Superradiance in volume diffraction grating ⋆

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Abstract

To simulate VFEL operation the superradiance from a short electron pulse moving in a volume diffraction grating is studied. It is supposed that Bragg condition for emitted photons is fulfilled and dynamical diffraction takes place. Spectral-angular distributions for transmitted and diffracted waves are derived. It is shown that the optimal geometry for superradiance exists and it is determined by the energy and transverse size of electron beam.

Key words: Volume Free Electron Laser (VFEL), Volume Distributed Feedback (VDFB), diffraction grating, Smith-Purcell radiation, electron beam instability
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1 Introduction

There are two ways to obtain coherent radiation: for unperturbed electron beam the coherent radiation appears as a result of bunching of electrons by ponderromotive wave. The second possibility exists at electron beam modulation or for short electron beam. If the size of electron beam is compared or less then wavelength $l < \lambda$, the intensity of radiation is proportional to N^2, where N is the number of electrons. As a result intensity of coherent radiation essentially exceeds the incoherent part (shot noise) in this case. Superradiance of electron beams was studied in a great number of works (see for example [1–3]). In present work the spectral-angular distributions of quasi-Cherenkov radiation in condition of volume distributed feedback for short electron beam is derived. Volume free electron lasers (VFEL) ([4]) use non-one-dimensional

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volume distributed feedback (VDFB), which essentually changes the dispersion characteristic of electromagnetic wave and retains radiation in interaction region. The sharp increase of amplification process take place in the points of degeneration ([5,6]). The spectral-angular behavour of superradiance in these point are modified also.

2 The spectral-angular distribution of quasi-Cherenkov superradiance

The spectral-angular distribution of radiation is as follows [7]:

$$W_{\mathbf{n}\omega} = \frac{e^2\omega^2}{4\pi^2 c^3}\left|\sum_i \int dt e^{i\omega t}\mathbf{v}_i(t)\mathbf{E}_{\mathbf{k}}^{(-)*}(\mathbf{r}_i(t);\omega)\right|^2 . \quad (1)$$

Here $\mathbf{E}_{\mathbf{k}}^{(-)}(\mathbf{r}_i(t);\omega)$ is the wave function of a photon, which asymptotically behaves as plane wave $\exp\{i\mathbf{kr}\}$ plus incoming spherical wave [7], $\mathbf{v}_i(t)$ is the velocity of i^{th} electron, ω and $\mathbf{k}$ are the frequency and wave vector of emitted photons. The integration over whole electron pass is performed in (1). Summing over all electrons in (1) one can obtain the following form-factor of electron beam:

$$\sum_{ij} \exp\{i\mathbf{k}_\perp(\mathbf{r}_{j0\perp}-\mathbf{r}_{i0\perp}) + ik_z^{(ch)}(z_{i0}-z_{j0})\} = \quad (2)$$
$$\int\int d\mathbf{r}_1 d\mathbf{r}_2 n(\mathbf{r}_1)n(\mathbf{r}_2)\exp\{i\mathbf{k}_\perp(\mathbf{r}_{1\perp}-\mathbf{r}_{2\perp}) + ik_z^{(ch)}(z_2-z_1)\},$$

where $\mathbf{r}_{i0} = (\mathbf{r}_{i0\perp}, z_{i0})$, $\mathbf{r}_{j0} = (\mathbf{r}_{j0\perp}, z_{j0})$ are the coordinates of electrons at $t=0$, $n(\mathbf{r}_1)$, $n(\mathbf{r}_2)$ are the microscopic density function $n(\mathbf{r}) = \sum\limits_i \delta(\mathbf{r}-\mathbf{r}_{i0})$. Averaging (2) over coordinates $\mathbf{r}_{i0}$ gives

$$\overline{n(\mathbf{r}_1)n(\mathbf{r}_2)} = N\delta(\mathbf{r}_1-\mathbf{r}_2) + N(N-1)f^{(2)}(\mathbf{r}_1,\mathbf{r}_2) \quad (3)$$

