Spin rotation and oscillation of high energy particles in storage ring

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The phenomenon of light polarization plane rotation (e.g., the Faraday effect, the natural rotation of a light polarization plane) as well as light birefringence (e.g., in matter placed in an electric field due to the Kerr effect) are the well known optical coherent phenomena. For the first glance they distinguish photons from other particles (nucleons, electrons, etc.) for which these effects for a long time have been considered nonexistent.

In [1-8] a wide range of phenom-ena similar to the effects of light polarization plane rotation and birefringence was shown to exist for particles other than photons. In particular it has been shown that as particles (neutrons, protons, neutrinos, etc.) pass through matter with polarized nuclei, the particle spin undergoes a rotation in an effective pseudomagnetic field of the matter induced by both strong and weak interactions. This effect is kinematically analogous to the phenomenon of light polarization plane rotation due to the Faraday effect. Experimentally, the effect of neutron spin precession in polarized target has been studied for neuterons [4-6].

As it was ascertained in [5-8] the analogue of birefringence phenomenon exists for particles, too.

As a matter of fact, the Faraday effect and the ef-fect of birefringence are caused by the dependence of the coherent photon-medium interaction energy on the photon spin state. This property unites the quasioptical phenomena discovered in [1-8] for interaction of spin-particles in matter with nuclei with the phenomena existing in light optics. However, attention should be drawn to the fact that whereas the photon spin is equal to unity, the particle (atom, nucleus) spin may take on different values. For particles of spin $S = 1/2$ there exists only one effect - that of spin rotation, i.e. a kinematic analog of the effect of light polarization plane rotation. An effect similar to birefringence exists for spin $S \geq 1$ particles. It is very interesting to mention that phenomena of rotation and oscillations of particle spin (birefringence effect) exist for particles with spin $S \geq 1$ in a medium with unpolarized scatterer spins, too. The fact that these effects are described by spin-dependent part of scattering amplitude allows to use them for the measurement of this amplitude at different energies of colliding particles.

0.1 Spin rotation of high-energy particles in polarized targets.

As a result of numerous studies (see, for example, [12]), a close connection between the coherent elastic scattering amplitude $f(0)$ and the refraction index of a medium N has been established:

$$N = 1 + \frac{2\pi\rho}{k^2} f(0) \qquad (1)$$

where ρ is the number of particles per cm^3, k is the wave number of a particle incident on a target. In 1964 it was shown [1] that while slow neutrons are propagating through the target with polarized nuclei a new effect of nucleon spin precession occured. It is stipulated by the fact that in a polarized target the neutrons are characterized by two refraction indices ($N_{\uparrow\uparrow}$ for neutrons with the spin parallel to the target polarization vector and $N_{\uparrow\downarrow}$ for neutrons with the opposite spin orientation , $N_{\uparrow\uparrow} \neq N_{\uparrow\downarrow}$). According to the [2], in the target with polarized nuclei there is a nuclear pseudomagnetic field and the interaction of an incident neutron with this field results in neutron spin rotation. The results obtained in [1], initiated experiments which proved the existence of this effect [9-11].

Thus, let us consider the amplitude of elastic coherent zero-angle scattering of nucleon by polarized nucleon (nucleous).

The general form of this amplitude with allowance for strong electromagnetic and weak interactions is given in [2]. Below we shall consider more concretely the effect of a relativistic nucleon spin rotation in the target with polarized protons (nuclei with spin 1/2), caused by strong interaction. In this case, the explicit structure of the elastic scattering amplitude of a particle with spin 1/2 by a particle with spin 1/2 (see, for example, [13, 14]) proceeds from the following simple discussions. In our case, the elastic scattering amplitude at zero angle depends on the spin operators $\frac{1}{2}\sigma$, $\frac{1}{2}\sigma_1$ of an incident particle and that of a target , and also on the momentum of the incident particle , that is, on $\vec{n} = \frac{\vec{k}}{k}$. Operators $\vec{\sigma}$, $\vec{\sigma_1}$ can be contained in the expression for the amplitude only in the first degree, as higher degrees of $\vec{\sigma}$ reduce either to a number or to $\vec{\sigma}$. The combinations $\vec{\sigma}$, $\vec{\sigma_1}$ and $\vec{n}$ must be such that the scattering amplitudes are a scalar and invariant in space and time reflections. These conditions defenetely determine its general form:

$$\widehat{F} = A + A_1 (\vec{\sigma} \cdot \vec{\sigma_1}) + A_2 (\vec{\sigma} \cdot \vec{n}) (\vec{\sigma_1} \cdot \vec{n}) . \qquad (2)$$

