

Time-reversal violating generation of static magnetic and electric fields and a problem of electric dipole moment measurement

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It is shown that in the experiments for search of EDM of an electron (atom, molecule) the T-odd magnetic moment induced by an electric field and the T-odd electric dipole moment induced by a magnetic field will be also measured. It is discussed how to distinguish these contributions.

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Nowadays there is an appreciable progress in the development of methods for ultraweak magnetic and electric field measurement. Therefore, new experiments for measurement of an electric dipole moment d (EDM) of electrons (atoms, molecules) [1-4] are being prepared and carried out.

The EDM of a particle exists if parity (P) and time-reversal (T) invariance are violated. Investigation of the EDM existence could provide knowledge about physics beyond the Standard Model [1-4].

F.L.Shapiro's idea [5] to measure electron EDM by applying a strong electric field to a substance that has an unpaired electron spin is being used for EDM search (see [1, 6]).

Interaction W_E of an electric dipole moment \vec{d} of an electron with an electric field \vec{E} depends on their orientation:

$$W_E = -\vec{d}\vec{E}, \quad (1)$$

where $\vec{d} = d\frac{\vec{J}}{J}$, \vec{J} is the atom spin, d is the EDM.

Spins of electrons (atoms) at low temperature appear to be polarized similar to the polarization (magnetization) of electrons by a magnetic field in paramagnetic substances due to the interaction W_B of an electron (atom) magnetic moment $\vec{\mu}$ with a magnetic field \vec{B}

$$W_B = -\vec{\mu}\vec{B}, \quad (2)$$

Spins of electrons (atoms) polarized by an electric field induce the magnetic field \vec{B}_E and change in the magnetic flux Φ at the surface of a flat sheet of material [1]:

$$\Delta\Phi = 4\pi\chi AdE^*/\mu_a, \quad (3)$$

$$B_E = \frac{\Delta\Phi}{A} = 4\pi\chi\frac{d}{\mu_a}E^*, \quad (4)$$

where χ is the magnetic susceptibility, $\chi \approx \frac{\rho\mu_a^2}{3k_B T}$, ρ is the number density of spins of interest, k_B is Boltzmann's constant and T is the sample temperature. In the cases where simple Langevin paramagnetism is applicable, E^* is the effective electric field at the location of the spins, $\mu_a = g\sqrt{J(J+1)}\mu_B$ where μ_B is the Bohr magneton, μ_a is the atomic or ionic magnetic moment, g is Lande factor and A is the sample area.

If an external magnetic field acts on either a para- or a ferromagnetic material, the spins in a substance become polarized due to substance magnetization. Therefore, the electric dipole moments appear polarized too. This results in the induction of an electric field \vec{E}_B (D.DeMille [1]):

$$E_B = 4\pi\rho dP(B), \quad (5)$$

where P represents the degree that the spins are polarized in the sample.

According to the analysis [1], modern methods for measurement of B_E and E_B provide sensitivity for electric dipole moment measurement about 10^{-32} e cm and in some cases even 10^{-35} e cm.

It is important to pay attention to another mechanism of time-reversal violating generation of magnetic and electric fields, which have been discussed in [7, 8]. According to the idea of [7, 8], an induced magnetic moment $\vec{\mu}(\vec{E})$ of a particle appears due to the action of a field \vec{E} under conditions of violation of P- and T-invariance (and similar, an

induced electric dipole moment \vec{d}_B of a particle appears due to the action of a field \vec{B}). This new effect does not depend on temperature. An effect magnitude is determined by a P-odd T-odd tensor polarisability β_{ik}^T of a particle (atom, molecule, nucleus, neutron, electron and so on). For an atom (molecule), β_{ik}^T arises due to P- and T-odd interaction of electrons with a nucleus. For the stationary state of an atom (molecule) $|N_0\rangle$ the tensor β_{ik}^T is as follows:

$$\beta_{ik}^T = \sum_F \frac{\langle N_0 | \hat{d}_i | F \rangle \langle F | \hat{\mu}_k | N_0 \rangle + \langle N_0 | \hat{\mu}_i | F \rangle \langle F | \hat{d}_k | N_0 \rangle}{E_F - E_{N_0}}, \quad (6)$$

where $|F\rangle$ is the wave function of a stationary state of the atom, considering T-odd interaction V_w^T , E_F and E_{N_0} are the energies of the atom (molecule) stationary states, \hat{d} and $\hat{\mu}$ are the operators of electric dipole moment and magnetic moment, respectively and $i, k = 1, 2, 3$ correspond to the axes x, y, z .

