Birefringence effect in the nuclear pseudoelectric field of matter and an external electric field for a deuteron (nucleus) rotating in a storage ring

V.G. Baryshevsky,
Research Institute for Nuclear Problems, Belarusian State University,
11 Bobruyskaya Str., Minsk 220050, Belarus,
e-mail: bar@inp.minsk.by

20th June 2012

Abstract

The birefringence effect in the nuclear pseudoelectric field of matter and an external electric field for a particle (deuteron, nucleus) moving in a storage ring is discussed. The influence of the birefringence effect on the EDM measurement experiments is considered. The attention is drawn to the possibility to measure the spin-dependent amplitude of the elastic coherent scattering of a deuteron by a nucleus, the electric polarizability of a deuteron (nucleus). Using a gas target with polarized nuclei also allows to study P-,T-odd interactions.

1 INTRODUCTION

The phenomena of spin rotation and spin dichroism (birefringence effect) for particles with the spin $S \geq 1$ in an unpolarized medium were theoretically described for the first time in [1, 2]. The peculiarity of this phenomenon is in conversion of the vector polarization to the tensor one and vice versa (similar the linear to circular polarization conversion for a photon in an optically anisotropic medium - the well known optical birefringence). However, unlike the optical birefringence, the birefringence effect for particles appear in isotropic matter (and even the spin of matter nuclei is either zero or unpolarized !). Anisotropy is provided by the particle itself (a particle with the spin $S \geq 1$ and mass $M \neq 0$ has the intrinsic anisotropy). Deuteron spin dichroism was recently observed with the 20 MeV accelerator [3]. Further investigations of this phenomenon are planned to be carry out with a storage ring and an external beam [4, 5]. Observation of particle spin rotation and spin dichroism (the birefringence effect) with a storage ring requires reducing of $(g-2)$ precession frequency (g is the gyromagnetic ratio). This precession appears due to interaction of the particle magnetic moment with an external electromagnetic field. The requirement for $(g-2)$ precession cancellation also arises when searching for a deuteron electric dipole moment (EDM) by the deuteron spin precession in an electric field in a storage ring[6, 7]. Some extremely interesting ideas providing noticeable cancellation for $(g-2)$ precession are proposed in [6, 7]

According to [6] balancing the energy of the particle and the strength of the electric field in the storage ring provides to reduce and even zeroize the $(g-2)$ precession frequency. As a result, EDM-caused spin rotation grows linearly with time [6, 7]. Note that, when $(g-2)$ precession is suppressed, the angle of spin rotation induced by the birefringence effect grows linearly with time, too.

The effect of deuteron (nucleus) birefringence in matter reveals itself in a storage ring due to presence of the residual gas inside the storage ring and use of a gas jet (gas target) for deuteron (nucleus) polarization analysis. Moreover, birefringence also occurs in the solid target, which is used for analysis of polarization of the deuteron (nucleus) beam outside the storage ring. Therefore, the phenomenon of birefringence in the gas medium and polarimeter would appear as a systematic error in the EDM measurements [7]. In addition, study of the birefringence phenomenon is of self-importance since it makes possible to measure the spin-depended part of the forward scattering amplitude.

Lastly, the action of the electric field on the deuteron rouses one more mechanism of deuteron spin rotation and oscillations (the phenomenon of birefringence in an electric field) conditioned by the deuteron tensor electric polarizability [8].

In this paper several our preceding works concerning the birefringence effect in the nuclear pseudoelectric field of matter and an external electric field [1, 2, 3, 4, 5, 8, 15, 16] are summarized for a particle (deuteron, nucleus) moving in a storage ring . The attention is drawn to the additional possibility to measure the spin-dependent amplitude of the elastic coherent scattering of a deuteron by a nucleus, the electric polarizability of a deuteron (nucleus). Use of a gas target with polarized nuclei also allows to study P-,T-odd interactions.

2 The index of refraction and effective potential energy of particles in medium.

Close connection between the coherent elastic scattering amplitude at zero angle $f(0)$ and the refraction index of medium N has been established as a result of numerous studies (see, for example, [9, 10]):

$$N = 1 + \frac{2\pi\rho}{k^2} f\,(0) \tag{1}$$

where ρ is the number of particles per cm^3, k is the wave number of a particle incident on a target.

The expression (1) was derived in assumption that $N - 1 \ll 1$. If $k \to 0$ then $(N-1)$ grows and expression for N has the form

$$N^2 = 1 + \frac{4\pi\rho}{k^2} f\,(0)$$

Let us consider particle refraction on the vacuum-medium boundary.

The wave number of the particle in vacuum is denoted k, $k' = kN$ is the wave number of the particle in the medium.

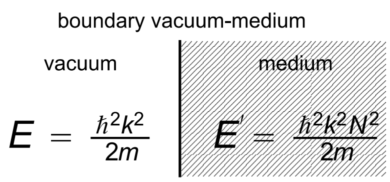

Figure 1: Kinetic energy of a particle in vacuum is not equal to that in medium.

As it can be seen, the kinetic energy of a particle in vacuum $E = \frac{\hbar^2 k^2}{2m}$ is not equal to that in the medium $E' = \frac{\hbar^2 k^2 N^2}{2m}$.

From the energy conservation condition we immediately obtain the necessity to suppose that the particle in the medium possesses an effective potential energy (see the detailed theory in [10].) This energy can be easily found from the evident equality

$$E = E' + V$$

i.e.

$$V = E - E' = -\frac{2\pi\hbar^2}{m}\rho f(0). \tag{2}$$

Above we considered the rest target. But in storage rings moving bunches can be used as a target. Therefore we should generalize the expressions (1,2) for this case. Thus, let us consider the collision of two bunches of particles. Suppose that in the rest frame of the storage ring the particles of the first beam have the energy E_1 and Lorentz-factor γ_1, whereas particles of the second beam are characterized by the energy E_2 and Lorentz-factor γ_2. Let us recollect that the phase of a wave in a medium is Lorentz-invariant. Therefore, we can find it by the following way. Let us choose the reference frame, where the second beam rests. As in this frame particles of the second beam rest, then the refraction index can be expressed in the conventional form (1):

$$N_1' = 1 + \frac{2\pi\rho_2'}{{k_1'}^2} f\left(E_1', 0\right), \tag{3}$$

where $\rho_2' = \gamma_2^{-1}\rho_2$ is the density of the bunch 2 in its rest frame and ρ_2 is the density of the second bunch in the storage ring frame, k_1' is the wavenumber of particles of the first bunch in the rest frame of the bunch 2. Let us denote the length of the bunch 2 in its rest frame as L, $L = \gamma_2\, l$, where l is the length of this bunch in the storage ring frame.

Now we can find the change of the phase of the wave caused by the interaction of the particle 1 with the particles of bunch 2:

$$\phi = k_1'(N_1' - 1)L = \frac{2\pi\rho_2'}{k_1'} f(E_1', 0)\ L = \frac{2\pi\rho_2}{k_1'} f(E_1', 0) k_1'\ l, \tag{4}$$

It is known that the ratio $\frac{f(E_1',0)}{k_1'}$ is invariant, so we can write $\frac{f(E_1',0)}{k_1'} = \frac{f(E_1,0)}{k_1}$, where $f(E,0)$ is the amplitude of elastic coherent forward scattering of particle 1 by the moving particle 2 in the rest frame of the storage ring.