By averaging the amplitude $\widehat{F}$ with the help of a spin matrix of the density of scatters ρ_s the elastic coherent scattering amplitude may be written as:

$$f = Sp\,\rho_s\,\widehat{F} = A + A_1 (\vec{\sigma} \cdot \vec{p}) + A_2 (\vec{\sigma} \cdot \vec{n}) (\vec{n} \cdot \vec{p}) \qquad (3)$$

where $\vec{p} = Sp\,\rho_s\,\vec{\sigma_1}$ is the polarization vector of a scatterer in a target.

Amplitude f can be expressed as

$$f = A + \vec{\sigma} \cdot \vec{g} \qquad (4)$$

where $\overrightarrow{g} = A_1 \overrightarrow{p} + A_2 \overrightarrow{n} \left(\overrightarrow{n} \cdot \overrightarrow{p} \right)$.

To simplify further reasoning let us consider the situation when vector $\overrightarrow{n}$ is either parallel to $\overrightarrow{p}$ ($\overrightarrow{n} \parallel \overrightarrow{p}$) or perpendicular to $\overrightarrow{p}$ ($\overrightarrow{n} \perp \overrightarrow{p}$).

In this case one has that $g \left(\overrightarrow{n} \parallel \overrightarrow{p} \right) = \left(A_1 + A_2 \right) \overrightarrow{p}$ and $g \left(\overrightarrow{n} \perp \overrightarrow{p} \right) = A_1 \overrightarrow{p}$. Thus in these cases vector $\overrightarrow{g}$ is directed along $\overrightarrow{p}$. Selecting quantization axes parallel to $\overrightarrow{p}$, one can see that scattering amplitude $f_{\uparrow\uparrow} = A + g$ of nucleon with spin parallel to $\overrightarrow{p}$ is not equal to scattering amplitude $f_{\uparrow\downarrow} = A - g$ of nucleon with spin antiparallel to $\overrightarrow{p}$. Hence, the corresponding refractive indices are not equal to each other (i.e. $N_{\uparrow\uparrow} \neq N_{\uparrow\downarrow}$).

Considering a wave passes through a layer of polarized medium with finite thickness one can express refractive index of nuclon with spin parallel to $\overrightarrow{p}$ as follows:

$$N_{\uparrow\uparrow} = 1 + \frac{2\pi\rho}{k^2} f_{\uparrow\uparrow} = 1 + \frac{2\pi\rho}{k^2} \left(A + g \right) \tag{5}$$

and for nucleon with opposite polarization

$$N_{\uparrow\downarrow} = 1 + \frac{2\pi\rho}{k^2} f_{\uparrow\downarrow} = 1 + \frac{2\pi\rho}{k^2} \left(A - g \right) \tag{6}$$

then the difference

$$\Delta N = N_{\uparrow\uparrow} - N_{\uparrow\downarrow} = \frac{2\pi\rho}{k^2} \left(f_{\uparrow\uparrow} - f_{\uparrow\downarrow} \right) = \frac{4\pi\rho}{k^2} g \tag{7}$$

is determined by the difference in correspondent coherent scattering amplitudes and differs from zero only in polarized medium.

