

The deuteron (nuclei) birefringence effect in a matter and in an electric field and the searches for an EDM of a deuteron (nucleus) rotating in a storage ring

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Abstract

The phenomena of deuteron birefringence in a matter and an electric field should be accurately considered when preparing experiments for the EDM search with a storage ring, because they could imitate the spin rotation due to the EDM. Moreover, study of these effects in such experiments could provide to measure both the spin-dependent part of the amplitude of the coherent elastic scattering of a deuteron by a nucleus at the zero angle and the tensor electric polarizability of a deuteron.

1 INTRODUCTION

The phenomena of spin rotation and spin dichroism (birefringence effect) for particles with the spin $S \geq 1$ in an unpolarized medium were theoretically described for the first time in [1, 2]. Deuteron spin dichroism was observed for the first time with the 20 MeV accelerator [3]. Further investigations of this phenomenon are planned to be carry out with a storage ring and an external beam [4, 5]. Observation of particle spin rotation and spin dichroism (birefringence effect) with a storage ring requires reducing of $(g-2)$ precession frequency (g is the gyromagnetic ratio). This precession appears due to interaction of the particle magnetic moment with an external electromagnetic field. The requirement for $(g-2)$ precession cancellation also arises when searching for a deuteron electric dipole moment (EDM) with a storage ring by the deuteron spin precession in an electric field [6, 7]. According to [6] balancing the energy of the particle and the strength of the electric field in a storage ring provides to reduce and even zeroize the $(g-2)$ precession frequency. As a result, the EDM-caused spin rotation grows linearly with time [6, 7]. Note that, when $(g-2)$ precession is suppressed, the angle of spin rotation induced by the birefringence effect grows linearly with time, too.

The effect of deuteron (nucleus) birefringence in a medium reveals itself in a storage ring due to presence of the residual gas inside the storage ring and use of a gas jet (gas target) for deuteron (nucleus) polarization analysis. Moreover, the birefringence also occurs in a solid target used for analysis of polarization of the deuteron (nucleus) beam outside the storage ring. Therefore, the phenomenon of birefringence in the gas medium and polarimeter would appear as a systematic error in the EDM measurements [7]. In addition, study of the birefringence phenomenon is of self-importance since it makes possible to measure the spin-dependend part of the forward scattering amplitude.

Lastly, the action of the electric field on the deuteron rouses one more mechanism of deuteron spin rotation and oscillations (the phenomenon of birefringence in an electric field) conditioned by the deuteron tensor electric polarizability [8].

In this paper the deuteron birefringence in a matter and in an electric field is considered for a particle moving in a storage ring. The equations describing the behavior of the deuteron spin in a storage ring including all the above mentioned contributions are derived. It is shown that the birefringence effect is noticeable and should be considered when carrying out the deuteron (nucleus) EDM searches at a storage ring.

2 THE PHENOMENON OF BIREFRINGENCE

According to the analysis [1, 2], when a particle with the spin $S \geq 1$ passes through an unpolarized medium, the medium refraction index depends on the particle spin orientation to its momentum. Therefore, the particle possesses some effective potential energy V in the medium and this energy depends on the spin orientation [1, 2, 4]

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma}Nf(\hat{0}), \quad (1)$$

where M is the particle mass, $f(\hat{0})$ is the spin dependent zero-angle elastic coherent scattering amplitude of the particle, N is the density of the scatterers in the matter (the number of scatterers in 1cm^3), γ is the Lorentz factor. Substituting $f(\hat{0})$ for a particle with the spin $S = 1$ in (1) in the explicit form one can obtain [1, 2, 4]

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma}N \left(d + d_1 \left(\vec{S}\vec{n} \right)^2 \right), \quad (2)$$

where \vec{n} is the unit vector along the particle momentum direction.

Let the quantization axis z is directed along \vec{n} and m denotes the magnetic quantum number. Then, for a particle in a state that is an eigenstate of the operator S_z of spin projection onto the z -axis, the efficient potential energy can be written as:

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma}N \left(d + d_1 m^2 \right). \quad (3)$$

According to (3) splitting of the deuteron energy levels in a matter is similar to splitting of atom energy levels in an electric field aroused by the quadratic Stark effect. Therefore, the above effect could be considered as caused by splitting of the spin levels of the particle in the pseudoelectric nuclear field of a matter.