As a result

$$\phi = \frac{2\pi\rho_2}{k_1} f(E_1, 0)\ l = \frac{2\pi\rho_2}{k_1} f(E_1, 0)\ v_{rel}\ t, \tag{5}$$

where v_{rel} is the velocity of relative motion of the particle 1 and bunch 2, if the first particle is nonrelativistic, whereas the second one is relativistic, then $v_{rel} \approx c$, where c is the speed of light, t is the time of interaction of the particle 1 with the bunch 2 in the rest frame of the storage ring.

The particle with the velocity $v_1 = \frac{\hbar k_1 c^2}{E_1}$ passes the distance $z = v_1\ t$ over the time t. It should be noted that the path length z differs from the length of bunch 2, because it moves. Expression (5) can be rewritten as:

$$\phi = \frac{2\pi\rho_2}{k_1} f(E_1, 0)\ \frac{v_{rel}}{v_1}\ z = k_1(N_1 - 1)z, \tag{6}$$

where the index of refraction of the particle 1 by the beam of moving particles 2 is:

$$N_1 = 1 + \frac{2\pi\rho_2}{{k_1}^2} \frac{v_{rel}}{v_1} f(E, 0) \tag{7}$$

If $v_2 = 0$, the the conventional expression (1) follows from (7)

So, let us consider particles captured to a trap. They are nonrelativistic. From (2) the effective potential energy V can be expressed as:

$$\begin{aligned} V &= -\frac{2\pi\hbar^2}{m_1}\rho_2 \frac{c}{v_1} f(E_1, 0) = \\ &= -2\pi\hbar\rho_2 c \frac{f(E_1, 0)}{k_1} = \\ &= -2\pi\hbar\rho_2 c \frac{f(E_1', 0)}{k_1'} = \\ &= -\frac{2\pi\hbar^2}{m_1\gamma_2}\rho_2 f(E_1', 0). \end{aligned} \tag{8}$$

where γ_2 is the Lorentz-factor of the bunch 2 and the Lorentz-factor of the particle 1 is $\gamma_1 = 1$ $\hbar k_1' = m_1 c \gamma_2$.

It should be noted that the amplitude of coherent elastic scattering at zero angle depends on the T-matrix as follows:

$$f(E_1', 0) = -\frac{(2\pi)^2 \hbar k_1'}{v_1 + v_2} T(E_1') = (2\pi)^2 m_1 \gamma_2 T(E_1') \tag{9}$$

i.e. the amplitude of forward scattering is proportional to the Lorentz-factor γ_2. As a result, the quantity $\gamma_2^{-1} f(E_1', 0)$ depends on the energy of the particle only due to possible dependence of T-matrix on energy. From (9,10) one can obtain

$$V = (2\pi)^3 \rho_2 T(E_1') \tag{10}$$

3 THE PHENOMENON OF ROTATION AND OSCILLATION OF DEUTERON (NUCLEAR) SPIN IN UNPOLARIZED MATTER (BIREFRINGENCE AND SPIN DICHROISM)

Let us consider the refraction of a particle with the spin $S \geq 1$ in matter.

According to [1, 2] the index of refraction for such a particle depends on the particle spin and can be written as:

$$\hat{N} = 1 + \frac{2\pi\rho}{k^2} \hat{f}(0), \tag{11}$$

where$\hat{f}(0) = Tr \hat{\rho}_J \hat{F}(0)$, ρ is the density of matter (the number of scatterers in 1 cm^3), k is the deuteron wave number, $\hat{\rho}_J$ is the spin density matrix of the scatterers, $\hat{F}(0)$ is the operator of the forward scattering amplitude, acting in the combined spin space of the particle and scatterer spin $\vec{J}$.

For a particle with the spin $S = 1$ (for example, deuteron) in an unpolarized target $\hat{f}(0)$ can be written as:

$$\hat{f}(0) = d + d_1 (\vec{S}\vec{n})^2. \tag{12}$$

where $\vec{S}$ is the deuteron spin operator, $\vec{n}$ is the unit vector along the deuteron momentum $\vec{k}$.

The axis of quantization z is directed along the particle wave vector $\vec{k}$. Considering only strong interactions, which are invariant to the parity transformation and time reversal, we may omit the terms containing S in odd degrees. Therefore, the refractive index for deuterons

$$\hat{N} = 1 + \frac{2\pi\rho}{k^2} \left(d + d_1 S_z^2\right) \tag{13}$$

depends on the deuteron spin orientation relative to the deuteron momentum.

The refractive index for a particle in the state, which is the eigenstate of the operator S_z of spin projection on the axis z:

$$\hat{N} = 1 + \frac{2\pi\rho}{k^2} \left(d + d_1 m^2\right), \tag{14}$$

m is the magnetic quantum number.

According to Eq.(14), the refractive indices for the states with $m = +1$ and $m = -1$ are the same, while those for $m = \pm 1$ and $m = 0$ are different ($\Re N(\pm 1) \neq \Re N(0)$ and $\Im N(\pm 1) \neq \Im N(0)$). This can be obviously explained as follows (see Fig.2): the shape of a deuteron in the ground state is non–spherical. Therefore, the scattering cross-section $\sigma_{||}$ for a deuteron with $m = \pm 1$ (deuteron spin is parallel to its momentum $\vec{k}$) differs from the scattering cross-section $\sigma_{\perp}$ for a deuteron with $m = 0$:

$$\sigma_{||} \neq \sigma_{\perp} \Rightarrow \Im f_{||}(0) = \frac{k}{4\pi}\sigma_{||} \neq \Im f_{\perp}(0) = \frac{k}{4\pi}\sigma_{\perp}. \tag{15}$$

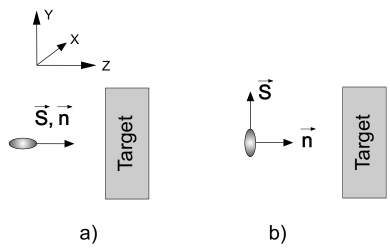

Figure 2: Two possible orientation of vectors $\vec{S}$ and $\vec{n}=\frac{\vec{k}}{k}$: a) $\vec{S}\|\vec{n}$; b) $\vec{S}\perp\vec{n}$

According to the dispersion relation $\Re f(0)\sim\Phi(\Im f(0))$, then $\Re f_{\perp}(0)\neq\Re f_{||}(0)$.

From the above it follows that deuteron spin dichroism appears even when the deuteron passes through an unpolarized target: due to different absorption, the initially unpolarized beam acquires polarization or, yet more precisely, alignment.

Let us consider the behavior of the deuteron spin in a target. The spin state of the deuteron is described by its vector and tensor polarization ($\vec{p}=\langle\vec{S}\rangle$ and $p_{ik}=\langle Q_{ik}\rangle$, respectively, here $\hat{Q}_{ij}=\frac{3}{2}\left(\hat{S}_i\hat{S}_j+\hat{S}_j\hat{S}_i-\frac{4}{3}\delta_{ij}\right)$. When the deuteron moves in matter its vector and tensor polarization appears changed. To calculate $\vec{p}$ and p_{ik} one need to know the explicit form of the deuteron spin wave function ψ

The wave function of the deuteron that has passed the distance z inside the target is:

$$\psi(z)=\exp\left(ik\hat{N}z\right)\psi_0, \tag{16}$$

where ψ_0 is the wave function of the deuteron before entering the target. The wave function ψ can be expressed as a superposition of the basic spin functions χ_m, which are the eigenfunctions of the operators $\hat{S}^2$ and $\hat{S}_z$ ($\hat{S}_z\chi_m=m\chi_m$):

$$\psi=\sum_{m=\pm1,0}a^m\chi_m. \tag{17}$$

Therefore,

$$\Psi=\left(\begin{array}{c}a^1\\a^0\\a^{-1}\end{array}\right)=\left(\begin{array}{c}ae^{i\delta_1}e^{ikN_1z}\\be^{i\delta_0}e^{ikN_0z}\\ce^{i\delta_{-1}}e^{ikN_{-1}z}\end{array}\right)=\left(\begin{array}{c}ae^{i\delta_1}e^{ikN_1z}\\be^{i\delta_0}e^{ikN_0z}\\ce^{i\delta_{-1}}e^{ikN_1z}\end{array}\right), \tag{18}$$

(according to the above $N_1=N_{-1}$).