Suppose that nucleon passes through polarized medium and their polarizations are oriented at certain angle to the vector $\overrightarrow{p}$. This state of nucleon can be described as superposition of two states with polarizations directed along and opposite to the vector $\overrightarrow{p}$. Initial wave function of nucleon can be expressed as:

$$\psi \left(\overrightarrow{r} \right) = e^{i \overrightarrow{k} \overrightarrow{r}} \chi_n, \ \chi_n = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{8}$$

or

$$\psi \left(\overrightarrow{r} \right) = c_1 e^{i \overrightarrow{k} \overrightarrow{r}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{i \overrightarrow{k} \overrightarrow{r}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{9}$$

Suppose quantization axes z coincides with vector $\overrightarrow{p}$ and particle falls onto the target orthogonally to its surface. As the state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ possesses refration index $N_{\uparrow\uparrow}$ and the state $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is characterized by $N_{\uparrow\downarrow}$, then the wave function of nucleon in polarized medium changes with penetration depth as follows:

$$\psi(\overrightarrow{r}) = \begin{pmatrix} c_1\psi_{\uparrow\uparrow}(\overrightarrow{r}) \\ c_2\psi_{\uparrow\downarrow}(\overrightarrow{r}) \end{pmatrix} = c_1\ e^{ikN_{\uparrow\uparrow}l} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2\ e^{ikN_{\uparrow\downarrow}l} \begin{pmatrix} 0 \\ 1 \end{pmatrix},. \qquad (10)$$

l is the pass length of nucleon in target.

Using (10) one can find nucleon polarization vector

$$\vec{p}_n = \langle\psi|\vec{\sigma}|\psi\rangle \qquad (11)$$

and as a result

$$\begin{aligned} p_{nx} &= 2Rec_1^* c_2 \psi_+^* \psi_- \langle\psi|\psi\rangle^{-1}, p_{ny} = 2Imc_1^* c_2 \psi_+^* \psi_- \langle\psi|\psi\rangle^{-1}, \qquad (12) \\ p_{nz} &= \left(|c_1\psi_+|^2 - |c_2\psi_-|^2\right) \langle\psi|\psi\rangle^{-1}. \end{aligned}$$

Suppose that nucleon spin in vacuum is perpendicular to the polarization vector of nuclei. Direction of nucleon spin in vacuum define as axes x. In this case $c_1 = c_2 = 1/\sqrt{2}$. Using (12) we obtain

$$\begin{aligned} p_{nx} &= \cos\left[kRe\left(N_{\uparrow\uparrow} - N_{\uparrow\downarrow}\right) l\right] e^{-kIm(N_{\uparrow\uparrow}-N_{\uparrow\downarrow})l} \langle\psi|\psi\rangle^{-1}, \\ p_{ny} &= -\sin\left[kRe\left(N_{\uparrow\uparrow} - N_{\uparrow\downarrow}\right) l\right] e^{-kIm(N_{\uparrow\uparrow}-N_{\uparrow\downarrow})l} \langle\psi|\psi\rangle^{-1}, \qquad (13) \\ p_{nz} &= \frac{1}{2}\left(e^{-2kImN_{\uparrow\uparrow}l} - e^{-2kImN_{\uparrow\downarrow}l}\right) \langle\psi|\psi\rangle^{-1} = \frac{e^{-2kImN_{\uparrow\uparrow}l} - e^{-2kImN_{\uparrow\downarrow}l}}{e^{-2kImN_{\uparrow\uparrow}l} + e^{-2kImN_{\uparrow\downarrow}l}}. \end{aligned}$$

$\langle\psi|\psi\rangle = \frac{1}{2}\left(e^{-2kImN_{\uparrow\uparrow}l} + e^{-2kImN_{\uparrow\downarrow}l}\right)$.

It should be reminded that $Im\, f(0) = \frac{k}{4\pi}\sigma_{tot}$, where σ_{tot} is the total cross-section of scatterring of nucleon by nucleon (nulei, atoms).

According to (13) when nucleon penetrate deep into target, its polarisation vector rotates around nuclei polarization vector at the angle

$$\vartheta = kRe\left(N_{\uparrow\uparrow} - N_{\uparrow\downarrow}\right) l = \frac{2\pi\rho}{k} Re\left(f_{\uparrow\uparrow} - f_{\uparrow\downarrow}\right) l \qquad (14)$$

This rotation is similar to the spin rotation appearing in magnetic field. Then we can conclude that polarized nuclear target acts on spin likewise the area occupied with nuclear pseudomagnetic field [1].