Let us place an atom (molecule) into an electric field \vec{E} . The induced magnetic dipole moment $\vec{\mu}(\vec{E})$ appears in this case [7, 8]:

$$\mu_i(\vec{E}) = \beta_{ik}^T E_k, \quad (7)$$

The tensor β_{ik}^T (like any tensor of rank two) can be expanded into scalar, symmetric and antisymmetric parts.

The antisymmetric part of the tensor β_{ik}^T is proportional to $e_{ikl} J_l$, where e_{ikl} is the totally antisymmetric tensor of rank three. The symmetric part of the tensor β_{ik}^T is proportional to the tensor of quadrupolarization $Q_{ik} = \frac{3}{2J(2J-1)} [J_i J_l + J_k J_l - \frac{2}{3} J(J+1) \delta_{ik}]$. As a result

$$\beta_{ik}^T = \beta_s^T \delta_{ik} + \beta_v^T e_{ikl} J_l + \beta_t^T Q_{ik}, \quad (8)$$

where $\beta_s^T, \beta_v^T, \beta_t^T$ are the scalar, vector and tensor P-, T-odd polarizabilities of the particle, respectively. For a substance with the nonpolarized spins $S\rho \rho(J) \vec{J} = 0$ and $S\rho \rho(J) Q_{ik} = 0$ (here $\rho(J)$ is the atom (molecule) spin density matrix). As a result for such a substance, β_{ik}^T appears to be a scalar $\beta_{ik}^T = \delta_{ik} \beta_s^T$.

Placement of a nonpolarized atom (molecule, nucleus) into an electric field induces the magnetic dipole moment $\vec{\mu}_E$ (see also Appendix):

$$\vec{\mu}(\vec{E}) = \beta_s^T \vec{E}, \quad (9)$$

where $\beta_s^T = \sum_F \frac{\langle N_0 | \hat{d}_z | F \rangle \langle F | \hat{\mu}_z | N_0 \rangle + \langle N_0 | \hat{\mu}_z | F \rangle \langle F | \hat{d}_z | N_0 \rangle}{E_F - E_{N_0}}$, \hat{d}_z and $\hat{\mu}_z$ are the z components of the operators of the electric dipole moment and magnetic moment, respectively with axis z parallel to the electric field \vec{E} . It should be emphasized that for strong fields (when the distance between atom (molecule) levels is comparable with the energy of interaction with an electric \vec{E} (magnetic \vec{B}) field) β_s^T depends on \vec{E} (\vec{B}). Weak interaction is much weaker than strong and electromagnetic interactions. Therefore, to find the wave function $|F\rangle$, the perturbation theory can be applied:

$$|F\rangle = |f\rangle + \sum_n \frac{\langle n | V_w^T | f \rangle}{E_f - E_n} |n\rangle = |f\rangle + \sum_n \eta_{nf} |n\rangle, \quad (10)$$

where $|f\rangle$ is the wave function of an atom in the absence of weak interactions and the mixing ratio is $\eta_{nf} = \frac{\langle n | V_w^T | f \rangle}{E_f - E_n}$. It should be mentioned that for theoretical analysis of β_s^T in a substance it is necessary to find a wave function of an excited state of an atom in the substance which is difficult to do.

It follows from (9) that in a substance placed into electric field the magnetic field is induced [7, 8]:

$$\vec{B}_E^{ind} = 4\pi\rho\beta_s^T \vec{E}^*. \quad (11)$$

Vice versa, if an atom (molecule, nucleus) is placed into a magnetic field, the induced electric dipole moment \vec{d}_B appears [7, 8]. The tensor polarizability associating \vec{d} with \vec{B} is $\chi_{ik}^T = \beta_{ki}^T$. If only contribution from the scalar polarizability is significant, then induced EDM can be represent as follows:

$$\vec{d}_B = \beta_s^T \vec{B}^*, \quad (12)$$

which leads to the induction of an electric field in the substance:

$$\vec{E}_B^{ind} = 4\pi\rho\beta_s^T \vec{B}^*, \quad (13)$$

where B^* (E^*) is the local field, acting on the considered particle in the substance.