Suppose plane (yz) coincides with the plane formed by the initial deuteron vector polarization $\vec{p}_0\neq0$ and the momentum $\vec{k}$ of the deuteron. In this case $\delta_1-\delta_0=\delta_0-\delta_{-1}=\frac{\pi}{2}$, and the components of the polarization vector at $z=0$ are $p_x=0, p_y\neq0,$ and $p_z\neq0$.

The components of the vector polarization $\vec{p}=\langle\vec{S}\rangle=\frac{\langle\Psi|\vec{S}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$ inside the target are:

$$\begin{aligned}
p_x &= \frac{\sqrt{2}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b\,(a-c)\sin\left(\frac{2\pi\rho}{k}\Re d_1 z\right)}{\langle \Psi \mid \Psi \rangle}, \\
p_y &= \frac{\sqrt{2}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b\,(a+c)\cos\left(\frac{2\pi\rho}{k}\Re d_1 z\right)}{\langle \Psi \mid \Psi \rangle}, \\
p_z &= \frac{e^{\rho\sigma_1 z}\left(a^2-c^2\right)}{\langle \Psi \mid \Psi \rangle}.
\end{aligned} \tag{19}$$

Similarly, the components of the tensor polarization $\hat{Q}_{ij} = \frac{3}{2}\left(\hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i - \frac{4}{3}\delta_{ij}\right)$ are expressed as:

$$\begin{aligned}
p_{xx} &= \frac{-\frac{1}{2}\left(a^2+c^2\right)e^{-\rho\sigma_1 z} + b^2 e^{-\rho\sigma_0 z} - 3ace^{-\rho\sigma_1 z}}{\langle \Psi \mid \Psi \rangle}, \\
p_{yy} &= \frac{-\frac{1}{2}\left(a^2+c^2\right)e^{-\rho\sigma_1 z} + b^2 e^{-\rho\sigma_0 z} + 3ace^{-\rho\sigma_1 z}}{\langle \Psi \mid \Psi \rangle}, \\
p_{zz} &= \frac{\left(a^2+c^2\right)e^{-\rho\sigma_1 z} - 2b^2 e^{-\rho\sigma_0 z}}{\langle \Psi \mid \Psi \rangle}, \\
p_{xy} &= 0, \\
p_{xz} &= \frac{\frac{3}{\sqrt{2}}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b\,(a+c)\sin\left(\frac{2\pi\rho}{k}\Re d_1 z\right)}{\langle \Psi \mid \Psi \rangle}, \\
p_{yz} &= \frac{\frac{3}{\sqrt{2}}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z} b\,(a-c)\cos\left(\frac{2\pi\rho}{k}\Re d_1 z\right)}{\langle \Psi \mid \Psi \rangle},
\end{aligned} \tag{20}$$

where $\langle \Psi \mid \Psi \rangle = \left(a^2+c^2\right)e^{-\rho\sigma_1 z} + b^2 e^{-\rho\sigma_0 z}$, $\sigma_0 = \frac{4\pi}{k}\Im f_0$, $\sigma_1 = \frac{4\pi}{k}\Im f_1$, $f_0 = d$, $f_1 = d + d_1$.

According to (19), (20) spin rotation occurs when the angle between the polarization vector $\vec{p}$ and momentum $\vec{k}$ of the particle differs from $\frac{\pi}{2}$.

For example, when $\Re d_1 > 0$, the angle between the polarization vector and momentum is acute (see Fig.3) and the spin rotates left-hand around the momentum direction, whereas an obtuse angle between the polarization vector and the momentum gives rise to right-hand spin rotation (see Fig.4).

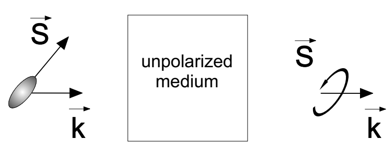

Figure 3:

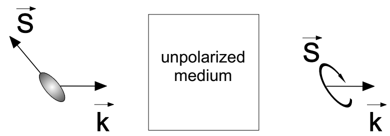

Figure 4:

When the polarization vector and momentum are perpendicular (a transversely polarized particle), then the components of the vector polarization at $z = 0$ are: $p_x = 0$, $p_y \neq 0$, and $p_z = 0$. In this case $a = c$ and dependance of the vector polarization on z can be expressed as:

$$
\begin{aligned}
p_x &= 0, \\
p_y &= \frac{\sqrt{2}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}2ba\cos\left(\frac{2\pi\rho}{k}\Re d_1 z\right)}{\langle\Psi \mid \Psi\rangle}, \\
p_z &= 0, \\
p_{xx} &= \frac{-4a^2e^{-\rho\sigma_1 z} + b^2e^{-\rho\sigma_0 z}}{\langle\Psi \mid \Psi\rangle}, \\
p_{yy} &= \frac{2a^2e^{-\rho\sigma_1 z} + b^2e^{-\rho\sigma_0 z}}{\langle\Psi \mid \Psi\rangle}, \\
p_{zz} &= \frac{2a^2e^{-\rho\sigma_1 z} - 2b^2e^{-\rho\sigma_0 z}}{\langle\Psi \mid \Psi\rangle}, \\
p_{xz} &= \frac{\frac{3}{\sqrt{2}}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}2ab\sin\left(\frac{2\pi\rho}{k}\Re d_1 z\right)}{\langle\Psi \mid \Psi\rangle}, \\
p_{yz} &= 0.
\end{aligned}
\tag{21}
$$

According to (22) the vector and tensor polarization oscillate when a transversely polarized deuteron passes through matter (see Fig.5) i.e. the vector polarization converts to the tensor one and vice versa (similar the linear to circular polarization conversion for a photon in an optically anisotropic medium - the well known optical birefringence). However, unlike the optical birefringence, the birefringence effect for particles appear in isotropic matter (and even the spin of matter nuclei is either zero or unpolarized !). Anisotropy is provided by the particle itself (a particle with the spin $S \geq 1$ and mass $M \neq 0$ has the intrinsic anisotropy).

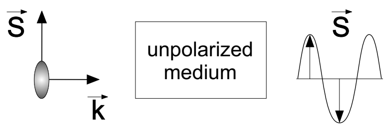

Figure 5:

According to the above (see (2)) a particle possesses some effective potential energy V in matter. If the particle spin $S \geq 1$, then this energy depends on the spin orientation [1, 2, 4]

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma}N\hat{f}(0), \tag{22}$$

where M is the particle mass, $\hat{f}(0)$ is the spin dependent zero-angle elastic coherent scattering amplitude of the particle, N is the density of the scatterers in the medium (the number of scatterers in $1cm^3$), γ is the Lorentz factor. Substituting $\hat{f}(0)$ for a particle with the spin $S = 1$ in (22) in the explicit form one can obtain [1, 2, 4]

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma}N\left(d + d_1\left(\vec{S}\vec{n}\right)^2\right), \tag{23}$$

where $\vec{n}$ is the unit vector along the particle momentum direction.