Let a real electric field \vec{E} acts on a deuteron (nucleus). The energy \hat{V}_E of deuteron beam in an external electric field due to the tensor electric polarizability can be written in the form

$$\hat{V}_E = -\frac{1}{2}\hat{\alpha}_{ik}E_iE_k, \quad (4)$$

where $\hat{\alpha}_{ik}$ is the deuteron tensor electric polarizability, E_i are the components of the electric field. This expression can be rewritten as follows:

$$\hat{V}_E = \alpha_S E^2 - \alpha_T E^2 \left(\vec{S}\vec{n}_E \right)^2, \quad (5)$$

where α_S is the deuteron scalar electric polarizability, α_T is the deuteron tensor electric polarizability, \vec{n}_E is the unit vector along \vec{E} .

Comparing (5) with (2) we can conclude that the effect of spin rotation and oscillations about the \vec{E} direction can be observed for particle with $S \geq 1$ in an electric field, too [8].

Thus, considering evolution of the spin of a particle in a storage ring one should take into account several interactions:

1. interactions of magnetic and electric dipole moments with an electromagnetic field;
2. interaction (5) of a particle with an electric field due to the tensor electric polarizability
3. interaction (2) of a particle with the pseudoelectric nuclear field of a matter.

Therefore, the equation for the particle spin wavefunction is:

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = \left(\hat{H}_0 + \hat{V}_d + \hat{V} + \hat{V}_E \right) \Psi(t) \quad (6)$$

where $\Psi(t)$ is the particle spin wavefunction, \hat{H}_0 is the Hamiltonian describing the spin behavior caused by interaction of the magnetic moment with the electromagnetic field (equation (6) with the only \hat{H}_0 summand converts to the Bargman-Myshel-Telegdy equation), \hat{V}_d describes interaction of the deuteron (nuclear) EDM with an electric field.

3 THE EQUATIONS FOR THE POLARIZATION VECTOR AND QUADRUPOLARIZATION TENSOR OF THE DEUTERON BEAM IN A STORAGE RING

Let us consider motion of a deuteron in a storage ring in external magnetic and electric fields. Particle spin precession induced by interaction of the magnetic moment of a particle with an external electromagnetic field can be described by the Bargman-Myshel-Telegdy equation [6, 9]

$$\frac{d\vec{p}}{dt} = [\vec{p} \times \vec{\Omega}_0], \quad (7)$$

where t is time in the laboratory system,

$$\vec{\Omega}_0 = \frac{e}{mc} \left[\left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma+1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right], \quad (8)$$

m is the mass of the particle, e is its charge, \vec{p} is the spin polarization vector, γ is the Lorentz-factor, $\vec{\beta} = \vec{v}/c$, \vec{v} is the particle velocity, $a = (g-2)/2$, g is the gyromagnetic ratio, \vec{E} and \vec{H} are the electric and magnetic fields in the point of particle location.

If a particle possesses an intrinsic dipole moment then the additional term that describes the spin rotation induced by the EDM should be added to (7) [6]

$$\frac{d\vec{p}_{edm}}{dt} = \frac{d}{\hbar} \left[\vec{p} \times (\vec{\beta} \times \vec{B} + \vec{E}) \right], \quad (9)$$

where d is the electric dipole moment of a particle.

As a result, evolution of the deuteron spin due to the magnetic and electric momenta can be described by the following equation:

$$\frac{d\vec{p}}{dt} = \frac{e}{mc} \left[\vec{p} \times \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma+1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right\} \right] + d \left[\vec{p} \times (\vec{\beta} \times \vec{B} + \vec{E}) \right]. \quad (10)$$

According to the section 2, the equation (10) does not describe particle spin evolution in a storage ring completely. The expression (10) should be supplemented with the additions given by interactions \hat{V}_E and \hat{V} (see (2-6)).

This additional contribution could be found by the aids of the particle spin wavefunction $\Psi(t)$ (see 6)).

The equations describing the time evolution of the spin and tensor of quadrupolarization caused by the phenomena of birefringence can be written as:

$$\begin{aligned}\frac{d\vec{p}}{dt} &= \frac{d}{dt} \frac{\langle \Psi(t) | \vec{S} | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle}, \\ \frac{dp_{ik}}{dt} &= \frac{d}{dt} \frac{\langle \Psi(t) | Q_{ik} | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle},\end{aligned}\quad (11)$$

where $\Psi(t)$ is the deuteron wave function, $\hat{Q}_{ik} = \frac{3}{2}(S_i S_k + S_k S_i - \frac{4}{3}\delta_{ik}\hat{I})$ is the tensor of rank two (tensor of quadrupolarization).

