Dependence of volume FEL (VFEL) threshold conditions on undulator parameters

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Abstract

Volume free electron laser uses volume distributed feedback (VDFB), which can essentially reduce the threshold current of generation and provides the possibility of smooth frequency tuning. An undulator VFEL is considered.

Key words: Volume Free Electron Laser (VFEL), Volume Distributed Feedback (VDFB), diffraction grating, Smith-Purcell radiation, electron beam instability
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1 Introduction

The advantages of distributed feedback (DFB) is well known in laser physics. In conventional lasers DFB is formed in such a way that diffracted and transmitted waves propagate along a line in opposite directions. The distinctive feature of volume free electron lasers (VFEL) is using of non-one-dimensional (volume) distributed feedback at which the diffracted wave propagates at the angle $\neq \pi$ to the transmitted wave. Firstly the idea to use volume distributed feedback (VDFB) for X-ray generation was proposed in [3]. VDFB causes sharp increase of amplification and instability increment of electron beam density is proportional to $\sim \rho^{1/(3+s)}$, where ρ is the electron beam density and s is the number of surplus waves appearing due to diffraction (for example, in case of two-wave Bragg diffraction $s = 1$, for three-wave diffraction $s = 2$ and so on). This dependence essentially differs from increment for conventional FEL in Compton regime, which is proportional to $\sim \rho^{1/3}$ [4]. Now the investigation of 4-wave diffraction in system with two-dimentional DFB is also started by [5].

Sharp increase of amplification caused by VDFB yields to noticeable reduction of threshold currents necessary for lasing start. This fact is particularly important for lasing in submillimetre and visible ranges and for shorter wavelengths. Explicit expressions VFEL threshold currents were obtained in [7]. In present work the dependence of VFEL starting current on undulator parameters is considered.

2 Generation and amplification laws of undulator VFEL

It is well known that to find amplification and starting current one should study the dispersion law of radiating system The set of equations describing interaction of relativistic electron beam, which propagates in spatially periodic structure in undulator is [7]:

$$\begin{array}{r} DE-\omega^2\chi_1 E_1-\omega^2\chi_2 E_2-...=0\\ -\omega^2\chi_{-1}E+D_1E_1-\omega^2\chi_{2-1}E_2-...=0\\ -\omega^2\chi_{-2}E-\omega^2\chi_{1-2}E_1+D_2E_2=0-...,\\ ... \end{array} \qquad (1)$$

where $D_\alpha = k_\alpha^2 c^2 - \omega^2\varepsilon + \chi_\alpha^{(b)}$, $\vec{k}_\alpha = \vec{k} + \vec{\tau}_\alpha$ are the wave vectors of photons diffracted by the crystal planes with corresponding reciprocal vectors $\vec{\tau}_\alpha$, $\varepsilon_0 = 1+\chi_0$, χ_α are the dielectric constants of a periodic structure. These constants can be obtained from the following representation of dielectric permiability of periodic structure:

$$\varepsilon(\vec{r},\omega) = 1 + \sum_{\{\tau\}} \exp(i\vec{\tau}\vec{r})\chi_\tau(\omega).$$

$\chi_\alpha^{(b)}$ is the part of dielectric susceptibility appearing from the interaction of an electron beam, propagating in undulator, with radiation:

$$\begin{array}{l} \chi_\alpha^{(b)} = \dfrac{\pi\Theta_s^2 c^2}{\gamma_z^2\gamma I_A}\dfrac{j_0}{(\omega-(\mathbf{k}+\mathbf{k}_w)\mathbf{u}_w)^2}\\ \qquad \textit{for the "cold" beam limit and}\\ \chi_\alpha^{(b)} = -i\sqrt{\pi}\dfrac{\pi\Theta_s^2 c^2}{\gamma_z^2\gamma I_A}\dfrac{j_0}{\sigma_\alpha^2}x_\alpha^t\exp[-(x_\alpha^t)^2]\\ \qquad \textit{for the "hot" beam limit,} \end{array} \qquad (2)$$