Let the quantization axis z is directed along $\vec{n}$ and m denotes the magnetic quantum number. Then, for a particle in a state that is an eigenstate of the operator S_z of spin projection onto the z-axis, the effective potential energy can be written as:

$$\hat{V} = -\frac{2\pi\hbar^2}{M\gamma} N \left(d + d_1 m^2\right). \tag{24}$$

According to (24) splitting of the deuteron energy levels in matter is similar to splitting of atom energy levels in an electric field aroused from the quadratic Stark effect. Therefore, the above effect could be considered as caused by splitting of the spin levels of the particle in the pseudoelectric nuclear field of matter.

Let an electric field acts on a deuteron (nucleus). The energy $\hat{V}_E$ of the deuteron beam in the external electric field $\vec{E}$ due to the tensor electric polarizability can be written in the form

$$\hat{V}_E = -\frac{1}{2}\hat{\alpha}_{ik} E_i E_k, \tag{25}$$

where $\hat{\alpha}_{ik}$ is the deuteron tensor electric polarizability, E_i are the components of the electric field. This expression can be rewritten as follows:

$$\hat{V}_E = \alpha_S E^2 - \alpha_T E^2 \left(\vec{S}\vec{n}_E\right)^2, \tag{26}$$

where α_S is the deuteron scalar electric polarizability, α_T is the deuteron tensor electric polarizability, $\vec{n}_E$ is the unit vector along $\vec{E}$.

Comparison of (26) with (23) provides to conclude that in an electric field we can observe the effect of spin rotation and oscillations about the $\vec{E}$ direction for a particle with $S \geq 1$, too [8].

4 THE EQUATIONS FOR THE POLARIZATION VECTOR AND QUADRUPOLARIZATION TENSOR OF THE DEUTERON BEAM IN A STORAGE RING

Considering evolution of the spin of a particle in a storage ring one should take into account several interactions:

1. interactions of the magnetic and electric dipole moments with an electromagnetic field;
2. interaction (26) of the particle with the electric field due to the tensor electric polarizability
3. interaction (23) of the particle with the pseudoelectric nuclear field of matter.

Therefore, the equation for the particle spin wavefunction is:

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = \left(\hat{H}_0 + \hat{V}_d + \hat{V} + \hat{V}_E\right)\Psi(t) \tag{27}$$

where $\Psi(t)$ is the particle spin wavefunction, $\hat{H}_0$ is the Hamiltonian describing the spin behavior caused by interaction of the magnetic moment with the electromagnetic field (equation (27) with the only $\hat{H}_0$ summand converts to the Bargman-Myshel-Telegdy equation), $\hat{V}_d$ describes interaction of the deuteron (nuclear) EDM with the electric field.

Let us describe motion of a deuteron in a storage ring in external magnetic and electric fields. Particle spin precession induced by interaction of the magnetic moment of a particle with an external electromagnetic field can be described by the Bargman-Myshel-Telegdy equation [6, 11]

$$\frac{d\vec{p}}{dt} = [\vec{p} \times \vec{\Omega}_0], \tag{28}$$

where t is time in the laboratory system,

$$\vec{\Omega}_0 = \frac{e}{mc}\left[\left(a+\frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right], \tag{29}$$

m is the mass of the particle, e is its charge, $\vec{p}$ is the spin polarization vector, γ is the Lorentz-factor, $\vec{\beta} = \vec{v}/c$, $\vec{v}$ is the particle velocity, $a = (g-2)/2$, g is the gyromagnetic ratio, $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields in the point of particle location.

If a particle possesses an intrinsic dipole moment then the additional term that describes the spin rotation induced by the EDM should be added to (28) [6]

$$\frac{d\vec{p}_{edm}}{dt} = \frac{d}{\hbar}\left[\vec{p}\times\left(\vec{\beta}\times\vec{B}+\vec{E}\right)\right], \tag{30}$$

where d is the electric dipole moment of a particle.

As a result, evolution of the deuteron spin due to the magnetic and electric momenta can be described by the following equation:

$$\frac{d\vec{p}}{dt} = \frac{e}{mc}\left[\vec{p}\times\left\{\left(a+\frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right\}\right] + d\left[\vec{p}\times\left(c\vec{\beta}\times\vec{B}+\vec{E}\right)\right]. \tag{31}$$

According to the section 2, the equation (31) does not describe the particle spin evolution in a storage ring completely. The expression (31) should be supplemented with the additions given by interactions $\hat{V}_E$ and $\hat{V}$ (see (23-27)) .

This additional contribution could be found by the aids of the particle spin wavefunction $\Psi(t)$ (see 27)).

The equations describing the time evolution of the spin and tensor polarization caused by the phenomena of birefringence can be written as:

$$\begin{aligned} \frac{d\vec{p}}{dt} &= \frac{d}{dt}\frac{\langle\Psi(t)|\vec{S}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)|}, \\ \frac{dp_{ik}}{dt} &= \frac{d}{dt}\frac{\langle\Psi(t)|Q_{ik}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)|}, \end{aligned} \tag{32}$$

where $\Psi(t)$ is the deuteron wave function, $\hat{Q}_{ik} = \frac{3}{2}\left(S_iS_k + S_kS_i - \frac{4}{3}\delta_{ik}\hat{\mathtt{I}}\right)$ is the tensor of rank two (the tensor polarization).

The equations (32) contain initial phases that determine the deuteron wave function. Therefore, a partly polarized beam can not be described by such equations. So the spin density matrix formalism should be used to derive equations describing the evolution of the deuteron spin [15].

The density matrix of the system "deuteron+target" is

$$\rho = \rho_d \otimes \rho_t, \tag{33}$$

where ρ_d is the density matrix of the deuteron beam

$$\rho_d = I(\vec{k})\left(\frac{1}{3}\hat{\mathtt{I}} + \frac{1}{2}\vec{p}(\vec{k})\vec{S} + \frac{1}{9}p_{ik}(\vec{k})\hat{Q}_{ik}\right), \tag{34}$$

$I(\vec{k})$ is the intensity of the beam, $\vec{p}$ is the polarization vector, p_{ik} is the tensor polarization of the deuteron beam, ρ_t is the density matrix of the target. For an unpolarized target $\rho_t = \hat{\mathtt{I}}$, where $\hat{\mathtt{I}}$ is the unit matrix in the spin space of target particle.