Hence, analysing the results of the experiment proposed in [1], one should consider that the appearance of the induced magnetic and electric fields is caused by:

1. A magnetic field is induced due to interaction of the electric dipole moment of an atom with an external electric field [1, 5] (see (3),(4))
2. A magnetic field is induced due to mechanism [7, 8] (see (11))
3. A magnetic field appears as a result of polarization (magnetization) of atom magnetic moments by the local induced magnetic field \vec{B}_{loc} due to interaction W of the magnetic dipole moment of an atom with this field

$$W = -\vec{\mu}_a \vec{B}_{loc}. \quad (14)$$

The local field \vec{B}_{loc} is the sum of two contributions:

$$\vec{B}_{loc} = \vec{B}_{E\ loc} + \vec{B}_{loc}^{ind}, \quad (15)$$

where the field $\vec{B}_{E\ loc}$ is the local magnetic field acting on an atom from the polarized (by mechanism [1,5], see (3),(4)) magnetic moments of the other atoms of the sample. This field depends on temperature and its contribution could be neglected for those temperature values, which provide $\chi \ll 1$. But for temperature $T < 1K$ the susceptibility $\chi \sim \frac{1}{T}$ becomes comparable with 1 and higher, and the energy of interaction of two magnetic dipoles for neighbour atoms occurs of order of $k_B T$ and greater. Thus, in this case, the collective effects, well-known in the theory of phase transition in magnetism, should be taken into account while considering magnetization by an electric field.

The field $\vec{B}_{loc}^{ind} = \vec{B}_{1\ loc}^{ind} + \vec{B}_{2\ loc}^{ind}$ does not depend on temperature.

The field $\vec{B}_{1\ loc}^{ind} = \chi_1^T \vec{E}^*$ is the local magnetic field produced in the point of the considered atom location by the magnetic moments of atoms of the substance (except for the considered atom) induced by the aid of mechanism [7,8] (see (6),(8)); $\chi_1^T \sim \rho \beta_s^T$ is the local P-,T-odd susceptibility of the substance (it depends on the substance density and sample shape: for sphere $\chi_1^T = \frac{8\pi}{3} \rho \beta_s^T$ for cylinder $\chi_1^T = 4\pi \rho \beta_s^T$).

The field $\vec{B}_{2\ loc}^{ind}$ is the self-induced magnetic field of the considered atom. The magnetic moment (T-odd current) of the atom induced by an electric field acting on the atom due to mechanism [7,8] causes appearance of the magnetic field inside the atom:

$$\vec{H}_E^T(\vec{r}) = rot \vec{A}_E^T(\vec{r}), \quad (16)$$

with vector potential

$$\vec{A}_E^T(\vec{r}) = \frac{1}{c} \int \frac{j_E^T(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r', \quad (17)$$

$j_E^T(\vec{r}')$ is the T-odd part of operator of the current density for an atom (molecule) placed in an electric field (it is calculated by the use of wavefunctions similar (10) ([12, 13])). The magnetic interaction hamiltonian of an atom with the field \vec{A}_E^T can be expressed as follows (see Appendix 2):

$$W_{2\ loc} = -\frac{1}{2c} \int (\vec{j}^0(\vec{r}) \vec{A}^T(\vec{r}) + \vec{A}^T(\vec{r}) \vec{j}^0(\vec{r})) d^3 r, \quad (18)$$

where $\vec{j}^0(\vec{r})$ is the atom current density operator calculated with the atom wavefunction without consideration of P-,T-odd interactions,

$$\vec{j}^0(\vec{r}) = c \ rot \vec{\mu}(\vec{r}), \quad (19)$$

and $\vec{\mu}(\vec{r})$ is the operator of magnetic moment density.

The above expressions allow us to rewrite W_2 as follows:

$$W_{2\ loc} = -\vec{\mu}_a \vec{B}_{loc}^{ind} = -\mu_a i \chi_{(at)ik}^T E_k^*,$$

where $\vec{\mu}_a$ is the operator of the magnetic moment of an atom, $\vec{\mu}_a = \mu_a \frac{\vec{j}}{j}$ and $\chi_{(at)ik}^T$ is the tensor of T-odd atom susceptibility, which does not depend on the substance density and sample shape. The scalar part χ_{at}^T of the T-odd atom susceptibility tensor is $\chi_{at}^T \sim \beta_s^T \frac{1}{a^3}$ (here a is the typical radius of distribution density of the magnetic moment induced by an electric field in the atom [7, 8]).