Here $f^{(2)}(\mathbf{r}_1,\mathbf{r}_2)$ is the two-particle distribution function. Neglecting corellation function one can write two-particle distribution function as $f^{(2)}(\mathbf{r}_1,\mathbf{r}_2) = f^{(1)}(\mathbf{r}_1)f^{(1)}(\mathbf{r}_2)$, where $f^{(1)}(\mathbf{r}_1)$ is the one-particle distribution function. Substitution of (3) in (2) gives:

$$\overline{\int\int d\mathbf{r}_1 d\mathbf{r}_2 n(\mathbf{r}_1)n(\mathbf{r}_2)\exp\{i\mathbf{k}_\perp(\mathbf{r}_{1\perp}-\mathbf{r}_{2\perp}) + ik_z^{(ch)}(z_2-z_1)\}} = \quad (4)$$
$$\left(N + N(N-1)\left|\int d\mathbf{r} f(\mathbf{r})\exp\{i\mathbf{k}^{(ch)}\mathbf{r}\}\right|^2\right)$$

Let us consider electron beam form-factors in some particular cases:

$$
\begin{array}{r}
for\ electron\ beam\ with\ rectangular\ cross\ section \\
N+N(N-1)\left(\frac{\sin\{k_x\Delta x/2\}}{k_x\Delta x/2}\frac{\sin\{k_y\Delta y/2\}}{k_y\Delta y/2}\frac{\sin\{k_z\Delta z/2\}}{k_z\Delta z/2}\right)^2 \\
for\ electron\ beam\ with\ circular\ cross\ section \\
N+N(N-1)\left(\frac{\sin\{\omega l/2u\}}{\omega l/2u}\frac{J_1(k_\perp R)}{k_\perp R}\right)^2 \\
for\ electron\ beam\ with\ Gauss\ profile \\
N+N(N-1)\exp\{-k_x^2\frac{\sigma_x^2}{2}\}\exp\{-k_y^2\frac{\sigma_y^2}{2}\}\exp\{-k_z^2\frac{\sigma_z^2}{2}\}
\end{array}
\tag{5}
$$

One can see from (5) that form-factor strongly depends on electron beam profile. For beam with Gaussian profile the form-factor depends on frequency steadily. In case of sharp boundary of electron beam form-factor is the oscillating function of frequency. Substituting (4) and the expression for $\mathbf{E}_{\mathbf{k}}^{(-)}(\mathbf{r}_i(t);\omega)$ in (1) one can obtain the following expression for spectral-angular distribution of photons in Bragg diffraction geometry [7]:

$$
\frac{d^2N_s}{d\omega d\Omega}=Q\frac{e^2\omega}{4\pi^2\hbar c^3}(\vec{e}_s\vec{v})^2\times \tag{6}
$$

$$
\left|\sum_{\mu=1,2}\gamma_{\mu s}^0 e^{i\frac{\omega}{c\gamma_0}\varepsilon_{\mu s}L}\left[\frac{1}{\omega-\vec{k}\vec{v}}-\frac{1}{\omega-\vec{k}_{\mu s}\vec{v}}\right]\left[e^{\frac{i(\omega-\vec{k}_{\mu s}\vec{v})L}{c\gamma_0}}-1\right]\right|^2,
$$

The denotions introducine in [7] is used in (6), Q is the form-factor. For photons with the polarization vector $\vec{e}_{\tau s}$ emitted at diffraction direction can be rewritten as follows [7]:

$$
\frac{d^2N_s}{d\omega d\Omega}=Q_\tau\frac{e^2\omega}{4\pi^2\hbar c^3}(\vec{e}_{\tau s}\vec{v})^2 \tag{7}
$$

$$
\left|\sum_{\mu=1,2}\gamma_{\mu s}^\tau\left[\frac{1}{\omega-\vec{k}_\tau\vec{v}}-\frac{1}{\omega-\vec{k}_{\mu\tau s}\vec{v}}\right]\left[e^{\frac{i(\omega-\vec{k}_{\mu\tau s}\vec{v})L}{c\gamma_0}}-1\right]\right|^2.
$$

In (7) Q_τ is the form-factor for diffracted photons:

$$
Q_\tau=N+N(N-1)\left|\int d\mathbf{r}f(\mathbf{r})\exp\{i\mathbf{k}_\tau^{(ch)}\mathbf{r}\}\right|^2 \tag{8}
$$