The equation for the deuteron beam density matrix can be written as:

$$\frac{d\rho_d}{dt} = -\frac{i}{\hbar}\left[\hat{H},\rho_d\right] + \left(\frac{\partial\rho_d}{\partial t}\right)_{col}, \tag{35}$$

where $\hat{H} = \hat{H}_0 + \hat{V}_d + \hat{V}_E$,

$$\begin{aligned} \hat{V}_d &= -d\left(\vec{\beta} \times \vec{B} + \vec{E}\right)\vec{S}, \\ \hat{V}_E &= \alpha_S\left(\vec{\beta} \times \vec{B} + \vec{E}\right)^2 - \alpha_T\left(\vec{\beta} \times \vec{B} + \vec{E}\right)^2\left(\vec{S}\vec{n}_E\right)^2, \\ \vec{n}_E &= \frac{\vec{E} + \vec{\beta} \times \vec{B}}{|\vec{E} + \vec{\beta} \times \vec{B}|}. \end{aligned} \tag{36}$$

The collision term $\left(\frac{\partial \rho_d}{\partial t}\right)_{col}$ can be found by the method described in [12]:

$$\left(\frac{\partial \rho_d}{\partial t}\right)_{col} = vNSp_t\left[\frac{2\pi i}{k}\left[F(\theta = 0)\rho - \rho F^{+}(\theta = 0)\right] + \int d\Omega F(\vec{k}')\rho(\vec{k}')F^{+}(\vec{k}')\right], \tag{37}$$

where $\vec{k}' = \vec{k} + \vec{q}$, $\vec{q}$ is the momentum carried over a nucleus of the matter from the incident particle, v is the speed of the incident particles, N is the atom density in the matter, F is the scattering amplitude depending on the spin operators of the deuteron and the matter nucleus (atom), F^{+} is the Hermitian conjugate of the operator F. The first term in (37) describes coherent scattering of a particle by matter nuclei, while the second term is for multiple scattering.

Let us consider the first term in (37):

$$\left(\frac{\partial \rho_d}{\partial t}\right)^{(1)}_{col} = vN\frac{2\pi i}{k}\left[\hat{f}(0)\rho_d - \rho_d\hat{f}(0)^{+}\right]. \tag{38}$$

The amplitude $\hat{f}(0)$ of deuteron scattering in an unpolarized target at the zero angle is

$$\hat{f}(0) = Sp_t F(0)\rho_t. \tag{39}$$

This amplitude can de rewritten according to (23) as

$$\hat{f}(0) = d + d_1(\vec{S}\vec{n})^2, \tag{40}$$

where $\vec{n} = \vec{k}/k$, $\vec{k}$ is the deuteron momentum.

As a result one can obtain:

$$\left(\frac{\partial \rho_d}{\partial t}\right)^{(1)}_{col} = -\frac{i}{\hbar}\left(\hat{V}\rho_d - \rho_d\hat{V}^{+}\right). \tag{41}$$

Finally, the expression (35) reads

$$\frac{d\rho_d}{dt} = -\frac{i}{\hbar}\left[\hat{H}, \rho_d\right] - \frac{i}{\hbar}\left(\hat{V}\rho_d - \rho_d\hat{V}^{+}\right) + vNSp_t\int d\Omega F(\vec{k}')\rho(\vec{k}')F^{+}(\vec{k}'). \tag{42}$$

The last term in the above formula, which is proportional to Sp_t, describes the multiple scattering process and spin depolarization aroused from it. Henceforward we consider such time of experiment (such effective length for a particle in a matter) that provides to neglect this term.

The intensity of the beam is

$$I = Sp_d\rho_d. \tag{43}$$

Consequently

$$\frac{dI}{dt} = vN\frac{2\pi i}{k}Sp_d\left[f(0)\rho_d - \rho_d f^{+}(0)\right]. \tag{44}$$

Substituting (34) and (40) into (44) we can get

$$\frac{dI}{dt} = \frac{\chi}{3}\left[2 + p_{ik}n_i n_k\right] I(t) + \alpha I(t), \tag{45}$$

where $\chi = -\frac{4\pi v N}{k}\mathtt{Im}d_1 = - - vN(\sigma_1 - \sigma_0)$, $\alpha = -\frac{4\pi v N}{k}\mathtt{Im}d = -vN\sigma_0$, σ_1 and σ_0 are the total cross-sections of deuteron scattering by a nonpolarized nucleus for the magnetic quantum numbers $m = 1$ and $m = 0$, respectively.

The vector polarization of the deuteron beam $\vec{p}$ is determined as

$$\vec{p} = \frac{Sp_d\rho_d\vec{S}}{Sp_d\rho_d} = \frac{Sp_d\rho_d\vec{S}}{I}. \tag{46}$$

From (46) one can get the differential equation for the beam polarization

$$\frac{d\vec{p}}{dt} = \frac{Sp_d(d\rho_d/dt)\vec{S}}{I(t)} - \vec{p}\frac{Sp_d(d\rho_d/dt)}{I(t)}. \tag{47}$$

The expression for the tensor polarization is

$$p_{ik} = \frac{Sp_d\rho_d Q_{ik}}{Sp_d\rho_d} = \frac{Sp_d\rho_d Q_{ik}}{I}, \tag{48}$$

where $\hat{Q}_{ik} = \frac{3}{2}\left(S_iS_k + S_kS_i - \frac{4}{3}\delta_{ik}\hat{\mathtt{I}}\right)$.

The change of the tensor polarization can be written as

$$\frac{dp_{ik}}{dt} = \frac{Sp_d(d\rho_d/dt)Q_{ik}}{I(t)} - p_{ik}\frac{Sp_d(d\rho_d/dt)}{I(t)}. \tag{49}$$

Using (34) and (28), (47) and (49) we can get the equation system for the time evolution of the deuteron polarization vector and quadrupolarization tensor ($\vec{n} = \vec{k}/k$, $\vec{n}_E = \frac{\vec{E}+\vec{\beta}\times\vec{B}}{|\vec{E}+\vec{\beta}\times\vec{B}|}$, $p_{xx} + p_{yy} + p_{zz} = 0$) [16]

$$\left\{\begin{array}{l}
\frac{d\vec{p}}{dt} = \frac{e}{mc}\left[\vec{p}\times\left\{\left(a+\frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right\}\right] + \\
+\frac{d}{\hbar}\left[\vec{p}\times\left(\vec{E}+\vec{\beta}\times\vec{B}\right)\right] + \frac{\chi}{2}(\vec{n}(\vec{n}\cdot\vec{p}) + \vec{p}) + \\
+\frac{\eta}{3}[\vec{n}\times\vec{n}'] - \frac{2\chi}{3}\vec{p} - \frac{\chi}{3}(\vec{n}\cdot\vec{n}')\vec{p} - \frac{2}{3}\frac{\alpha_T E_{eff}^2}{\hbar}[\vec{n}_E\times\vec{n}_E'], \\
\frac{dp_{ik}}{dt} = -\left(\varepsilon_{jkr}p_{ij}\Omega_r + \varepsilon_{jir}p_{kj}\Omega_r\right) + \\
+\chi\left\{-\frac{1}{3} + n_in_k + \frac{1}{3}p_{ik} - \frac{1}{2}(n_i'n_k + n_in_k') + \frac{1}{3}(\vec{n}\cdot\vec{n}')\delta_{ik}\right\} + \\
+\frac{3\eta}{4}\left([\vec{n}\times\vec{p}]_in_k + n_i[\vec{n}\times\vec{p}]_k\right) - \frac{\chi}{3}(\vec{n}\cdot\vec{n}')p_{ik} - \\
-\frac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar}\left([\vec{n}_E\times\vec{p}]_in_{E,k} + n_{E,i}[\vec{n}_E\times\vec{p}]_k\right),
\end{array}\right. \tag{50}$$

where $\vec{E}_{eff} = (\vec{E} + \vec{\beta}\times\vec{B})$, $\eta = -\frac{4\pi N}{k}\mathtt{Re}d_1$, $n_i' = p_{ik}n_k$, $n_{E,i}' = p_{ik}n_{E,k}$, $\Omega_r(d)$ are the components of the vector $\vec{\Omega}(d)$ ($r = 1,2,3$ corresponds x,y,z):

$$\begin{aligned}
\vec{\Omega}(d) &= \frac{e}{mc}\left\{\left(a+\frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right\} + \\
&+ \frac{d}{\hbar}\left(\vec{E}+\vec{\beta}\times\vec{B}\right).
\end{aligned} \tag{51}$$

Then we consider the spin rotation about the particle momentum. According to [6, 7] the spin precession caused by the magnetic moment ($(g-2)$ precession) can be minimized and even zeroized by applying a radial electric field.