As a result

$$\vec{B}_{loc}^{ind} = (\chi_{1\ loc}^T + \chi_{at}^T) \vec{E}_{loc}^* = \chi_{loc(subst)}^T \vec{E}_{loc}^*, \quad (20)$$

$$\chi_{loc(subst)}^T = \chi_{1\ loc}^T + \chi_{at}^T. \quad (21)$$

The interaction (11) of the magnetic moment of an atom with the induced magnetic field causes the appearance of the magnetic field due to different population of magnetic levels of the atom in the field \vec{B}_{loc} in thermal equilibrium

$$\vec{B}'^{ind} = 4\pi\chi\vec{B}_{loc} \approx 4\pi\chi\vec{B}_{loc}^{ind} = 4\pi\frac{\rho\mu_a^2}{3k_B T}\chi_{loc(subst)}^T \vec{E}^*, \quad (22)$$

the field $\vec{B}_{loc} = \vec{B}_{E\ loc} + \vec{B}_{loc}^{ind}$, but $\vec{B}_{E\ loc}$ contribution could be neglected for those temperature values, which provide $\chi \ll 1$, and it is omitted here.

Therefore, the flux $\Delta\Phi$, which is going to be measured in the experiment proposed in [1] should be written as:

$$\begin{aligned} \Delta\Phi &= AB_E = 4\pi A(\chi\frac{d}{\mu_a} + \rho\beta_s^T + \chi\chi_{loc(subst)}^T)E^* = \\ &= 4\pi A[\chi(\frac{d}{\mu_a} + \chi_{loc(subst)}^T) + \rho\beta_s^T]E^*, \end{aligned} \quad (23)$$

$$\begin{aligned} \vec{B}_E &= 4\pi(\chi\frac{d}{\mu_a} + \rho\beta_s^T + \chi\chi_{loc(subst)}^T)\vec{E}^* = \\ &= 4\pi[\chi(\frac{d}{\mu_a} + \chi_{loc(subst)}^T) + \rho\beta_s^T]E^*, \end{aligned} \quad (24)$$

where $\chi = \frac{\rho\mu_a^2}{3k_B T}$.

The electric field measured in the experiment [1] is as follows (see (9),(10)):

$$E_B = 4\pi\rho(dP(B) + \beta_s^T B^*). \quad (25)$$

The term proportional to the interaction of the electric dipole moment of an atom with the electric field induced by mechanism [7, 8] is small (it is of the second order over T-odd interaction) and is neglected.

Thus, measurement of $\Delta\Phi$ and E_B provides knowledge about the sum of quantities d , $\chi_{loc(subst)}^T$ and β_s^T . To distinguish these contributions one should consider the fact that χ and $P(B)$ depend on temperature, while β_s^T does not. Therefore, studying B_E and E_B dependence on temperature allows one to evaluate contributions from d , $\chi_{loc(subst)}^T$ and β_s^T to the measured effect.

It should be particularly emphasized that $\Delta\Phi$ and E_B differs from zero even when EDM d is equal to zero.

According to [1] we can expect a magnetic induction sensitivity about $3 \times 10^{-15} G/\sqrt{Hz}$. In ten days of averaging the sensitivity is $\sim 10^{-18} G$. This leads to the sensitivity of about $10^{-32} e\ cm$ for d . Such sensitivity of magnetic induction measurement provides for polarizability measurement the sensitivity $\beta_s^T \sim 10^{-43} cm^3$ (see (24)) and for mixing ratio the sensitivity $\eta_T \sim 10^{-17}$. This value for η_T is significantly lower than the limitation for mixing ratio $\eta_T \sim 10^{-14}$ [7-10], which could be obtained from the results of measurements of atom dipole moment that have been done earlier (see, for example, [11]).

It should be emphasized that the polarizability β_s^T and the susceptibility $\chi_{loc(subst)}^T$ differs from zero even for atoms with the zero spin, for which EDM is absent.