The diffraction anomalies appear in spectral-angular distributions of spontaneous radiation near the point of root degeneration in conditions of dynamical diffraction [8]. Two factors arouse these peaks. The first one is the decreasing of the wave group velocity:

$$v_z^{(gr)} = \frac{2\gamma_0 c}{1 + \beta \mp \frac{\Delta}{\varkappa}(1-\beta)}, \tag{9}$$

where $\Delta = -\chi_0(1-\beta) - \beta\alpha$, $\varkappa = \sqrt{\Delta^2 + 4\beta r}$. The second condition for apearance of diffraction spectral-angular anomaly is the phase condition:

$$(k_{1z} - k_{2z})L = 2\pi n, \tag{10}$$

where n is integer, k_{1z} and k_{2z} are the roots of dispersion equation for two wave dynamical diffraction. In these conditions the additional factor $(k\chi L/4\pi)^2 \gg 1$ appears in expressions for spectral-angular density. The remarkable property of points of diffraction describing by (10) is the fact that (10) corresponds to maximal radiation intensity and maximal value of amplification due to stronger interaction of photons with spartially periodic medium. The same conditions (9), (10) correspond to more intensive interaction in two wave VFEL ([6])

There are optimal parameteres of diffraction geometry, for which the spectral-angular density of photon in the superradiance process increases. The dependence of form-factor on asymmetry factor $\beta = \gamma_0/\gamma_1$ is presented on fig.1.

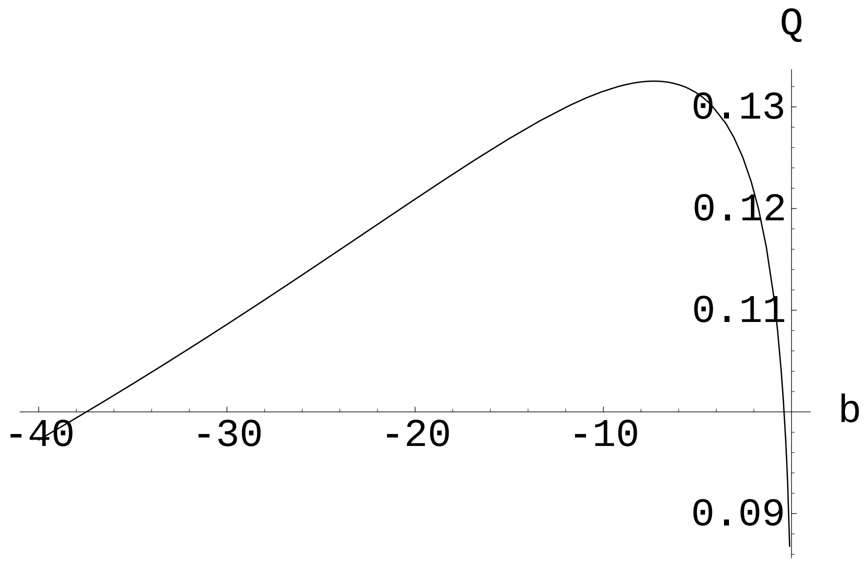

Fig. 1. Dependence of form-factor of quasi-Cherenkov superradiation on asymmetry factor β.

The optimal geometry is determined by many factors such as electron beam profile and energy, electrodynamical properties of grating, electron bunch length and so on.

3 Conclusion

Quasi-Cherenkov superradiance in the volume diffraction grating can be obtained at the absence of Cherenkov superradiance, it is possible if:
1) Cherenkov condition for radiation is not fulfilled, but quasi-cherenkov condition is satisfied [7] or
2) cross section of electron beam meets the condition $k\Theta_{ch}R \gg 1$, where R is electron beam radius. Then there is possibility to observe superradiance at angles lower than Θ_{ch} for quasi-Cherenkov radiation mechanism due to equality

$$\Theta^2_{q-ch} = \Theta^2_{ch} + \frac{-\alpha \pm \sqrt{\alpha^2 + 4r}}{2}$$

one can obtain superradiance at range of Bragg parameteres corresponding to angles $k\Theta_{q_ch}R \sim 1$ The similarity between quasi-Cherenkov lasing process and quasi-Cherenkov radiation from short electron beam allows the experimental simulation of induced radiation in condition of VDFB. For example by using the volume diffraction grating [9] with spatial period of about few milllimeters we can study the quasi Cherenkov superradiance, to check the dependence of quasi Cherenkov radiation on parameter β and to observe the spectral angular diffraction anomalies corresponding to (9) and (10) in millimeter wavelength range. For volume grating with length $L = 10$ cm the angular and spectral resolution $\Delta\Theta \sim \Delta\omega/\omega \sim 10^{-3}$ for observing of diffraction anomalies is needed.

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