The angles of spin rotation caused by both the EDM and birefringence effect are small for the considered experiment duration. Therefore, the perturbation theory can be used for (50) solution.

$$\vec{p}(t) = \vec{p}^0 + \frac{e}{mc}\left[\vec{p}^0 \times \left\{a\vec{B} + \left(\frac{1}{\gamma^2-1} - a\right)\vec{\beta}\times\vec{E}\right\}\right]t +$$
$$+\frac{d}{\hbar}\left[\vec{p}^0 \times \left(\vec{E} + \vec{\beta}\times\vec{B}\right)\right]t + \{\frac{\chi}{2}(\vec{n}(\vec{n}\cdot\vec{p}^0) + \vec{p}^0)t + \frac{\eta}{3}[\vec{n}\times\vec{n}_0^{'}]t -$$
$$-\frac{2\chi}{3}\vec{p}^0 t - \frac{\chi}{3}(\vec{n}\cdot\vec{n}_0^{'})\vec{p}^0 t\} - \frac{2}{3}\frac{\alpha_T E_{eff}^2}{\hbar}[\vec{n}_E \times \vec{n}_{E\,0}^{'}]t, \quad (52)$$

$$p_{ik}(t) = p_{ik}^0 - \frac{e}{mc}(\varepsilon_{jkr}p_{ij} + \varepsilon_{jir}p_{kj})\left\{a\vec{B} + \left(\frac{1}{\gamma^2-1} - a\right)\vec{\beta}\times\vec{E}\right\}_r t -$$
$$-\frac{d}{\hbar}(\varepsilon_{jkr}p_{ij} + \varepsilon_{jir}p_{kj})\left(\vec{E} + \vec{\beta}\times\vec{B}\right)_r t +$$
$$+\{\chi\left[-\frac{1}{3} + n_i n_k + \frac{1}{3}p_{ik}^0 - \frac{1}{2}(n_{i0}^{'}n_k + n_i n_{k0}^{'}) + \frac{1}{3}(\vec{n}\cdot\vec{n}_0^{'})\delta_{ik}\right]t +$$
$$+\frac{3\eta}{4}\left([\vec{n}\times\vec{p}^0]_i n_k + n_i[\vec{n}\times\vec{p}^0]_k\right)t - \frac{\chi}{3}(\vec{n}\cdot\vec{n}_0^{'})p_{ik}^0 t\} -$$
$$-\frac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar}\left([\vec{n}_E\times\vec{p}^0]_i n_{E,k} + n_{E,i}[\vec{n}_E\times\vec{p}^0]_k\right)t, \quad (53)$$

where $\vec{p}^0$ is the beam polarization at $t_0 = 0$, $n_{i0}^{'} = p_{ik}^0 n_k$, $n_{E\,0,i}^{'} = p_{ik}^0 n_{E,k}$, p_{ik}^0 are the components of the tensor polarization at the initial moment of time.

In real situation, even when $(g-2)$ precession is suppressed, nevertheless, the rotation angle can appear large enough (during the experiment the spin can rotate several turns [7]). Absorption can also appear significant. In this case one should analyze the system (50) instead of perturbation theory results (52).

Thus, according to (52), (53) the spin behavior of a deuteron rotating in a storage ring is caused by several contributions:
1. spin rotation which is described by Bargman-Myshel-Telegdy equation;
2. rotation due to the deuteron EDM;
3. rotation and dichroism due to the the phenomena of birefringence in matter;
and
4. spin rotation due to the phenomena of birefringence in an electric field.

Let us consider some particular cases.

Case I. Suppose the vector polarization is parallel to the the z axis, i.e. $p_x^0 = p_y^0 = 0$, $p_z^0 \neq 0$, $p_{ik}^0 = 0$, if $i \neq k$, $p_{xx} \neq 0$, $p_{yy}^0 \neq 0$, $p_{zz}^0 = 0$.

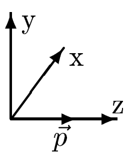

Figure 6: The original orientation of the polarization vector

Suppose $(g-2)$ spin precession caused by the magnetic moment is zero, then one can obtain

$$\begin{aligned}
p_x(t) &= 0 \\
p_y(t) &= -\frac{d}{\hbar} p_z^0 \, (E+\beta B)\, t, \\
p_z(t) &= p_z^0 + \frac{1}{3}\chi p_z^0 t, \\
p_{xx}(t) &= p_{xx}^0 + \frac{\chi}{3}\left(-1+p_{xx}^0\right) t, \\
p_{yy}(t) &= p_{yy}^0 + \frac{\chi}{3}\left(-1+p_{yy}^0\right) t, \\
p_{zz}(t) &= \frac{2\chi}{3} t, \\
p_{xy}(t) &= p_{xz}(t) = 0, \\
p_{yz}(t) &= \frac{d}{\hbar} p_{yy}\, (E+\beta B)\, t, \\
p_{xy} &= \frac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar} p_z^0
\end{aligned} \tag{54}$$

The solution (55) shows that even in this practically ideal case (when the polarization vector is exactly parallel to $\vec{n}$) change of polarization due to the birefringence effect leads to the appearance of additional components of $\vec{p}$ and p_{ik} along with the components aroused from the deuteron EDM.

Case II. Let us consider now the more real case.

Suppose the angle between the initial polarization vector and the z axis is acute (Fig.7) Let us

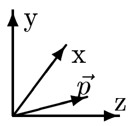

Figure 7: The original orientation of the polarization vector

express the effect caused by the gas inside the storage ring. Then the solution of (52,53) can be written as follows:

$$\begin{aligned}
p_x(t) &= p_x^0 - \frac{\chi}{6} p_x^0\, t - \frac{\chi}{3} p_{zz}^0 p_x^0\, t - \frac{\eta}{3} p_{yz}^0 t \\
p_y(t) &= p_y^0 - \frac{d}{\hbar}(E+\beta B)\, p_z^0\, t - \frac{\chi}{6} p_y^0\, t - \frac{\chi}{3} p_{zz}^0 p_y^0\, t + \frac{\eta}{3} p_{xz}^0 t \\
p_z(t) &= p_z^0 + \frac{d}{\hbar}(E+\beta B)\, p_y^0\, t + \frac{\chi}{3} p_z^0\, t - \frac{\chi}{3} p_{zz}^0 p_z^0\, t \\
p_{xx}(t) &= p_{xx}^0 - \frac{\chi}{3}\left(1+p_{yy}^0\right) t - \frac{\chi}{3} p_{zz}^0 p_{xx}^0\, t \\
p_{yy}(t) &= p_{yy}^0 - 2\frac{d}{\hbar} p_{yz}\,(E+\beta B) - \frac{\chi}{3}\left(1+p_{xx}^0\right) t - \frac{\chi}{3} p_{zz}^0 p_{yy}^0\, t \\
p_{zz}(t) &= p_{zz}^0 + 2\frac{d}{\hbar} p_{yz}\,(E+\beta B) + \frac{\chi}{3}\left(2-p_{zz}^0\right) t - \frac{\chi}{3} p_{zz}^{0\,2}\, t \\
p_{xy}(t) &= p_{xy}^0 - \frac{d}{\hbar} p_{xz}\,(E+\beta B) + \frac{\chi}{3} p_{xy}^0\, t - \frac{\chi}{3} p_{zz}^0 p_{xy}^0\, t \\
p_{xz}(t) &= p_{xz}^0 + \frac{d}{\hbar} p_{xy}\,(E+\beta B) - \frac{\chi}{6} p_{xz}^0\, t - \frac{3\eta}{4} p_y^0\, t - \frac{\chi}{3} p_{zz}^0 p_{xz}^0\, t \\
p_{yz}(t) &= p_{yz}^0 + \frac{d}{\hbar}(p_{yy}-p_{zz})\,(E+\beta B) - \frac{\chi}{6} p_{yz}^0\, t + \frac{3\eta}{4} p_x^0\, t - \frac{\chi}{3} p_{zz}^0 p_{yz}^0\, t.
\end{aligned} \tag{55}$$