Conclusion

In [1] the experiments for search of the EDM of an electron (atom) by measurement of induced magnetic and electric fields are discussed. In the present paper it is shown that in this experiments the T-odd magnetic moment induced by an electric field and the T-odd electric dipole moment induced by a magnetic field will be also measured. The T-odd scalar polarizability β_s^T and the susceptibility $\chi_{loc(subst)}^T$ (χ_{at}^T) will be measured too. It is shown that study of temperature dependence of $\Delta\Phi$ and E_B allows us to distinguish the contribution provided by $\chi_{loc(subst)}^T$ (χ_{at}^T) and the electric dipole moment d provided by β_s^T .

Since external electric and magnetic fields are used in the experiments [2-4], then the described mechanism of induction of magnetic and electric fields [8] should be taken into account for understanding the EDM contribution in the above experiments.

For example, in [2] it is proposed to detect EDM by measurement of Zeeman precession frequency of ^{199}Hg nuclear spins in parallel electric \vec{E} and magnetic \vec{B} fields. The measurements are simultaneously performed in two cells with oppositely directed electric fields to reduce the frequency noise due to magnetic field fluctuations. According to [2] a difference between the Zeeman frequencies in these two cells is

$$\hbar(\omega_1 - \omega_2) = 4dE. \quad (26)$$

But it should be considered that an electric field induces magnetic moment and, consequently, the additional local magnetic field

$$\vec{B}_{loc}^{ind} = \chi_{loc}^T(\text{subst})\vec{E}.$$

acts on the nucleus spin.

The magnetic moment of a Hg atom interacts with this field:

$$W = -\vec{\mu}_{Hg}\vec{B}_{loc} = -\vec{\mu}_{Hg}\chi_{loc}^T(\text{subst})\vec{E}. \quad (27)$$

$\chi_{loc}^T(\text{subst})$ is the local P-,T-odd susceptibility of the substance.

Vice versa, the magnetic field \vec{B} induces the electric dipole moment, which creates the electric field that interacts with the EDM (if it exists). But this effect contributes only a little bit to the frequency difference, because it is of the second order over T-odd interaction.

Thus, frequency difference can be presented as follows:

$$\hbar(\omega_1 - \omega_2) = 4(d + \mu_{Hg}\chi_{loc}^T(\text{subst}))E. \quad (28)$$

Note that for gases $\chi_{loc}^T(\text{subst}) \approx \chi_{at}^T$.

It is evident that the experimentally observed value again includes contributions from two effects: EDM and the effect described in [7, 8].

The similar situation is for [3]. Here the parameter χ_{at}^T appears as an additional term in the coefficient ε (see eq.(2) in [3]).

For the experiment with molecules [4] an external electric field also induces the magnetic moment (by the mechanism described in [7, 8]), due to which the field $\vec{B} + \mu_B\chi_{at}^T\vec{E}$ acts on the magnetic moment of the molecule instead of the external field \vec{B} .

Therefore, the difference in phase shifts 2ϕ considered in [4]:

$$2\phi = (2\mu_B B + d_e \eta E) \frac{T}{\hbar}$$

should be replaced by

$$2\phi = [2\mu_B B + (d_e \eta + 2\mu_B \chi_{at}^T)E] \frac{T}{\hbar}. \quad (29)$$

Note that contribution from χ_{at}^T in 2ϕ differs from zero even if an electron's EDM d_e is equal to zero.

It is also important to draw attention that in the interferometric method [4] molecules pass through external electric and magnetic fields. Hence, this experimental setup practically realizes the experimental setup proposed in [14] for the method of atomic spin interferometry. According to [14], even interactions, which do not depend on spin orientation, contribute to the spin rotation in a field providing the phase difference 2ϕ between different spin states. For example, conventional electromagnetic interaction $U = -\frac{1}{2}\alpha E^2$ also contributes in the difference of phase shifts 2ϕ (here α is the conventional P-,T-invariant polarizability, which does not even depend on spin orientation). It means that one should change the sign of the electric field to eliminate contribution proportional to E^2 and other similar contributions (scattering by the residual gas in setup and even interaction of a molecule with gravitational field).

It should be noted that the magnetic field induced by an electric field appears on electron due to its possible polarizability. From the dimensionality consideration it follows that this contribution to the levels splitting $W_e = -\vec{\mu}_e \chi_e \vec{E}$ is comparable with $W = -\vec{d}_e \vec{E}$.