It is important to emphasize that in experiments with gas target scattering amplitude $f(0)$, being contained in expression for N, is the elastic coherent amplitude of zero-angle scattering of a nucleon by an atom (of hydrogen, deuterium and so on). This fact should be taken into consideration at particular analysis, because atom electrons can contribute to the rotation angle.

Let us consider proton beam passing through polarized gas hydrogen target. Let polarized atoms of hydrogen in magnetic field are described by the magnetic quantum number $M = 1$. It means that in hydrogen atom spins of proton and electron are parallel. Polarized atoms of hydrogen possess magnetic moment directed along spin, which produces magnetic field $B_H \sim \rho\, \mu_H$, where ρ is the number of atoms of hydrogen in $1\, cm^3$, μ_H is the magnetic moment of hydrogen atom. As the magnetic moment of proton is much less than that of electron, then magnetic field B_H of polarized hydrogen atoms is mainly produced by their electrons.

Hence the angle of rotation of proton spin can be expressed as a sum of two additives ϑ_B and ϑ_S

$$\vartheta = \vartheta_B + \vartheta_S,$$

where ϑ_B is the angle of rotation caused by magnetic field and ϑ_S is the angle of rotation appearing due to strong interactions i.e. nuclear pseudomagnetic field.

Thus, considering rotation phenomena as a tool for experimental investigation of zero-angle nucleon-nucleon scattering amplitude one come to neccessity to extract addition caused by magnetic field of the target $\vec{B} = \vec{H} + \vec{B_H}$, here $\vec{H}$ is the external magnetic field. This can be done:

1. by calculation using equations of Bargman-Michel-Telegdi (BMT) [14] as magnetic moments of proton and electron (hydrogen atom) are known with high accuracy

or

2. by experimental separating of additions caused by magnetic and nuclear pseudomagnetic fields. This possibility is due to the fact that magnetic field induced by polarized magnetic moment depends on the shape of the target because of long-distance action of electromagnetic interaction. Whereas the nuclear pseudomagnetic field (being short-range action) does not depend on the shape of the target.

Moreover, from the analysis of BMT-equations follows that if proton velocity is directed along $\vec{B}$ then angle of rotation of proton spin orthogonal to $\vec{B}$ for nonrelativistic protons is determined by magnetic moment. If proton velocity is orthogonal to $\vec{B}$ than spin rotation in magnetic field is determined by anomalous magnetic moment, since due to cyclotron movement of proton in magnetic field the addition to the angle of rotation, caused by Dirac magnetic moment (equal to the nuclear magneton), yields to the conservation of angle between proton spin and momentum (if the anomalous magnetic moment $\Delta\mu$ is equal to zero). Hence the observed deviation of proton spin in magnetic field $\vec{B}$ is conditioned by $\Delta\mu$.

Let us evaluate the effect magnitude for particular setup [15]. According to [15] target thickness is $n = \rho l = 10^{14}\, atoms/cm^2$, a revolution frequency of proton beam $\nu \sim 10^6\, s^{-1}$.

From (14) we can obtain for rotation angle caused by nuclear pseudomagnetic field :

$$\begin{aligned}\vartheta_S &= \frac{2\pi\rho l}{k} Re\left(f_{\uparrow\uparrow}^{NN}(0) - f_{\uparrow\downarrow}^{NN}(0)\right)\cdot \nu T = \frac{4\pi\, n\nu T}{k} Reg = \\ &= \frac{4\pi n\nu T}{\sqrt{\frac{2ME}{\hbar^2}}} Reg = \frac{4\pi\hbar n\nu RegT}{\sqrt{2ME}} \cong 1\, rad\end{aligned}$$

T is the observation time. (for example, $T = 10\ hours = 3.6\cdot 10^4\, s$, $g \cong 2\cdot 10^{-13}\, cm$)

It is interesting to note that we can obtain simple estimation, showing relation between angle of proton spin rotation caused by magnetic field $\overrightarrow{B_e}$, produced by polarized magnetic moment of electrons (atoms), and that in nuclear pseudomagnetic field. It should be reminded that $B_e = \eta 4\pi\rho\mu_e$, where η depends on the target shape (for example, for sphere $\eta = \frac{2}{3}$) [16].