$\Theta_s = eH_w/(mc^2\gamma k_w)$, $\gamma_z^{-2} = \gamma^{-2}+\Theta_s^2$, $k_w = 2\pi/\lambda_w$, λ_w is undulator period,H_w is undulator field, $x_\alpha^t = (\omega - (\mathbf{k}+\mathbf{k}_w)\mathbf{u}_w)/\sqrt{2}\sigma_\alpha$, $\sigma_\alpha^2 = (k_{\alpha 1}^2\Psi_1^2 + k_{\alpha 2}^2\Psi_2^2 + k_{\alpha 3}^2\Psi_3^2)u^2$ and $\vec{\Psi} = \Delta\vec{u}/u$ is the velocity spread. If the inequality $x_\alpha^t \gg 1$ is fulfilled, all the electrons interact with electromagnetic wave and the "cold" limit is realized. In the opposite case ("hot" limit) $x^t_\alpha < 1$ only small part of electron beam interacts with electromagnetic wave. Setting the determinant of linear system (1) equal to zero one can obtain the dispersion equation for the system "electromagnetic wave + undulator + electron beam + periodic structure". In case of two-wave dynamical diffraction this equation has the following form:

$$DD_1 - \omega^4 \chi_1 \chi_{-1} = 0 \tag{3}$$

For the system with finite interaction length the solution of boundary problem can be presented as a sum:

$$\mathbf{E} + \mathbf{E}_1 = \sum_i c_i \exp\{i\mathbf{k}_i\mathbf{r}\}(\mathbf{e} + \mathbf{e}_1 s_1^{(i)} \exp\{i\tau\mathbf{r}\}), \tag{4}$$

here $s_1^{(i)} = \frac{k_i^2 c^2 - \omega^2 \varepsilon_0}{\omega^2 \chi_1}$ are the coupling coefficients between the diffracted and transmitted waves ($E^{(1)} = s_1 E$) and $\vec{k}_i$ are the solutions of dispersion equation (3). To determine coefficients c_i it is necessary to write the boundary conditions on the system ends $z = 0$ and $z = L$. For Bragg geometry, when transmitted and diffracted waves has the opposite signs of wave vector projections on the axis z, these conditions are as follows:

$$\sum_i c_i = a \quad \sum_i \frac{c_i}{\delta_i} = 0 \quad \sum_i \frac{c_i}{\delta_i^2} = 0 \quad \sum_i s_1^{(i)} c_i \exp\{ik_{iz}L\} = b \tag{5}$$

In (5) the wave vector is represented as: $\mathbf{k} = \mathbf{k}_0 + \frac{\omega}{c}\delta$, where $\mathbf{k}_0$ satisfies undulator synchronism condition. The boundary conditions (5) are written for the "cold" electron beam. In this case the dispersion equation has four roots (δ_i, $i = 1 \div 4$). The first and the fourth conditions in (5) correspond to continuity of transmitted wave at $z = 0$ and diffracted wave at boundary $z = L$ (it is supposed that the wave with wave vector $\vec{k}$ and amplitude a is falling on boundary $z = 0$ and the wave with wave vector $\vec{k}_1$ and amplitude b is falling on boundary $z = L$). The second and the third conditions in (5) accord with the requirement that the electron beam is unpertubed before entering the interaction region. The part of electron beam energy converting to radiation can be expressed as:

$$I \sim \gamma_0 |E(z = L)|^2 + |\gamma_1||E_1(z = 0)|^2 = \tag{6}$$
$$(\gamma_0|a|^2 + |\gamma_1||b|^2) \left(\frac{\gamma_0 c}{\vec{n}\vec{u}}\right)^3 \frac{16\pi^2 n^2}{-\beta(k|\chi_1|L_*)^2 k L_* (\Gamma_{start} - \Gamma)},$$

where L is the length of interaction in undulator,

$$\Gamma_{start} = \left(\frac{\gamma_0 c}{\vec{n}\vec{u}}\right)^3 \frac{16\pi^2 n^2}{-\beta(k|\chi_1|L_*)^2} - \chi" \left(1 - \beta \pm \frac{r"\sqrt{-\beta}}{|\chi_1|\chi"}\right)$$