And similar, the magnetic field induced by an electric field appears on nucleus due to its possible polarizability. From the dimensionality consideration it follows that this contribution to the levels splitting $W_{nucl} = -\vec{\mu}_{nucl}\chi_{nucl}\vec{E}$ is comparable with $W = -\vec{d}_{nucl}\vec{E}$.

Attention also should be paid to the fact that elementary particles also can possess nonzero susceptibility χ . As an example, let us consider an experiment for measurement of electric dipole moment of neutron. Suppose neutron is placed into an electric field. If neutron possesses electric dipole moment, then, at first sight, the energy of interaction of this dipole moment with the electric field is $W_{n1} = -d_n\vec{\sigma}\vec{E}$, where d_n is the electric dipole moment of the neutron, $\vec{\sigma}$ are the Pauli matrices. But, in accordance with the above, the electric field induces the magnetic field on the neutron $\vec{B}_{ind} = \chi_n\vec{E}$ and the magnetic moment of the neutron interacts with this magnetic field $W_{n2} = -\vec{\mu}_n\chi_n\vec{E} = -\mu_n\chi_n\vec{\sigma}\vec{E}$. As a result, total energy of interaction of a neutron with an electric field is:

$$W_n = -(d_n + \mu_n\chi_n)\vec{\sigma}\vec{E} \quad (30)$$

It is evident that even at $d_n = 0$ W_n differs from zero. It means that in experiments for dipole electric moment measurement the sum $(d_n + \mu_n\chi_n)$ will be measured. Experimental estimations for electric dipole moment give value $d_n \sim 10^{-26} \div 10^{-27}$. But according to (30) this value concerns the sum $(d_n + \mu_n\chi_n)$ i.e. $(d_n + \mu_n\chi_n) \leq 10^{-26} \div 10^{-27}$. Then, considering $d_n = 0$, the limit for χ_n can be obtained: $\chi_n = \frac{(10^{-26} \div 10^{-27})e}{\mu_n} \leq 10^{-13}$.

Appendix 1

To illustrate the mechanism of the magnetic field induction by an electric field let us consider a simple model at first [12]. Suppose an atom is in the $s_{1/2}$ state and we place it to an electric field. Taking into account the admixture of the nearest $p_{1/2}$ state due to P - and T -odd interactions of an electron with a nucleus and interaction with the electric field one can represent the wave function of an atom in the form:

$$|\tilde{s}_{1/2}\rangle = \frac{1}{\sqrt{4\pi}}[R_0(r) - R_1(r)(\vec{\sigma}\vec{n})\eta_T - R_1(r)(\vec{\sigma}\vec{n})(\vec{\sigma}\vec{E})\delta]|\chi_{1/2}\rangle \quad (31)$$

Here $\vec{\sigma}$ are the Pauli matrices, $\vec{n} = \vec{r}/r$ is the unit vector along \vec{r} , \vec{E} is the electric field strength, R_0 and R_1 are radial parts of $s_{1/2}$ and $p_{1/2}$ wave functions respectively, $|\chi_{1/2}\rangle$ is the spin part of wave function, η_T is the mixing coefficient describing P and T noninvariant mixing of S and P states, δ describes mixing of S and P states by the Stark effect and \vec{E} is the electric field strength.

Interference of Stark and PT - odd terms changes the electron spin direction $\Delta\vec{s}$ in the point \vec{r} as follows:

$$\begin{aligned} \Delta\vec{s}(\vec{r}) &= \frac{\eta_T\delta}{8\pi}R_1^2(r)\langle\chi_{1/2}|(\vec{\sigma}\vec{n})\vec{\sigma}(\vec{\sigma}\vec{n})(\vec{\sigma}\vec{E}) \\ &\quad + (\vec{\sigma}\vec{E})(\vec{\sigma}\vec{n})\vec{\sigma}(\vec{\sigma}\vec{n})|\chi_{1/2}\rangle \\ &= \frac{\eta_T\delta R_1^2(r)}{8\pi}\left(4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}\right) \end{aligned} \quad (32)$$

The vector field $4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}$ is shown in Fig. 1. Since $\Delta\vec{s}$ does not depend on the initial direction of the atomic spin, this spin structure appears even in a nonpolarized atom. The spin vector averaged over spatial variables differs from zero and is directed along \vec{E} . As a result, the magnetic moment of an atom also differs from zero.