The equations (11) contain initial phases that determine the deuteron wave function. Therefore, a partly polarized beam can not be described by such equations. So the spin density matrix formalism should be used to derive equations describing the evolution of the deuteron spin.

The density matrix of the system "deuteron+target" is

$$\rho = \rho_d \otimes \rho_t, \quad (12)$$

where ρ_d is the density matrix of the deuteron beam

$$\rho_d = I(\vec{k}) \left(\frac{1}{3}\hat{I} + \frac{1}{2}\vec{p}(\vec{k})\vec{S} + \frac{1}{9}p_{ik}(\vec{k})\hat{Q}_{ik} \right), \quad (13)$$

$I(\vec{k})$ is the intensity of the beam, \vec{p} is the polarization vector, p_{ik} is the quadrupolarization tensor of the deuteron beam, ρ_t is the density matrix of the target. For an unpolarized target $\rho_t = \hat{I}$, where I is the unit matrix in the spin space of target particle.

The equation for the deuteron beam density matrix can be written as:

$$\frac{d\rho_d}{dt} = -\frac{i}{\hbar} [\hat{H}, \rho_d] + \left(\frac{\partial \rho_d}{\partial t} \right)_{col}, \quad (14)$$

where $\hat{H} = \hat{H}_0 + \hat{V}_d + \hat{V}_E$,

$$\begin{aligned}\hat{V}_d &= -d(\vec{\beta} \times \vec{B} + \vec{E})\vec{S} \\ \hat{V}_E &= \alpha_S(\vec{\beta} \times \vec{B} + \vec{E})^2 - \alpha_T(\vec{\beta} \times \vec{B} + \vec{E})^2(\vec{S}\vec{n}_E)^2, \\ \vec{n}_E &= \frac{\vec{E} + \vec{\beta} \times \vec{B}}{|\vec{E} + \vec{\beta} \times \vec{B}|}.\end{aligned}\quad (15)$$

The collision term $\left(\frac{\partial \rho_d}{\partial t} \right)_{col}$ can be found by the method described in [10]

$$\left(\frac{\partial \rho_d}{\partial t} \right)_{col} = vN Sp_t \left[\frac{2\pi i}{k} [F(\theta=0)\rho - \rho F^+(\theta=0)] + \int d\Omega F(\vec{k}')\rho(\vec{k}')F^+(\vec{k}') \right], \quad (16)$$

where $\vec{k}' = \vec{k} + \vec{q}$, \vec{q} is the momentum carried over a nucleus of the matter from the incident particle, v is the speed of the incident particles, N is the atom density in the matter, F is the scattering amplitude depending on the spin operators of the deuteron and the matter nucleus (atom), F^+ is the Hermitian conjugate of the operator F . The first term in (16) describes coherent scattering of a particle by matter nuclei, while the second term is for multiple scattering.

Let us consider the first term in (16):

$$\left(\frac{\partial \rho_d}{\partial t} \right)_{col}^{(1)} = vN \frac{2\pi i}{k} [\hat{f}(0)\rho_d - \rho_d \hat{f}(0)^+]. \quad (17)$$

The amplitude $\hat{f}(0)$ of deuteron scattering in an unpolarized target at the zero angle is

$$\hat{f}(0) = Sp_t F(0) \rho_t. \quad (18)$$

This amplitude can be rewritten according (2) as

$$\hat{f}(0) = d + d_1 (\vec{S} \vec{n})^2, \quad (19)$$

where $\vec{n} = \vec{k}/k$, \vec{k} is the deuteron momentum.

As a result one can obtain:

$$\left(\frac{\partial \rho_d}{\partial t} \right)_{col}^{(1)} = -\frac{i}{\hbar} (\hat{V} \rho_d - \rho_d \hat{V}^+). \quad (20)$$

Finally, the expression (14) reads

$$\frac{d\rho_d}{dt} = -\frac{i}{\hbar} [\hat{H}, \rho_d] - \frac{i}{\hbar} (\hat{V} \rho_d - \rho_d \hat{V}^+) + vN Sp_t \int d\Omega F(\vec{k}') \rho(\vec{k}') F^+(\vec{k}'). \quad (21)$$

The last term in the above formula, which is proportional to Sp_t , describes the multiple scattering process and spin depolarization aroused from it. Henceforward we consider such time of experiment (such effective length for a particle in a matter) that provides to neglect this term.