Suppose polarization vector $\overrightarrow{p}$ is orthogonal to the particle momentum. In this case the addition to the rotation angle caused by B_e is expressed

$$\vartheta_{Be} = \frac{k\Delta\mu_p B_e l}{E} = \eta\frac{k\Delta\mu_p\, 4\pi\rho\mu_e l}{E},$$

where $\Delta\mu_p$ is the anomalous magnetic moment of proton , l is the length of gas target , E is the particle energy.

Then we obtain

$$\frac{\vartheta_{Be}}{\vartheta_s} = \eta\frac{\Delta\mu_p\,\mu_e}{g\frac{\hbar^2}{2M}} \cong 0.9\,\eta\,\frac{r_0}{g},$$

where $r_0 = \frac{e^2}{mc^2}$ is the electromagnetic radius of electron. And for $g = 2\cdot 10^{-13}$ and $r_0 = 2.8\cdot 10^{-13}$ ratio $\dfrac{\vartheta_{Be}}{\vartheta_s} \cong \eta$.

0.2 Deuteron spin rotation and oscillations in a nonpolarized target

According to the above, the refractive index of neutral and charged particles of spin S can be written as

$$\widehat{N} = 1 + \frac{2\pi\rho}{k^2}\widehat{f}\,(0) \tag{15}$$

where $f(0)$ is the amplitude of particle zero-angle elastic coherent scattering by a scattering center, which is an operator acting in the particle spin space, $\widehat{f}\,(0) = Sp\,\widehat{\rho}_J\,\widehat{F}\,(0)$; $\widehat{\rho}_J$ is the spin density matrix of the scatterer; $\widehat{F}\,(0)$ stands for the forward scattering operator amplitude acting in the spin space of the particle and the scatterer of spin $\overrightarrow{J}$. If the wave function of the particle entering a target is ψ_0, then that after travelling a distance z in the target is written as

$$\psi(z) = \exp\left(i\,k\,\widehat{N}\,z\right)\psi_0 \tag{16}$$

The explicit form of the amplitude $\widehat{f}(0)$ for particles with arbitrary spin S has been obtained in [6]. According to these articles even for an unpolarized target is a function of the incident particle spin operator and can be written as

$$\widehat{f}(0) = d + d_1 S_z^2 + d_2 S_z^4 + ... + d_s S_z^{2s} \tag{17}$$

The quantization axis z is directed along $\vec{n} = \frac{\vec{k}}{k}$. Consider a specific case of strong interactions invariant under space and time reflections. For this reason, the terms containing odd powers of S are neglected. Correspondingly, the refractive index

$$\widehat{N} = 1 + \frac{2\pi\rho}{k^2}\left(d + d_1 S_z^2 + d_2 S_z^4 + ... + d_s S_z^{2s}\right) \tag{18}$$

From eq. (18) one can draw an important conclusion about the refractive index being dependent on the spin orientation with respect to the pulse direction. Let m denote a magnetic quantum number, then for a particle in a state that is an eigenstate of the spin projection operator onto the z axis, S_z, the refractive index is written as

$$N(m) = 1 + \frac{2\pi\rho}{k^2}\left(d + d_1 m^2 + d_2 m^4 + ... + d_s m^{2s}\right) \tag{19}$$

According to eq. (19), the particle states with quantum numbers m and $-m$ have the same refractive indices. For a spin-1 particle (for example, a J/ψ particle, a deuteron) and a spin-$\frac{3}{2}$ particle (i.g. an Ω^- hyperon).

$$N(m) = 1 + \frac{2\pi\rho}{k^2}\left(d + d_1 m^2\right)$$

As seen,

$$\begin{aligned}
Re\,N(\pm 1) &\neq Re\,N(0)\,,\\
Im\,N(\pm 1) &\neq Im\,N(0)\,,\\
Re\,N\left(\pm\frac{3}{2}\right) &\neq Re\,N\left(\pm\frac{1}{2}\right)\\
Im\,N\left(\pm\frac{3}{2}\right) &\neq Im\,N\left(\pm\frac{1}{2}\right)
\end{aligned}$$

Since we have obtained the explicit spin structure of the refractive index and the wave function (16) is known, in every specific case we can find all spin properties of a beam in the target at depth z.