$$\Gamma = \frac{\pi^2 n^2}{4} \frac{\pi \Theta_s^2 c^2 j_0}{\gamma_z^2 \gamma I_A \omega^2} k^2 L_*^2 q^2 f(y),$$

$$f(y) = \sin y \frac{(2y + \pi n)\sin y - y(y + \pi n)\cos y}{y^3 (y + \pi n)^3}$$

is the function of generation dependence on detuning from synhronism condition, $y = (\omega - \mathbf{k}\mathbf{v}_w - \Omega)L/(2u_z)$ is detuning factor, $\beta = \gamma_0/\gamma_1$ is diffraction asymmetry factor, γ_0, γ_1 are diffraction cosines, $\chi" = Im\ \chi_0$. The function $f(y)$ is presented in fig.1 One can see from fig.1 that dependence on detuning

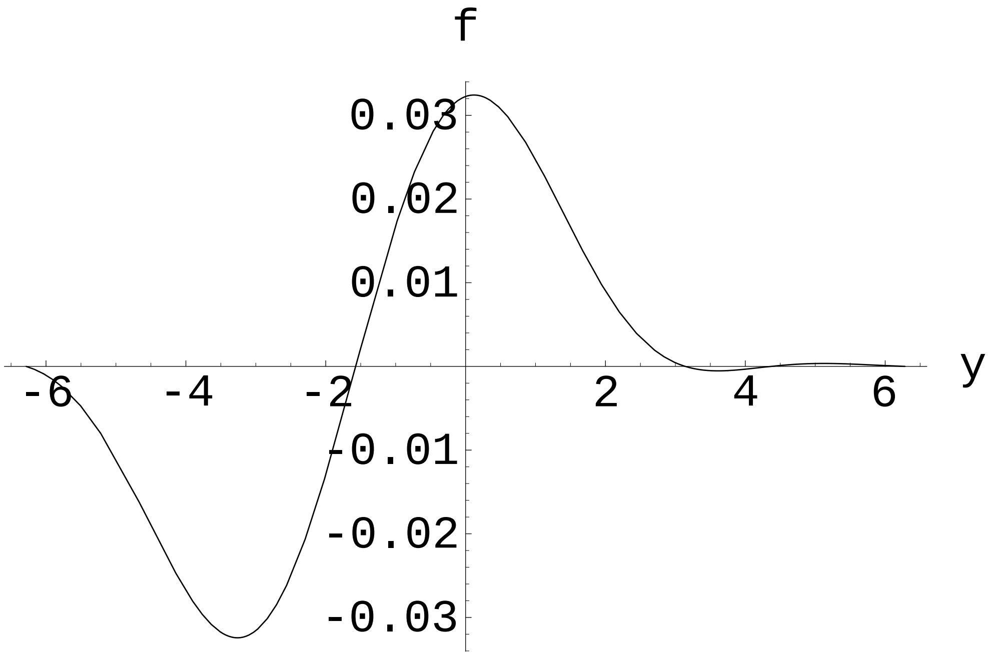

Fig. 1. Dependence of induced radiation on detuning factor y in the condition of two-wave diffraction.

factor y is not asymmetric. This distinguishes lasing in the range of roots degeneration from generation process in conventional undulator FEL. The latter has the following dependence on detuning factor [4]:

$$g(y) = \frac{\sin y}{y} \frac{y \cos y - \sin y}{y^2}.$$

This difference ensues from interference of contribution to radiation of two diffraction roots. From (6) follows that: 1. the starting current in case of two-wave diffraction is proportional to $j_{start} \sim (kL)^{-1}(k\chi_1 L)^{-2}$;
2. non-one dimensional VDFB provides the possibility to decrease the starting current of generation by varying of the angles between the waves. The dependence of $\Gamma_{start}(\beta)/\Gamma_{start}(\beta = 0)$ on asymmetry factor β is presented in fig.2.