The intensity of the beam is

$$I = Sp_d \rho_d. \quad (22)$$

Consequently

$$\frac{dI}{dt} = vN \frac{2\pi i}{k} Sp_d [f(0) \rho_d - \rho_d f^+(0)]. \quad (23)$$

Substituting (13) and (19) into (23) we can get

$$\frac{dI}{dt} = \frac{\chi}{3} [2 + p_{ik} n_i n_k] I(t) + \alpha I(t), \quad (24)$$

where $\chi = -\frac{4\pi v N}{k} \text{Im} d_1 = -vN(\sigma_1 - \sigma_0)$, $\alpha = -\frac{4\pi v N}{k} \text{Im} d = -vN\sigma_0$, σ_1 and σ_0 are the total cross-sections of deuteron scattering by a nonpolarized nucleus for the magnetic quantum numbers $m = 1$ and $m = 0$, respectively.

Polarization vector of the deuteron beam \vec{p} is determined as

$$\vec{p} = \frac{Sp_d \rho_d \vec{S}}{Sp_d \rho_d} = \frac{Sp_d \rho_d \vec{S}}{I}. \quad (25)$$

From (25) one can get the differential equation for the beam polarization

$$\frac{d\vec{p}}{dt} = \frac{Sp_d (d\rho_d/dt) \vec{S}}{I(t)} - \vec{p} \frac{Sp_d (d\rho_d/dt)}{I(t)}. \quad (26)$$

The expression for the quadrupolarization tensor is

$$p_{ik} = \frac{Sp_d \rho_d Q_{ik}}{Sp_d \rho_d} = \frac{Sp_d \rho_d Q_{ik}}{I}, \quad (27)$$

where $\hat{Q}_{ik} = \frac{3}{2} (S_i S_k + S_k S_i - \frac{4}{3} \delta_{ik} \hat{\mathbf{I}})$.

The change of the quadrupolarization tensor can be written as

$$\frac{dp_{ik}}{dt} = \frac{Sp_d(d\rho_d/dt)Q_{ik}}{I(t)} - p_{ik} \frac{Sp_d(d\rho_d/dt)}{I(t)}. \quad (28)$$

Using (13) and (7), (26) and (28) we can get the equation system for the time evolution of the deuteron polarization vector and quadrupolarization tensor ($\vec{n} = \vec{k}/k$, $\vec{n}_E = \frac{\vec{E} + \vec{\beta} \times \vec{B}}{|\vec{E} + \vec{\beta} \times \vec{B}|}$, $p_{xx} + p_{yy} + p_{zz} = 0$)

$$\left\{ \begin{array}{l} \frac{d\vec{p}}{dt} = \frac{e}{mc} \left[\vec{p} \times \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma+1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right\} \right] + \\ + \frac{d}{\hbar} \left[\vec{p} \times (\vec{E} + \vec{\beta} \times \vec{B}) \right] + \frac{\chi}{2} (\vec{n} (\vec{n} \cdot \vec{p}) + \vec{p}) + \\ + \frac{\eta}{3} [\vec{n} \times \vec{n}'] - \frac{2\chi}{3} \vec{p} - \frac{\chi}{3} (\vec{n} \cdot \vec{n}') \vec{p} - \frac{2}{3} \frac{\alpha_T E^2}{\hbar} [\vec{n}_E \times \vec{n}'_E], \\ \frac{dp_{ik}}{dt} = -(\varepsilon_{jkr} p_{ij} \Omega_r + \varepsilon_{jir} p_{kj} \Omega_r) + \\ + \chi \left\{ -\frac{1}{3} + n_i n_k + \frac{1}{3} p_{ik} - \frac{1}{2} (n'_i n_k + n_i n'_k) + \frac{1}{3} (\vec{n} \cdot \vec{n}') \delta_{ik} \right\} + \\ + \frac{3\eta}{4} ([\vec{n} \times \vec{p}]_i n_k + n_i [\vec{n} \times \vec{p}]_k) - \frac{\chi}{3} (\vec{n} \cdot \vec{n}') p_{ik} - \\ - \frac{3}{2} \frac{\alpha_T E^2}{\hbar} ([\vec{n}_E \times \vec{p}]_i n_{E,k} + n_{E,i} [\vec{n}_E \times \vec{p}]_k), \end{array} \right. \quad (29)$$

where $\eta = -\frac{4\pi N}{k} \text{Re}d_1$, $n'_i = p_{ik} n_k$, $n'_{E,i} = p_{ik} n_{E,k}$, $\Omega_r(d)$ are the components of the vector $\vec{\Omega}(d)$ ($r = 1, 2, 3$ corresponds x, y, z):

$$\begin{aligned} \vec{\Omega}(d) &= \frac{e}{mc} \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma+1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \vec{\beta} \times \vec{E} \right\} + \\ &+ \frac{d}{\hbar} (\vec{E} + \vec{\beta} \times \vec{B}). \end{aligned} \quad (30)$$

