[e^{\frac{i(\omega - \vec{k}_{\mu\tau s}\vec{v})L}{c\gamma_0}} - 1 \right] \right|^2, \quad (1)$$

$$\gamma^{\tau}_{1(2)s} = \frac{-\beta_1 C_s \chi_{\tau}}{(2\varepsilon_{2(1)s} - \chi_0) - (2\varepsilon_{1(2)s} - \chi_0)\exp\left(i\frac{\omega}{c\gamma_0}(\varepsilon_{2(1)s} - \varepsilon_{1(2)s})L\right)},$$

where eQ -particle charge, $C_s = \vec{e}_s\vec{e}_{\tau s}$, $e_{\vec{\tau}1} \parallel \left[\vec{k}\vec{\tau}\right]$, $e_{\vec{\tau}2} \parallel \left[\vec{k}e_{\vec{\tau}1}\right]$– the unit vectors of radiation polarization, $\vec{e}_s$ – the unit polarization vector of incident wave, $\vec{k_{\mu s}} = \vec{k} + \frac{\omega}{c\gamma_0}\varepsilon_{\mu s}\vec{N}$, $\vec{N}$ - unit vector of the normal to the entrance surface of a crystal plate, directed inside the crystal, χ_0, χ_τ, $\chi_{-\tau}$ – Fourier-components of complex crystal susceptibilities,

$$\varepsilon_{\mu s} = \frac{1}{4}\left(-\alpha_B\beta_1 + \chi_0(\beta_1 + 1) \pm \sqrt{\left[-\alpha_B\beta_1 + \chi_0(\beta_1 - 1)\right]^2 + 4\beta_1\chi^s_{\tau}\chi^s_{-\tau}}\right), \quad (2)$$

$$\alpha_B = \frac{2\vec{k}\vec{\tau} + \tau^2}{k^2}, \tag{3}$$

α_B is the Bragg-off parameter ($\alpha_B = 0$ in case of exact fulfillment of Bragg's condition) $\beta_1 = \gamma_0/\gamma_1$, $\gamma_0 = \vec{n}_\gamma \vec{N}$, $\vec{n}_\gamma = \frac{\vec{k}}{k}$, $\vec{n}_{\gamma\tau} = \frac{\vec{k}+\vec{\tau}}{|\vec{k}+\vec{\tau}|}$, $\gamma_1 = \vec{n}_{\gamma\tau}\vec{N}$, L_0 - the thickness of the crystal along the direction of a charged particle velocity $L_0 = L/\gamma_0$. The expression (1) has transparent physical sense: in case of two-beam diffraction the crystal has two reflex indexes $n_{\mu s} = \frac{k_{z\mu s}}{k_z} = 1 + \frac{\kappa_{\mu s}}{k_z}$, axis z is directed along vector $\vec{N}$, $\kappa_{\mu s} = \frac{\omega}{c\gamma_0}\varepsilon_{\mu s}$.

Every item in (1) describes well-known radiation amplitude $A_{\mu s}$ of photon arising as a result of charged particle movement through the crystal target of L thickness. Such as there are two reflex indexes, that the total radiation density expressed through the square of module of amplitudes sum, that is $\frac{d^2 N_s}{d\omega d\vec{O}} \sim |A_{1s} + A_{2s}|^2$.

Such as $\chi_0' < 0$ though from the Vavilov-Cherenkov condition it follows that only for a single root ($\mu = 1$) the real part of refraction index $n' > 1$. As a result the difference $\omega - \vec{k}_{1\tau s}\vec{v}$ can turn into zero and the term of the expression (1) comprising this difference in a denominator, begins to grow proportionally L. At first sight it means that the term, containing this difference (quasi-Cherenkov term), will give the main contribution into the radiation when increasing the thickness of the crystal along the particle velocity. However, in case of Bragg diffraction there's a considerable distinction of physical phenomenon, taking place in the crystal, from the case of Laue diffraction, namely, in some area of frequencies and angles the phenomenon of total reflection takes place. In this area of angles and frequencies the wave vectors in crystal lattice become imaginary values at conditions of absorption absence. In the area of total reflection, stipulated by the existence of the heterogeneous wave in the crystal, it is necessary to take into account of both dispersion branches during calculation of the radiation intensity in Bragg diffraction scheme. Although the structure of expression (1) is very simple, but in order to obtain quantitative data it is necessary to performed correct calculations on formula (1) with taking into account all terms of it, since presence oscillating ones and its interference can easily bring to wrong results if some terms are neglected.