Let us come to consideration of deuteron passing through medium (particle with spin = 1). The wave function can be represented as a superposition of basis spin wave functions χ_m, which are eigenfunctions of the operators $\widehat{S}^2$ and $\widehat{S}_z$, $\widehat{S}_z \chi_m = m\chi_m$:

$$\psi = \sum_{m=\pm 1,0} a^m \chi_m.$$

Let us look for the mean value $\langle \vec{S} \rangle = \langle \psi \left| \widehat{\vec{S}} \right| \psi \rangle / \langle \psi | \psi \rangle$ of the spin operator in state ψ.

Suppose the particle enter the target at $z = 0$. Wave function of the particle insid the medium at the depth z can be expressed as:

$$\Psi = \left\{ \begin{array}{c} a^1 \\ a^0 \\ a^{-1} \end{array} \right\} = \left\{ \begin{array}{c} a\, e^{i\delta_1} e^{ikN_1 z} \\ b\, e^{i\delta_0} e^{ikN_0 z} \\ c\, e^{i\delta_{-1}} e^{ikN_{-1} z} \end{array} \right\} = \left\{ \begin{array}{c} a\, e^{i\delta_1} e^{ikN_1 z} \\ b\, e^{i\delta_0} e^{ikN_0 z} \\ c\, e^{i\delta_{-1}} e^{ikN_1 z} \end{array} \right\}$$

it should be mentioned that $N_1 = N_{-1}$

Let us choose coordinate system in which plane (xz) coincides with that formed by vector $\langle \overrightarrow{S} \rangle$ ($<\vec{S}>=\frac{<\psi|\vec{S}|\psi>}{|\psi|^2}$) before entering the target and deuteron momentum. In this case $\delta_1 - \delta_0 = \delta_{-1} - \delta_0 = 0$ and components of vector at $z = 0 \quad \langle S_x \rangle \neq 0, \langle S_y \rangle = 0$.

As a result we obtain:

$$\begin{aligned} < \quad S_x >&= \sqrt{2} e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b(a+c) \cos[\frac{2\pi\rho}{k} Re d_1\, z]/|\psi|^2, \\ < \quad S_y >&= -\sqrt{2} e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b(a-c) \sin[\frac{2\pi\rho}{k} Re d_1 z]/|\psi|^2, \\ < \quad S_z >&= e^{\rho\sigma_1 z}(a^2-c^2)/|\psi|^2, \end{aligned} \tag{20}$$

Particle with spin 1 also possesses tensor polarization i.e. tensor of quadrupolarization $\hat{Q}_{ij} = 3/2(\hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i - 4/3\delta_{ij})$

for it we can obtain

$$
\begin{aligned}
< \quad Q_{xx} >&= \left\{ -\left[a^2+c^2\right] \frac{1}{2}\, e^{-\rho\sigma_1\, z} + b^2\, e^{-\rho\sigma_0\, z} + 3e^{-\rho\sigma_1\, z}\, ac\, \cos\left[\delta_1 - \delta_{-1}\right] \right\} /|\psi|^2 \\
< \quad Q_{yy} >&= \left\{ -\left[a^2+c^2\right] \frac{1}{2}\, e^{-\rho\sigma_1\, z} + b^2\, e^{-\rho\sigma_0\, z} - 3e^{-\rho\sigma_1\, z}\, ac\, \cos\left[\delta_1 - \delta_{-1}\right] \right\} /|\psi|^2 \\
< \quad Q_{zz} >&= \left\{ \left[a^2+c^2\right] \frac{1}{2}\, e^{-\rho\sigma_1\, z} - 2b^2\, e^{-\rho\sigma_0\, z} \right\} /|\psi|^2 \ , \qquad (21) \\
< \quad Q_{xy} >&= -3e^{-\rho\sigma_1\, z}\, ac\, \sin\left[\delta_1 - \delta_{-1}\right] /|\psi|^2, \\
< \quad Q_{xz} >&= \frac{3}{\sqrt{2}} e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b(a-c) \cos[\frac{2\pi\rho}{k} Red_1 z]/|\psi|^2, \\
< \quad Q_{yz} >&= -\frac{3}{\sqrt{2}} e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b(a+c) \sin[\frac{2\pi\rho}{k} Red_1 z]/|\psi|^2 \ ,
\end{aligned}
$$