According to (56) the change in components of vector and tensor polarization caused by the EDM is mixed with the contributions from the birefringence effect to the same components. In much the same mixing of contributions from the EDM and the tensor electric polarizability α_T appear, but here we does not dwell on them because of the expressions unhandiness.

Thus, the changes in the deuteron vector and tensor polarization are the result of several mechanisms:

- rotation of the spin in the horizontal plane $(\vec{E}, \vec{n})$ (Fig.8).

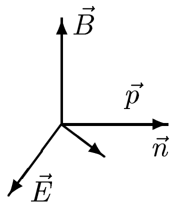

Figure 8: The respective orientation of $\vec{E}$, $\vec{B}$, and $\vec{n}$

This rotation is risen due to interaction of the magnetic moment with external fields. The rotation frequency is expressed as:

$$\vec{\omega}_a = \frac{e}{mc}\left\{a\vec{B} + \left(\frac{1}{\gamma^2-1} - a\right)\vec{\beta}\times\vec{E}\right\}; \tag{56}$$

- rotation of the spin in the vertical plane $(\vec{B}, \vec{n})$ caused by the electric dipole moment (Fig.9);

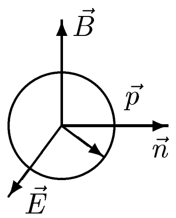

Figure 9: Rotation of the polarization vector due to birefringence in an electric field

The rotation frequency is

$$\vec{\omega}_d = d\frac{c}{\hbar}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right) \tag{57}$$

- rotation caused by the phenomenon of birefringence in a medium, this is precession in the vertical plane $(\vec{B}, \vec{E})$ (Fig.10);

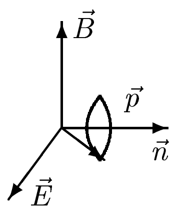

Figure 10: Rotation of the polarization vector due to the phenomena of birefringence

The rotation frequency is

$$\omega = \frac{2\pi N}{M\gamma}\,\hbar\,\mathtt{Re}d_1. \tag{58}$$

- rotation due to the phenomenon of birefringence in an electric field in the vertical plane $(\vec{B}, \vec{n})$ (Fig.11), i.e. in the same plane as the rotation caused by the EDM.

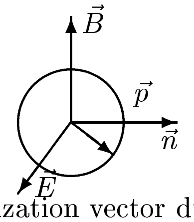

Figure 11: Rotation of the polarization vector due to birefringence in an electric field

The rotation frequency is

$$\omega_E = \frac{\alpha_T E_{eff}^2}{\hbar} \tag{59}$$

Besides the above rotations **the transitions from the vector polarization into the tensor one and spin dichroism occur.**

Moreover, the spin dichroism leads to the appearance of the tensor polarization.

Let us compare the frequency and the angle of polarization vector rotation caused by the EDM with those caused by the birefringence effect.

1. The spin rotation frequency caused by the EDM is determined by the formula (30)

$$\omega_{edm} = \frac{d\,E}{\hbar} + \frac{d}{\hbar}\beta B. \tag{60}$$

We can get $\omega_{edm} \approx 3 \cdot 10^{-7} rad/s$ for the storage ring with $E = 3.5\,MV/m$, $B = 0.2\,T$ and expected value of the deuteron EDM $d \sim 10^{-27} e \cdot cm$ and $\omega_{edm} \approx 3 \cdot 10^{-9} rad/s$ for EDM $d \sim 10^{-29} e \cdot cm$.

2. The spin rotation frequency caused by the phenomena of birefringence in a residual gas:

$$\omega = \frac{2\pi N}{M\gamma}\,\hbar\,\mathtt{Re}d_1. \tag{61}$$

Using the last formula one can get $\omega \approx 2 \cdot 10^{-7}\, rad/s$ for $N = 10^9\, cm^{-3}$ (suppose the pressure inside the storage ring $\sim 10^{-7}$ Torr), $\mathtt{Re}d_1 \sim 10^{-13}$. This effect depends on the density N (depends on the pressure inside the storage ring).

3. The spin rotation frequency caused by the phenomenon of birefringence in the gas jet (gas target), which is used for the beam extraction to the polarimeter [7] (Fig.12):

In this case the effective rotation frequency (or the rotation angle for $\tau = 1s$) is determined as follows:

$$\omega_{t\,eff} \equiv \varphi_t = \frac{2\pi N_t}{k}\, l\, \mathtt{Re}d_1\, \nu, \tag{62}$$

where $\omega_{t\,eff}$ is the effective frequency of spin rotation , φ_t is the deuteron spin rotation angle for $\tau = 1s$, N_t is the density of the target, l is the length of the target, ν is the frequency of beam rotation in the storage ring, $k = \frac{M\gamma v}{\hbar}$ is the deuteron wave number.

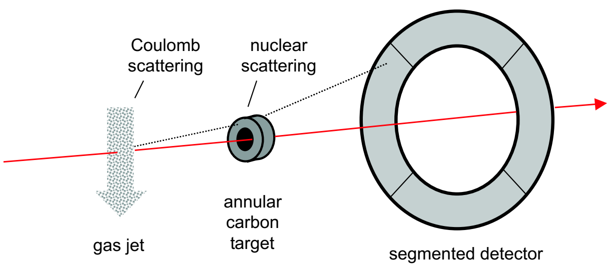

Figure 12: Polarimeter [7]

Really, the frequency ω_t of the spin rotation in matter is

$$\omega_t = \frac{2\pi N_t}{M\gamma}\,\hbar\,\mathrm{Re}d_1 \qquad (63)$$

The spin rotation angle $\theta_\tau = \omega_t\tau_t$, where $\tau_t = \frac{l}{v}$ is the deuteron flying time in the target. The angle of rotation at 1 second is $\theta = \omega_t\tau_t\nu$ (during a second the deuteron passes through the gas target ν times). As a result,

$$\omega_{t\,eff} = \omega_t\,\tau_t\,\nu, \qquad (64)$$

so one can get

$$\omega_{t\,eff} = \frac{2\pi N_t}{M\gamma}\,\hbar\,\mathrm{Re}d_1\frac{l}{v}\,\nu = \frac{2\pi j}{k}\,\mathrm{Re}d_1\nu, \qquad (65)$$

where $j = N_t\,l$. Then using the experimental parameters [7] $j = 10^{15}\,cm^{-2}$, $\nu \approx 10^5 - 10^6$ we have $\omega_{t\,eff} \approx 10^{-5} - 10^{-4}\,rad/s$.