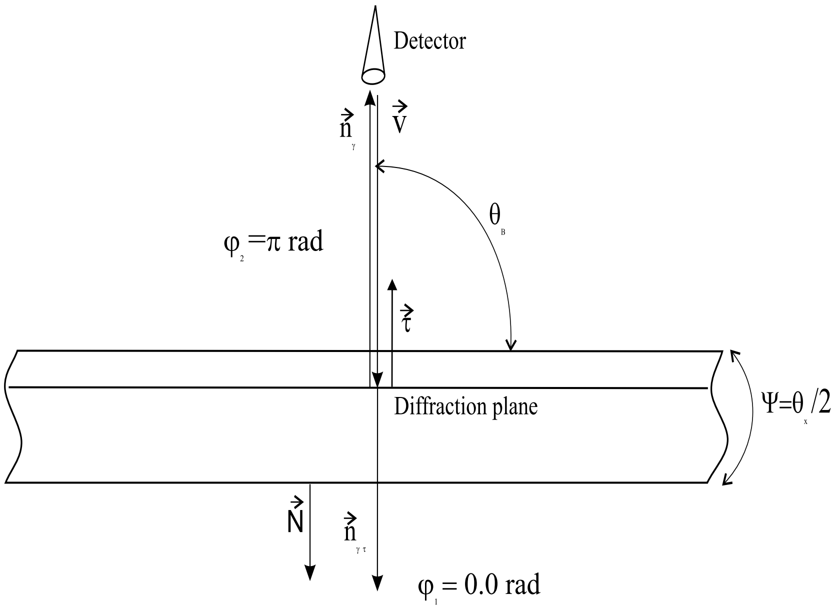

Figure 2: **The scheme of backward diffraction experiment, the angular distribution is taken as the function of the tilt angle ψ.**

2 PXR in backward geometry

Recently there were conducted the experiments on observation of X-ray radiation, generated by the charged particles in scheme of backward diffraction [13] - see the Fig 2.

There were taken the angular dependence of radiation intensity as the function of the tilt angle ψ relatively the direction of charged particles movement, for the tilt angle $\psi = 0$ this direction coincides with the direction of charged particles movement, the Bragg's angle $\theta_B = 90°$, the radiation is detected at the angle $2\theta_B = 180°$ relative to this direction. At rotation of the crystal at the angle ψ the Bragg's angle becomes equal to $\theta'_B = \theta_B + \psi$, the radiation angle – $2\theta'_B = 2\theta_B + \theta_X$, where $\theta_X = 2\psi$.

The scheme of this kind (Fig.2) presents a keen interest, because the theoretical description of the radiation intensity cannot be given in scope of kinematical diffraction, such as existence of the inhomogeneous wave brings to the possibility of realization of the effect of the total reflection. Let us make a comparison of

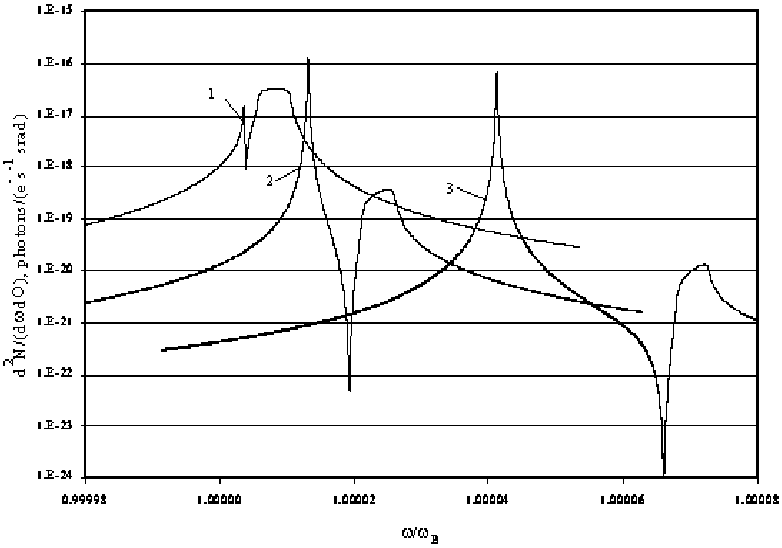

Figure 3: **The spectra of the PXR radiation, received for the tilt angles of the crystal: 1 - 0,3 mrad; 2 - 2,5 mrad; 3 - 5,0 mrad.**

numerical calculations results basing on the formula (1) with the results, received in the experiment on the microtone in Mainz. The measurements were taken in a silicon crystal plate thick $L = 525\ \mu$m, the energy of electron beam 855 MeV. The temperature of the target was maintained at 120 K. There was studied the radiation in reflexes (111), (333), (444), (555), (777), (888). At the Fig.3 there are presented spectral-angular distributions of PXR for the reflex (444), received basing on the formula (1) for the tilt angles of the crystal 0,3 mrad, 2,5 mrad and 5 mrad.