Appendix 2

Considering matrix elements W for different orientations of spin of an atom (molecule) with respect to the direction of electric field one can obtain the shift of atom (molecule) levels depending on spin orientation.

The above semiclassical reasoning provides to obtain an expression for the shift of atom (molecule) levels, which can be defined more accurately and more exactly from analysis of quantum-electrodynamic radiation correction to the levels of an atom (molecule) placed into an external field.

In this case the hamiltonian of the system contains not only the hamiltonian of the atom in the external field H_A^T , but also the hamiltonian of photons H_γ and hamiltonian of interaction of the atom with photons $H_{A\gamma}$

$$H = H_A^T + H_\gamma + H_{A\gamma}, \quad (33)$$

$$H_A^T = H_{0A} + V_{ext} + V_w = H_A^{ext} + V_w, \quad (34)$$

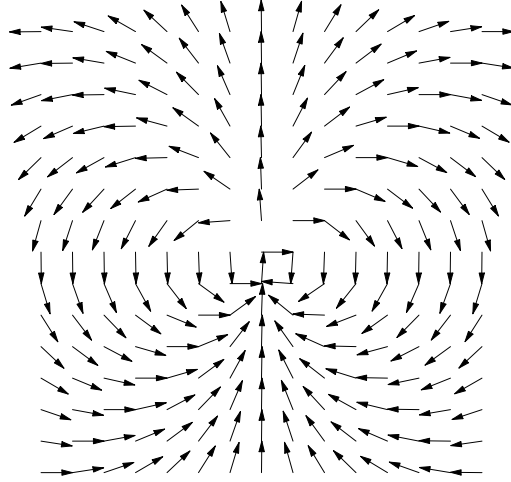


FIG. 1: Vector field $4\vec{n}(\vec{n}\vec{E}) - 2\vec{E}$. Vectors on figure show direction of atomic spin in $s_{1/2}$ state if we take into account the admixture of the $p_{1/2}$ state due to PT noninvariant interactions and external electric field.

where H_{0A} is the hamiltonian of an atom (molecule) in the absence of external fields and weak interactions, V_{ext} is the energy of interaction of an atom (molecule) with an external field, V_w is the energy weak interaction of electrons with nucleus and with each other, $H_A^{ext} = H_{0A} + V_{ext}$.

The studied induced levels shift in external fields appears in the second order of quantum electrodynamic perturbation theory over interaction of atoms with photons $H_{A\gamma}$.

Following the receipt [15] one can obtain that the magnitude of radiation shift $\delta\varepsilon_N$ of a level is defined by an expression similar to formula obtained in [15] even in presence of T-odd interactions:

$$\delta\varepsilon_N = \frac{e^2}{16\pi^3} P \int \frac{d^3k}{|\vec{k}|} \sum_F \frac{\bar{\Psi}_N(\vec{r}) \gamma_\mu e^{i\vec{k}\vec{r}} \Psi_F(\vec{r}) \bar{\Psi}_F(\vec{r}') \gamma_\mu e^{-i\vec{k}\vec{r}'} \Psi_N(\vec{r}')}{E_N - E_F - \frac{E_F}{|E_F|} |\vec{k}|} d^3r d^3r', \quad (35)$$

where Ψ_N are the eigenfunctions of Hamiltonian H_A^T

$$\begin{aligned} H_A^T \Psi_N &= E_N \Psi_N, \\ \bar{\Psi}_N &= \Psi_N^\dagger \beta; \quad \gamma_\mu, \beta \text{ are the Dirac matrices, } \mu = 1, 2, 3, 4. \end{aligned}$$

The symbol Pf denotes the principal value of integral.