Then we consider the spin rotation about the particle momentum. According to [6, 7] the spin precession caused by the magnetic moment ($(g - 2)$ precession) can be minimized and even zeroized by applying a radial electric field.

The angles of spin rotation caused by both the EDM and birefringence effect are small for the considered experiment duration. Therefore, the perturbation theory can be used for (29) solution.

$$\begin{aligned} \vec{p}(t) &= \vec{p}^0 + \frac{e}{mc} \left[\vec{p}^0 \times \left\{ a \vec{B} + \left(\frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} \right\} \right] t + \\ &+ \frac{d}{\hbar} \left[\vec{p}^0 \times (\vec{E} + \vec{\beta} \times \vec{B}) \right] t + \left\{ \frac{\chi}{2} (\vec{n} (\vec{n} \cdot \vec{p}^0) + \vec{p}^0) t + \frac{\eta}{3} [\vec{n} \times \vec{n}'_0] t - \right. \\ &\quad \left. - \frac{2\chi}{3} \vec{p}^0 t - \frac{\chi}{3} (\vec{n} \cdot \vec{n}'_0) \vec{p}^0 t \right\} - \frac{2}{3} \frac{\alpha_T E^2}{\hbar} [\vec{n}_E \times \vec{n}'_{E0}] t, \end{aligned} \quad (31)$$

$$\begin{aligned} \mathfrak{R}_k(t) &= p_{ik}^0 - \frac{e}{mc} (\varepsilon_{jkr} p_{ij} + \varepsilon_{jir} p_{kj}) \left\{ a \vec{B} + \left(\frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} \right\}_r t - \\ &\quad - \frac{d}{\hbar} (\varepsilon_{jkr} p_{ij} + \varepsilon_{jir} p_{kj}) (\vec{E} + \vec{\beta} \times \vec{B})_r t + \\ &+ \left\{ \chi \left[-\frac{1}{3} + n_i n_k + \frac{1}{3} p_{ik}^0 - \frac{1}{2} (n'_{i0} n_k + n_i n'_{k0}) + \frac{1}{3} (\vec{n} \cdot \vec{n}'_0) \delta_{ik} \right] t + \right. \\ &\quad \left. + \frac{3\eta}{4} ([\vec{n} \times \vec{p}^0]_i n_k + n_i [\vec{n} \times \vec{p}^0]_k) t - \frac{\chi}{3} (\vec{n} \cdot \vec{n}'_0) p_{ik}^0 t \right\} - \\ &\quad - \frac{3}{2} \frac{\alpha_T E^2}{\hbar} ([\vec{n}_E \times \vec{p}^0]_i n_{E,k} + n_{E,i} [\vec{n}_E \times \vec{p}^0]_k) t, \end{aligned} \quad (32)$$

where \vec{p}^0 is the beam polarization at $t_0 = 0$, $n'_{i0} = p_{ik}^0 n_k$, $n'_{E0,i} = p_{ik}^0 n_{E,k}$, p_{ik}^0 is the components of the quadrupolarization tensor at the initial moment of time.

In real situation, even when $(g - 2)$ precession is suppressed, nevertheless, the rotation angle can appear large enough (during the experiment the spin can rotate several turns [7]). Absorption can also appear significant. In this case one should analyze the system (29) instead of perturbation theory results (31).

Thus, according to (31), (32) the spin behavior of a deuteron rotating in the storage ring is caused by several contributions:

1. spin rotation which is described by Bargman-Myshel-Telegdy equation;
2. rotation due to the deuteron EDM,
3. rotation and dichroism due to the the phenomena of birefringence in a medium and
4. spin rotation due to the phenomena of birefringence in an electric field.

Let us consider some particular cases.

Case I. Suppose the vector polarization is parallel to the the z -axis, i.e. $p_x^0 = p_y^0 = 0$, $p_z^0 \neq 0$, $p_{ik}^0 = 0$, if $i \neq k$, $p_{xx} \neq 0$, $p_{yy} \neq 0$, $p_{zz} = 0$.