As far as the tilt angle gets greater there's a shift of the spectrum towards increasing of the frequency. The narrow maximum in the radiation spectrum corresponds at calculation by (1) to the term, proportional to $\frac{1}{\omega - \vec{k}_{1\tau\sigma}\vec{v}}$, which denominator can turn into 0, and so this peak can be interpreted as conditioned by quasi-Cerenkov radiation mechanism.

With increasing of the tilt angle of the crystal ψ (what corresponds to the polar angle of the radiation 2ψ) the spectral-angular intensity of this maximum increases and at some angle $\sim \vartheta_{ph} = \sqrt{\gamma^{-2} - \chi_0'}$ (γ – Lorentz factor of charged particle) becomes to exceed the intensity of radiation maximum, intensity of which can be re-

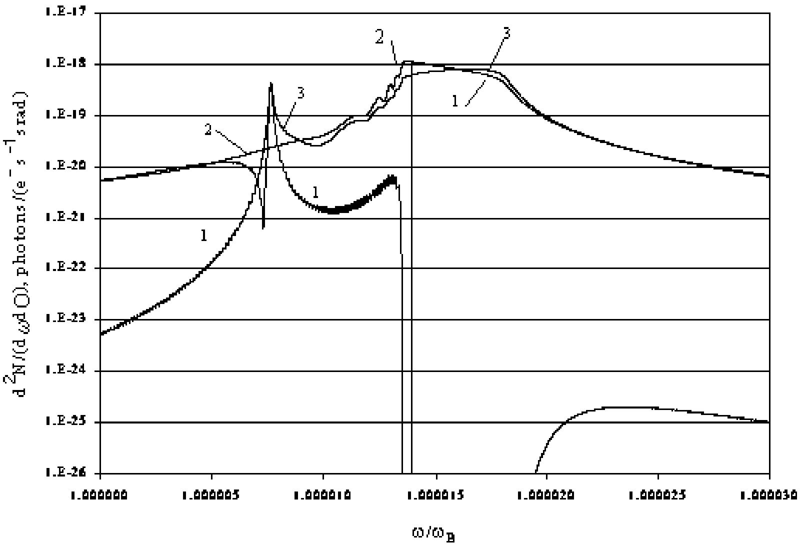

Figure 4: **The spectrum of the radiation, calculated by the formula (1) for 1 - $\mu = 1$; 2 - $\mu = 2$; 3 - the sum radiation.**

ceived using simplified theory [9] based on simple presentation of ordinary transition radiation diffraction on crystallographic planes, and maximum corresponds to area of maximal effectiveness of the X-ray reflection on the crystal surfaces (the radiation, emitted at angles and frequencies, for which Vavilov-Cherenkov condition does not fulfill, however the coefficient of the X-ray radiation reflection is maximum). It's necessary to note, that the spectral width of the quasi-Cerenkov maximum is defined by the expression $\frac{\Delta\omega}{\omega_B} \sim \frac{c}{L_{eff}\omega_B \sin^2\theta_B}$ [10], here $\omega_B = \frac{\pi c}{d \sin\theta_B}$ – Bragg frequency, d – interplanar distance, $L_{eff} = min\,(L_0, L_{abs})$, L_{abs} – absorption length, while the width of maximum, corresponds to area of total reflection $\frac{\Delta\omega}{\omega_B} \sim \frac{|\chi_\tau|}{\sin^2\theta_B}$.

For the reflex (444) the width of the quasi-Cherenkov maximum is in order $4 \times 10^{-7}\omega_B$, what more than one degree narrow of the width of peak in total reflection area, equal to $\sim 5 \times 10^{-6}\omega_B$. At the Fig.4 for the reflex (555) there were demonstrated the spectral-angular distributions of "forming" the sum radiation components (the radiation, corresponding to different branches of dispersion curves ($\mu = 1, 2$). It's clear from the picture, that the quasi-Cherenkov maximum belongs to the first dispersion branch $\mu = 1$.