Let us consider the shift of the level N of the ground state of an atom (molecule). Integrating (35) over d^3k one obtains

$$\begin{aligned} \delta\varepsilon_N &= \frac{1}{8\pi} \sum_{N'} \int \frac{j_{NN'\mu}(\vec{r}) j_{N'N\mu}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + \\ &+ \frac{1}{4\pi^2} \sum_{F \neq N} \int j_{NF\mu}(\vec{r}) \Phi_{FN}(|\vec{r} - \vec{r}'|) j_{FN\mu}(\vec{r}') d^3r d^3r', \end{aligned} \quad (36)$$

here $\sum_{N'}$ is the sum over degenerate spin states of the level N ;

transition current

$$j_{NF\mu}(\vec{r}) = i e \bar{\Psi}_N(\vec{r}) \gamma_\mu \Psi_N(\vec{r}), \quad (37)$$

$$\Phi_{FN}(|\vec{r} - \vec{r}'|) = -\frac{4\pi}{(|\vec{r} - \vec{r}'|)} \int_0^\infty \frac{\sin k |\vec{r} - \vec{r}'|}{E_F - E_N + k} dk, \text{ for } E_F > 0, \quad (38)$$

$$\Phi_{FN}(|\vec{r} - \vec{r}'|) = \frac{4\pi}{(|\vec{r} - \vec{r}'|)} \int_0^\infty \frac{\sin k |\vec{r} - \vec{r}'|}{E_F + E_N + k} dk, \text{ for } E_F < 0 \quad (39)$$

Integrals in (38), (39) can be computed explicitly

$$\int_0^{\infty} \frac{\sin \alpha x}{x + \beta} = ci(\alpha\beta) \sin(\alpha\beta) - \cos(\alpha\beta) si(\alpha\beta), \quad |arg \beta| < \pi, \quad \alpha > 0. \quad (40)$$

Let us consider the first term in (37) more attentively

$$\begin{aligned} \delta\varepsilon_N^{(1)} &= \frac{1}{8\pi} \int \frac{j_{NN'\mu}(\vec{r}) j_{N'N\mu}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' = \\ &= \frac{1}{8\pi} \int \frac{\vec{j}_{NN'}(\vec{r}) \vec{j}_{N'N}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' - \\ &- \frac{1}{8\pi} \int \frac{\Psi_N^+(\vec{r}) \Psi_{N'}(\vec{r}) \Psi_{N'}^+(\vec{r}') \Psi_N(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r', \end{aligned} \quad (41)$$

We are studying T-odd addition to the shift of level energy. To find wave functions one can use the first order of perturbation theory due to the weakness of interaction V_w (see (10)). Transition currents in the first order over V_w can be found with these wavefunctions:

$$j_{NF} = j_{NF}^0 + j_{NF}^T, \quad (42)$$

where j_{NF}^0 is the transition current in the absence of weak T-odd interactions (when $V_w = 0$), j_{NF}^T is the correction to transition current caused by the presence of V_w and calculated using wavefunctions (10).

Let us consider in details the part of $\delta\varepsilon_N^{(1)}$ caused by currents $\vec{j}_{NN'}$:

$$\delta\varepsilon_N^{\vec{j}} = \frac{1}{8\pi} \int \frac{\vec{j}_{NN'}(\vec{r}) \vec{j}_{N'N}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' \quad (43)$$

Substitution of (42) to (43) gives

$$\begin{aligned} \delta\varepsilon_N^{\vec{j}} &\approx \frac{1}{8\pi} \int \frac{\vec{j}_{NN'}^0(\vec{r}) \vec{j}_{N'N}^0(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' + \\ &+ \frac{1}{8\pi} \int \frac{\vec{j}_{NN'}^0(\vec{r}) \vec{j}_{N'N}^T(\vec{r}') + \vec{j}_{NN'}^T(\vec{r}) \vec{j}_{N'N}^0(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r d^3r' \end{aligned} \quad (44)$$

The first term in (44) describes conventional T-even contribution to radiation correction, while the second term is the T-odd contribution to the radiation shift of the level $\delta\varepsilon_N^{\vec{j}^T}$.

Remembering that a vector-potential $\vec{A}(\vec{r})$ satisfies an equation [15]:

$$\Delta \vec{A} = -\frac{1}{c}(\vec{j}), \quad (\text{here } \Delta \text{ is the Laplace operator}) \quad (45)$$

one can obtain:

$$\vec{A}(\vec{r}) = -\frac{1}{4\pi c} \int \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'. \quad (46)$$

Therefore, the expression similar to (??) can be written:

$$\delta\varepsilon_N^{\vec{j}^T} = \frac{1}{2c} \int [\vec{j}_{NN'}^0(\vec{r}) \vec{A}_{N'N}^T(\vec{r}) + \vec{A}_{NN'}^T(\vec{r}) \vec{j}_{N'N}^0(\vec{r})] d^3r \quad (47)$$

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