3. if electron beam current is less than starting value $j < j_{start}$ then energy in electromagnetic wave at the system entrance can be written in the form:

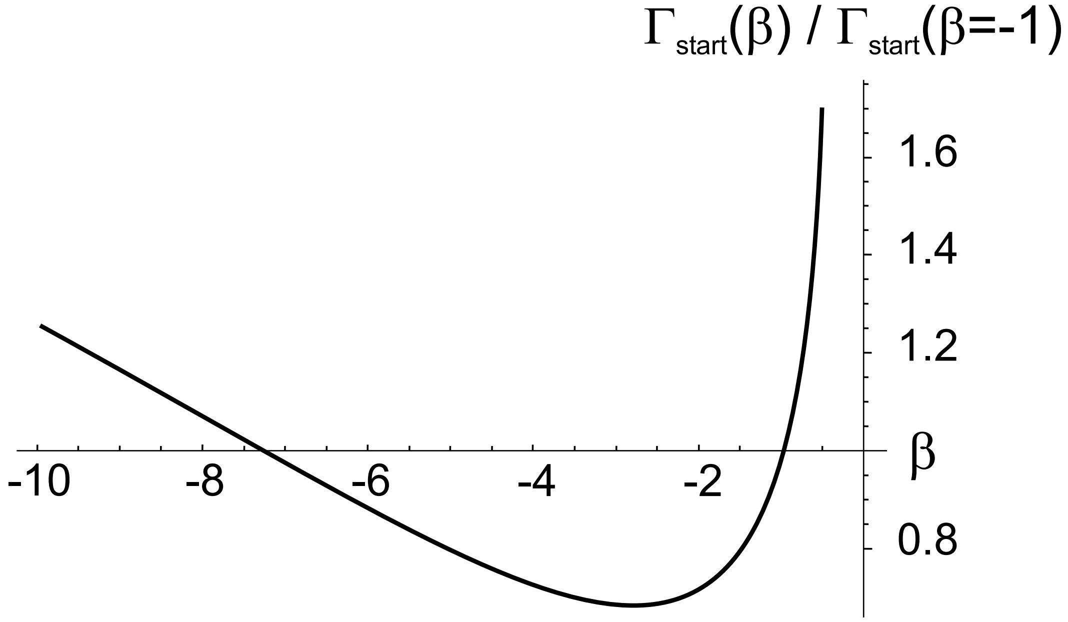

Fig. 2. Dependence of form-factor of quasi-Cherenkov superradiation on asymmetry factor β.

$$I/(\gamma_0|a|^2 + |\gamma_1||b|^2) = 1 - \beta\frac{\pi^2 n^2}{4}\frac{\pi\Theta_s^2 j_0 c^2}{\gamma_z^2\gamma I_A\omega^2}(kL)^3\left(\frac{k\chi_\tau L}{4\pi}\right)^2 f(y) \qquad (7)$$

The conventional FEL gain is proportional to $(kL)^3$ [4], but as follows from (7) in case of two-wave diffraction the gain gets an additional factor $\sim \left(\frac{k\chi_\tau L}{4\pi}\right)^2$, which noticeably exceeds the unity in conditions of dynamical diffraction. Such increase of radiation output in degeneration point can be explained by the reduction of wave group velocity, which can be written as:

$$v_g = -\left(\frac{\partial D}{\partial k_z}\right)/\left(\frac{\partial D}{\partial \omega}\right) \sim \prod_{i<j}(k_{zi} - k_{zj}) \qquad (8)$$

It follows from (8) that for multi-wave dynamical diffraction in the s-fold-degeneration point $v_g \sim v_0/(kL)^s$, the starting current $j_{start} \sim (kL)^{-3}(k\chi_1 L)^{-2s}$ and amplification is proportional to $(kL)^3(k\chi_1 L)^{2s}$. It should be noted that considered effects take place in wide spectral range for wavelengths from centimeters to X-ray ([3,6–9]) and influence of effect increases with the frequency growth.

3 Conclusion

The generation threshold in undulator VFEL in case of VDFB can be achived at lower electron beam current and shorter undullator length when special conditions of roots degeneration are fulfilled. Change of VDFB conditions by varying the volume geometry parameters (for example, the angle between wave vectors) gives the possibility to increase Q-factor and decrease starting current (see fig. 2) and, hence, the efficiency of generation can be increased.

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