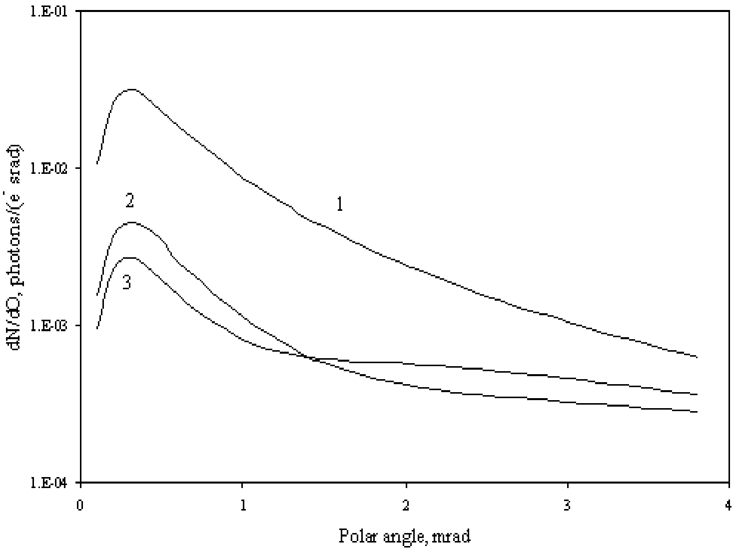

Figure 5: The angular distribution in the reflexes: 1 - (111), the energy resolution of the detector $\Delta\omega = 358$eV, 2 - (333), $\Delta\omega = 402$ eV, 3 - (444), $\Delta\omega = 400$ eV.

The distribution at the Fig.4 was received for the tilt angle of the crystal $\psi = 0,3$ mrad, at such angles the intensity of the maximum, corresponding to area of total reflection is considerably exceeding the intensity of the quasi-Cherenkov maximum. The angular intensity at such angles is fully defined by total reflection area, as the frequency's width of this maximum is much greater the width of quasi-Cherenkov maximum.

At the fig.5 there are presented the angular distributions of the radiation as a function of the tilt angle for reflexes (111), (333) and (444). The form of distributions coincides well with the experimental curves, the value of the angular distribution in the maximum for the reflex (111) is different from the experimental one at 10%, for the reflex (333) at 18%, for (444) - at 21%. So, this difference appears at small angles - less and order of 0,3 mrad. The difference increases with increasing of the energy of photons being emitted. Possible explanation of this effect can be the influence of the multiple scattering, and exactly, the additional contribution in the intensity of the bremsstrahlung radiation, emitted at small angles.

When decreasing the energy of charged particles, the contribution of the bremsstrahlung radiation increases considerably. This is explained by fact that coherent length of bremsstrahlung radiation $L_{Br} = \sqrt{\frac{4c}{\omega\overline{\theta_s^2}}}$, $\overline{\theta_s^2}$ - root-mean-square angle of multiple scattering (MS), becomes less then coherent length of PXR. In the paper [13] there is described an experiment also in backward geometry, only at the energy of the electron beam one degree lower ($E_p = 80,5$ and $86,5$ MeV). In this energy region coherent length of bremsstrahlung radiation $L_{Br} \ll L$, and the account of bremsstrahlung radiation contribution is very important. Our calculations showed that the intensity of angular distribution in the maximum, received by the integration of the expression (1) over frequency in the range $\Delta\omega = 10^{-3}\omega_B$, is two times lower the experimental value. However, it is necessary to take into consideration the contribution of the bramsstrahlung radiation into the sum radiation at such energies of electron beam (estimations were made basing on the formulas from the paper [10]. In that way, it's possible to state that received results for the angular intensity of the radiation agree well with the experimental data.

Conclusion

Conducted comparisons of theoretical and experimental distributions of PXR in the backward geometry showed a good coincidence of results of the theory, developed in [11] and the experiment. It allows to state, that only the approach of dynamical theory of the diffraction allows to interpret experiment. The other simplified approaches, such as kinematical theory, approach in which diffraction maximum is considered as diffracted on crystal planes transition radiation (it is called in some works diffraction transition radiation (DTR)) and which does not take into account interference between different dispersion branches [15, 16], don't bring to successful interpretation of experimental data.

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