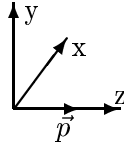


Figure 1: The initial orientation of the polarization vector

Suppose $(g - 2)$ spin precession caused by the magnetic moment is zero, then one can obtain

$$\begin{aligned}
p_x(t) &= 0 \\
p_y(t) &= -\frac{d}{\hbar} p_z^0 (E + \beta B) t, \\
p_z(t) &= p_z^0 + \frac{1}{3} \chi p_z^0 t, \\
p_{xx}(t) &= p_{xx}^0 + \frac{\chi}{3} (-1 + p_{xx}^0) t, \\
p_{yy}(t) &= p_{yy}^0 + \frac{\chi}{3} (-1 + p_{yy}^0) t, \\
p_{zz}(t) &= \frac{2\chi}{3} t, \\
p_{xy}(t) &= p_{xz}(t) = 0, \\
p_{yz}(t) &= \frac{d}{\hbar} p_{yy} (E + \beta B) t, \\
p_{xy} &= \frac{3}{2} \frac{\alpha_T E^2}{\hbar} p_z^0
\end{aligned} \tag{33}$$

The solution (34) shows that even in this practically ideal case (when the polarization vector is exactly parallel to \vec{n}) change of polarization due to birefringence effect leads to the appearance of additional components of \vec{p} and p_{ik} along with the components aroused by the deuteron EDM.

Case II. Let us consider now the more real case.

Suppose the angle between the initial polarization vector and the z axis is acute

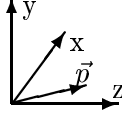


Figure 2: The initial orientation of the polarization vector

then the solution of (30,31) can be written as follows:

$$\begin{aligned}
p_x(t) &= p_x^0 - \frac{\chi}{6} p_x^0 t - \frac{\chi}{3} p_{zz}^0 p_x^0 t - \frac{\eta}{3} p_{yz}^0 t \\
p_y(t) &= p_y^0 - \frac{d}{\hbar} (E + \beta B) p_z^0 t - \frac{\chi}{6} p_y^0 t - \frac{\chi}{3} p_{zz}^0 p_y^0 t + \frac{\eta}{3} p_{xz}^0 t \\
p_z(t) &= p_z^0 + \frac{d}{\hbar} (E + \beta B) p_y^0 t + \frac{\chi}{3} p_z^0 t - \frac{\chi}{3} p_{zz}^0 p_z^0 t \\
p_{xx}(t) &= p_{xx}^0 - \frac{\chi}{3} (1 + p_{yy}^0) t - \frac{\chi}{3} p_{zz}^0 p_{xx}^0 t \\
p_{yy}(t) &= p_{yy}^0 - 2 \frac{d}{\hbar} p_{yz} (E + \beta B) - \frac{\chi}{3} (1 + p_{xx}^0) t - \frac{\chi}{3} p_{zz}^0 p_{yy}^0 t \\
p_{zz}(t) &= p_{zz}^0 + 2 \frac{d}{\hbar} p_{yz} (E + \beta B) + \frac{\chi}{3} (2 - p_{zz}^0) t - \frac{\chi}{3} p_{zz}^0 t \\
p_{xy}(t) &= p_{xy}^0 - \frac{d}{\hbar} p_{xz} (E + \beta B) + \frac{\chi}{3} p_{xy}^0 t - \frac{\chi}{3} p_{zz}^0 p_{xy}^0 t \\
p_{xz}(t) &= p_{xz}^0 + \frac{d}{\hbar} p_{xy} (E + \beta B) - \frac{\chi}{6} p_{xz}^0 t - \frac{3\eta}{4} p_y^0 t - \frac{\chi}{3} p_{zz}^0 p_{xz}^0 t \\
p_{yz}(t) &= p_{yz}^0 + \frac{d}{\hbar} (p_{yy} - p_{zz}) (E + \beta B) - \frac{\chi}{6} p_{yz}^0 t + \frac{3\eta}{4} p_x^0 t - \frac{\chi}{3} p_{zz}^0 p_{yz}^0 t.
\end{aligned} \tag{34}$$

According to (35) the change in components of polarization vector and tensor of quadrupolarization caused by the EDM is mixed with the contributions from the birefringence effect to the same components.