where $|\psi|^2 = 2(a^2+c^2)\, e^{-\rho\sigma_1\, z} + b^2\, e^{-\rho\sigma_0\, z}$ and z is the length of particle path inside a medium.

According to (20,21) rotation appears if angle between polarization vector and momentum of particle differs from $\frac{\pi}{2}$. At this for acute angle between polarization vector and momentum the sign of rotation is opposite than that for obtuse angles.

If spin is orthogonal to momentum then ($a = c$) particle spin (tensor of quadrupolarization) oscillate (do not rotate)

$$
\begin{aligned}
< \quad S_x >&= \sqrt{2} e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} 2 \cos[\frac{2\pi\rho}{k} Red_1 z]/|\psi|^2, \\
< \quad S_y >&= 0, \\
< \quad S_z >&= 0,
\end{aligned}
$$

And tensor of quadrupolarization:

$$
\begin{aligned}
< \quad Q_{xx} >&= \left\{ 2^2\, e^{-\rho\sigma_1\, z} + {}^2\, e^{-\rho\sigma_0\, z} \right\}/|\psi|^2 \ , \\
< \quad Q_{yy} >&= \left\{ -4^2\, e^{-\rho\sigma_1\, z}/2 + {}^2\, e^{-\rho\sigma_0\, z} \right\}/|\psi|^2 \ , \\
< \quad Q_{zz} >&= \left\{ 2^2\, e^{-\rho\sigma_1\, z} - 2\, {}^2\, e^{-\rho\sigma_0\, z} \right\}/|\psi|^2 \ , \qquad (22) \\
< \quad Q_{xy} >&= 0, \\
< \quad Q_{xz} >&= 0, \\
< \quad Q_{yz} >&= \left\{ -\frac{3}{\sqrt{2}} e^{(\sigma_0+\sigma_1)z}\, 2ab\, \sin[\frac{2\pi\rho}{k} Red_1 z] \right\}/|\psi|^2 \ ,
\end{aligned}
$$

where $|\psi|^2 = 2^2\, e^{-\rho\sigma_1\, z} + {}^2\, e^{-\rho\sigma_0\, z}$ and z is the length of particle path inside a medium.

Let us evaluate phase of oscillation

$$\varphi = \frac{2\pi\rho\ z}{k} Red_1$$

for COSY. In this case total phase storing during experiment $\Phi = \varphi\nu T$. For particular conditions $\rho\ z = n = 10^{14} cm^{-2}$, $Red_1 \sim 10^{-13}$ (for scattering of deuteron by hydrogen), $\nu \sim 10^6, T \sim 70\ hours = 2.52 \cdot 10^5\ s$ and Φ can be estimated as $\Phi \sim 1\,rad.$

As you can see the effect magnitude is large enough to be observable at COSY. It is very important that in considered case (deuterons passing through nonpolarized medium) there are no magnetic fields at the target area.

Experimental study of $f\,(0)$ at different particle energies is an important tool for investigation of particles interaction properties. Phenomena of spin rotation and oscillation, described above, give the basis for methods of investigation of amplitude $f\,(0)$ spin-dependendent part, examination of dispesion relations for it and testing of P-,T-violations at particle interactions for different energies. Of the essence, interactions at final state make the measurement of T-odd contributions difficult at experimental studies of T-violation by scattering of particles by each other. This problem is absent for T-odd particle spin rotation due to refraction, because it is determined by the elastic coherent zero-angle scattering amplitude. In this case initial and final states of target coincides and the above difficulty does not occur. As a result, methods based on the measurement of spin rotation angle (and $f\,(0)$ measurement, as a sequence) can provide the most tight restrictions to the possible value of T-odd interactions.