Thus, the changes in the deuteron polarization vector and quadrupolarization tensor are the result of several mechanisms:

- the rotation of the spin in the horizontal plain (\vec{E}, \vec{n}) .

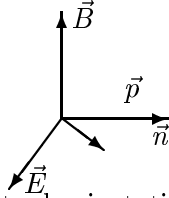


Figure 3: Mutual orientation of \vec{E} , \vec{B} , and \vec{n}

This rotation is risen due to interaction of the magnetic moment with external fields. The rotation frequency is expressed as:

$$\vec{\omega}_a = \frac{e}{mc} \left\{ a \vec{B} + \left(\frac{1}{\gamma^2 - 1} - a \right) \vec{\beta} \times \vec{E} \right\}; \tag{35}$$

- the rotation of the spin in the vertical plane (\vec{B}, \vec{n}) caused by the electric dipole moment;

The rotation frequency is

$$\vec{\omega}_d = \frac{d}{\hbar} \left(E + \vec{\beta} \times \vec{B} \right) \tag{36}$$

- the rotation caused by the phenomenon of birefringence in a medium, this is precession in the vertical plane (\vec{B}, \vec{E}) ;

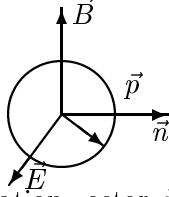


Figure 4: Rotation of the polarization vector due to birefringence in an electric field

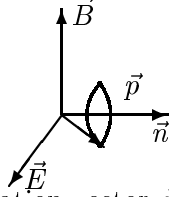


Figure 5: Rotation of the polarization vector due to the phenomena of birefringence

The rotation frequency is

$$\omega = \frac{2\pi N}{M\gamma} \hbar \text{Red}_1. \quad (37)$$

-the rotation due to the phenomenon of birefringence in an electric field in the vertical plane (\vec{B}, \vec{n}) , i.e. in the same plane as the rotation caused by the EDM.

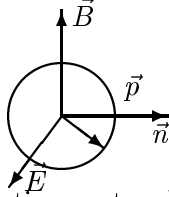


Figure 6: Rotation of the polarization vector due to birefringence in an electric field

The rotation frequency is

$$\omega_E = \frac{\alpha_T E^2}{\hbar} \quad (38)$$

Besides the rotations the transitions from the vector polarization into the tensor one and spin dichroism appear. Moreover, the spin dichroism leads to the appearance of the tensor polarization.

Let us compare the frequency and the angle of polarization vector rotation caused by the EDM with those caused by the birefringence effect.

1. The spin rotation frequency caused by the EDM is determined by the formula (9)

$$\omega_{edm} = \frac{dE}{\hbar} + \frac{d}{\hbar} \beta B. \quad (39)$$

We can get $\omega_{edm} \approx 3 \cdot 10^{-7} \text{ rad/s}$ for the storage ring with $E = 3.5 \text{ MV/m}$, $B = 0.2 \text{ T}$ and expected value of the deuteron EDM $d \sim 10^{-27} \text{ e} \cdot \text{cm}$ and $\omega_{edm} \approx 3 \cdot 10^{-9} \text{ rad/s}$ for EDM $d \sim 10^{-29} \text{ e} \cdot \text{cm}$.

2. The spin rotation frequency caused by the phenomena of birefringence in a residual gas:

$$\omega = \frac{2\pi N}{M\gamma} \hbar \text{Red}_1. \quad (40)$$

Using the last formula one can get $\omega \approx 2 \cdot 10^{-7} \text{ rad/s}$ for $N = 10^9 \text{ cm}^{-3}$ (suppose the pressure inside the storage ring $\sim 10^{-7} \text{ Torr}$), $\text{Red}_1 \sim 10^{-13}$. This effect depends on the density N (depends on the pressure inside the storage ring).

3. The spin rotation frequency caused by the phenomenon of birefringence in the gas jet (gas target), which is used for the beam extraction to the polarimeter [7] (Fig.7):

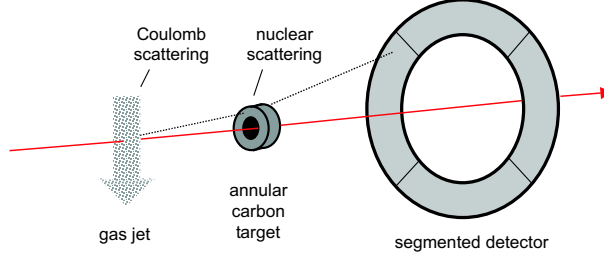


Figure 7: Polarimeter [7]

In this case the effective rotation frequency (or the rotation angle for $\tau = 1s$) is determined as follows:

$$\omega_{t\,eff} \equiv \varphi_t = \frac{2\pi N_t}{k} l \text{Re}d_1 \nu, \quad (41)$$

where $\omega_{t\,eff}$ is the effective frequency of spin rotation, φ_t is the deuteron spin rotation angle for $\tau = 1s$, N_t is the density of the target, l is the length of the target, ν is the frequency of beam rotation in the storage ring, $k = \frac{M\gamma v}{\hbar}$ is the deuteron wave number.

Really, the frequency ω_t of the spin rotation in a matter is

$$\omega_t = \frac{2\pi N_t}{M\gamma} \hbar \text{Re}d_1 \quad (42)$$

The spin rotation angle $\theta_\tau = \omega_t \tau_t$, where $\tau_t = \frac{l}{v}$ is the deuteron flying time in the target. The angle of rotation at 1 second is $\theta = \omega_t \tau_t \nu$ (during a second the deuteron passes through the gas target ν times). As a result,

$$\omega_{t\,eff} = \omega_t \tau_t \nu, \quad (43)$$

so one can get

$$\omega_{t\,eff} = \frac{2\pi N_t}{M\gamma} \hbar \text{Re}d_1 \frac{l}{v} \nu = \frac{2\pi j}{k} \text{Re}d_1 \nu, \quad (44)$$

where $j = N_t l$. Then using the experimental parameters [7] $j = 10^{15} \text{ cm}^{-2}$, $\nu \approx 10^5 - 10^6$ we have $\omega_{t\,eff} \approx 10^{-5} - 10^{-4} \text{ rad/s}$.

So the angle of polarization vector rotation for $\tau = 1s$ caused by the phenomenon of birefringence in the gas jet of polarimeter appears by two orders of magnitude greater than the angle of rotation due to the EDM. The additional contribution to spin rotation is also provided by the solid carbon target of polarimeter (see Figure.7).

4. The estimation for the spin rotation frequency caused by the birefringence in an electric field according to (36) for $\alpha_T \sim 10^{-37} \text{ cm}^3$ and $E = 3.5 \text{ MV/m}$ is $\omega_E \sim 10^{-6} \text{ rad/s}$.

Let us estimate the value of spin dichroism. This characteristic is given by the parameter $\chi = -vN(\sigma_1 - \sigma_0)$ (see expressions (31,32)):

- in the case of the scattering by the residual gas we have $|\chi| \sim 0.5 \cdot 10^{-6}$ for $N \sim 10^9 \text{ cm}^{-3}$;
- if a deuteron passes through the gas jet (gas target) for $\tau = 1s$ is

$$\chi_t = j(\sigma_1 - \sigma_0) \nu, \quad (45)$$

then for $j = 10^{15} \text{ cm}^{-2}$ and $\nu \approx 6 \cdot 10^5$ one can get $\chi_t \sim 3.4 \cdot 10^{-5}$. So we can conclude that there is the significant beam spin dichroism.

4 CONCLUSION

The above analysis shows that the phenomena of deuteron birefringence in a matter and an electric field should be accurately considered when preparing experiments for the EDM search with a storage ring,

because they could imitate the spin rotation due to the EDM. Moreover, study of these effects in such experiments could provide to measure both the spin-dependent part of the amplitude of the coherent elastic scattering of a deuteron by a nucleus at the zero angle and the tensor electric polarizability of a deuteron.

It should be also mentioned that if the nuclei in the gas jet are polarized, then according to [4] the P-,T-odd spin rotation and dichroism appear in the storage ring. They are caused by the T-odd nucleon-nucleon interaction of a deuteron with a polarized nucleus, in particular, interaction described as $V_{P,T} \sim \vec{S} [\vec{p}_N \times \vec{n}]$, where \vec{p}_N is the polarization vector of gas target.

P-even T-odd spin rotation and dichroism of deuterons (nuclei) caused by the interaction either $V_T \sim (\vec{S}[\vec{p}_N \times \vec{n}])(\vec{S}\vec{n})$ or $V_T' \sim S\rho\rho_J[(\vec{S} \times \vec{n})\vec{J}](\vec{J}\vec{n})$ also could be observed [4] (here $J \geq 1$ is the spin of the polarized target nuclei, ρ_J is the spin matrix density of the target nuclei).

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