One more promising aria of experimental investigations can be mentined, where phenomena, caused by nuclear optics of polarized medium, can appear efficient. Storage rings become now the more and more important investigation tool. Life-time of particle beam in storage ring can reach several hours and even days. During this time particles circulate in storage ring with frequency of several MHz and even small spin-dependent interactions of beams with each other can significantly influence on polarization state of beams.

We considered the rest target above. But in storage rings moving bunch is usually used as a target. No problems appear at study of spin rotation effect in this case - you should consider the effect in the rest frame of one of the beams and then reduce the result to the laboratory frame.

As an example let us consider cross-collision of two bunches of polarized particles. Suppose particles of the first beam have mass m_1, energy $E_1$and Lorentz factor γ_1, whereas particles of the second beam are characterized by mass m_2, energy E_2 and Lorentz factor γ_2. Choose the frame where the second beam rest. In this frame the energy of the particles of the first beam $E_1{}' = E_1\gamma_2 = m_1c^2\gamma_1\gamma_2$. Refractive index is expressed in conventional form:

$$\widehat{N} = 1 + \delta\widehat{N} + \frac{2\pi\rho_2'}{k_1'^2}\widehat{f}\,(E_1{}', 0)$$

where $\delta\widehat{N}$ is the contribution to refractive index caused by refraction of particles in electric and magnetic fields of the bunch, $\rho_2' = \gamma_2^{-1}\rho_2$ is the density of bunch 2 in its rest frame and ρ_2 is the density of the second bunch in laboratory frame, $k_1' = k_1\gamma_2$. is the wave number of particles of the first bunch in the rest frame of bunch 2 and k_1 is the wave number of these particles in laboratory frame. Contribution to the particle spin rotation angle caused by $\widehat{f}\,(E_1{}', 0)$ is expressed:

$$\vartheta' = \frac{2\pi\rho_2'}{k_1'}\left(Re\, f_{\uparrow\uparrow}\,(E_1{}', 0) - Re\, f_{\uparrow\downarrow}\,(E_1{}', 0)\right) L,$$

where $L = \gamma_2 l$ is the length of the bunch 2 in its rest frame, l is the length of this bunch in laboratory frame. Lorentz factor of particle 1 in rest frame of particle 2 is $\gamma = \gamma_1\gamma_2$ and this factor can be educed in scattering amplitude: $f\,(E_1{}', 0) = \gamma_1\gamma_2\, f'\,(E_1{}', 0)$. As a result:

$$\vartheta' = 2\pi\rho_2\lambda_{1c}\left(Re\, f\,{}'_{\uparrow\uparrow}\,(E_1{}', 0) - Re\, f\,{}'_{\uparrow\downarrow}\,(E_1{}', 0)\right) l,$$

where $\lambda_{1c} = h/m_1 c$.

Angle of rotation is invariant. Then in laboratory frame it will be the same. Evidently, particles 2 also experience refraction on the bunch 1. As a result mutual rotation of polarizatuion vectors of both bunches occurs. Taking into consideration the fact that in storage ring particle passes through the target (bunch) many times one can see that total angle of spin rotation, caused by refraction, many times rises. The above makes measurement of zero-angle scattering amplitude in wide particle energy range very promising.

Very interesting possibility appears when one of the beams (e.g. beam 2) has very low energy. It can be beam of ultracold atoms. Then index of refraction of beam 2 by beam 1 is high and we can apply atom interferometry methods for scattering amplitude measurement.

It should be mentioned that unique possuibility to measure spin-independent zero-angle scattering amplitude at high energies also appears.

It is important to note that photon beam formed by laser wave can be used as one of the beams. In this case effect of spin rotation is determined by elastic coherent amplitude of zero-angle scattering of particle by photon. According to the above analysis both electromagnetic and P-,T- odd weak interactions contribute to the refration index of particle in the area occupied by photons [17]. As a result area occupied by photons can be described as optically anisotropic medium.

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