So the angle of polarization vector rotation for $\tau = 1s$ caused by the phenomenon of birefringence in the gas jet of polarimeter appears by two orders of magnitude greater than the angle of rotation due to the EDM. The additional contribution to spin rotation is also provided by the solid carbon target of polarimeter (see Figure.12).

4. The estimation for the spin rotation frequency caused by the birefringence in an electric field according to (57) for $\alpha_T \sim 10^{-37}cm^3$ and $E = 3.5MV/m$ is $\omega_E \sim 10^{-6}rad/s$.

Let us estimate the value of spin dichroism. This characteristic is given by the parameter $\chi = -vN(\sigma_1 - \sigma_0)$ (see expressions (52), (53)):

- in the case of the scattering by the residual gas we have $|\chi| \sim 0.5\cdot 10^{-6}$ for $N \sim 10^9$ cm^{-3};
- if a deuteron passes through the gas jet (gas target) for $\tau = 1$s is

$$\chi_t = j(\sigma_1 - \sigma_0)\,\nu, \qquad (66)$$

then for $j = 10^{15}\,cm^{-2}$ and $\nu \approx 6\cdot 10^5$ one can get $\chi_t \sim 3.4\cdot 10^{-5}$. So we can conclude that there is the significant beam spin dichroism.

It should be especially mentioned that the polarimeter (Fig.12) includes a solid target. Therefore, the birefringence effect in the matter of the solid target arises, too.

The possible influence of P-odd spin rotation and spin dichroism (i.e. different absorbtion cross-sections in matter for a deuteron with the spin parallel and antiparallel to the momentum $\sigma_{\uparrow\uparrow} \neq \sigma_{\uparrow\downarrow}$) aren't to be forgotten.

5 CONCLUSION

The above analysis shows that the phenomena of deuteron birefringence in matter and an electric field should be studied in experiments for the EDM search with a storage ring. Moreover, they could imitate the spin rotation due to the EDM. Study of the birefringence effects in such experiments could provide to measure both the spin-dependent part d_1 of the amplitude of the coherent elastic scattering of a deuteron by a nucleus at the zero angle and the tensor electric polarizability of a deuteron.

It should be also mentioned that if the nuclei in the gas jet are polarized, then according to [4] the P-,T-odd spin rotation and dichroism appear in the storage ring. They are caused by the T-odd nucleon-nucleon interaction of a deuteron with a polarized nucleus and, in particular, interaction described by $V_{P,T} \sim \vec{S}\,[\vec{p}_N \times \vec{n}]$, where $\vec{p}_N$ is the polarization vector of gas target.

P-even T-odd spin rotation and dichroism of deuterons (nuclei) caused by the interaction either $V_T \sim (\vec{S}[\vec{p}_N \times \vec{n}])(\vec{S}\vec{n})$ or $V_T' \sim Sp\rho_J[([\vec{S}\times\vec{n}]\vec{J})(\vec{J}\vec{n})]$ also could be observed [4] (here $J \geq 1$ is the spin of the polarized target nuclei, ρ_J is the spin matrix density of the target nuclei).

6 Appendix

Let us consider the amplitude of deuteron scattering by a proton depending on spin orientation with respect to the deuteron momentum [2].

For fast deuterons the scattering amplitude can be found in the eikonal approximation [13, 14]. According to [14], the amplitude of zero-angle coherent scattering in this approximation can be written as follows:

$$f(0) = \frac{k}{2\pi\ i} \int \left(e^{i\chi_D\left(\vec{b},\vec{r}\right)} - 1 \right) d^2b \left|\varphi\left(\vec{r}\right)\right|^2 d^3r \tag{67}$$

where k is the deuteron wavenumber, $\vec{b}$ is the impact parameter, i.e. the distance between the deuteron and the proton centers of gravity in the normal plane; $\varphi\left(\vec{r}\right)$ is the wavefunction of the deuteron in the ground state; $\left|\varphi\left(\vec{r}\right)\right|^2$ is the probability to find proton and neutron (in the deuteron) at the distance $\vec{r}$ apart. The phase shift due to the deuteron scattering by a proton is

$$\chi_D = -\frac{1}{\hbar v} \int_{-\infty}^{+\infty} V_D\left(\vec{b}, z', \vec{r}_\perp\right) dz' \tag{68}$$

$\vec{r}_\perp$ is the $\vec{r}$ component perpendicular to the momentum of the incident deuteron, v is the deuteron velocity. The phase shift $\chi_D = \chi_1 + \chi_2$, where χ_1 and χ_2 are the phase shifts caused by proton-proton and neutron-proton interactions, respectively.

For the polarized deuteron under consideration the probability $\left|\varphi\left(\vec{r}\right)\right|^2$ is different for the different spin states of the deuteron. Thus for states with the magnetic quantum number $m = \pm 1$, the probability is $\left|\varphi_{\pm1}\left(\vec{r}\right)\right|^2$, whereas for $m = 0$, it is $\left|\varphi_0\left(\vec{r}\right)\right|^2$. Owing to the additivity of the phase shifts, the equation (67) can be rewritten as

$$f\left(0\right) = \frac{k}{\pi} \int \left\{ t_1\left(\vec{b} - \frac{\vec{r}_\perp}{2}\right) + t_2\left(\vec{b} + \frac{\vec{r}_\perp}{2}\right) + 2it_1\left(\vec{b} - \frac{\vec{r}_\perp}{2}\right) t_2\left(\vec{b} + \frac{\vec{r}_\perp}{2}\right) \right\} \left|\varphi\left(\vec{r}\right)\right|^2 d^2b d^3r \tag{69}$$

where

$$t_{1(2)} = \frac{e^{i\chi_{1(2)}} - 1}{2i}.$$

Attention should be given to the fact that the latter expression is valid if one neglects spin dependence of nuclear forces between the colliding proton (neutron) and proton. When this dependence is taken into account, the phase shift χ_D is an operator acting in the spin space of colliding particles and in the general case the expansion (69) is not valid. However, to estimate the magnitude of the effect of deuteron spin oscillations, spin dependence of nuclear forces may be neglected (spin dependent contribution was considered in [16]). Let us also omit terms caused by the Coulomb interaction. From (69) it follows

$$f(0) = f_1(0) + f_2(0) + \frac{2ik}{\pi} \int t_1\left(\vec{b} - \frac{\vec{r}_\perp}{2}\right) t_2\left(\vec{b} + \frac{\vec{r}_\perp}{2}\right) \left|\varphi\left(\vec{r}_\perp, z\right)\right|^2 d^2b d^2r_\perp dz \tag{70}$$

where

$$f_{1(2)}(0) = \frac{k}{\pi} \int t_{1(2)}(\vec{\xi}) d^2\xi = \frac{m_D}{m_{1(2)}}\ f_{p(n)}(0)$$

and $f_{p(n)}(0)$ is the amplitude of the proton (neutron)-proton zero-angle elastic coherent scattering.

The expression (70) can be rewritten as

$$f(0) = f_1(0) + f_2(0) + \frac{2ik}{\pi} \int t_1(\vec{\xi})\ t_2(\vec{\eta}) \left|\varphi\left(\vec{\xi} - \vec{\eta}, z\right)\right|^2\ d^2\xi\ d^2\eta\ dz \tag{71}$$

Then from (71)

$$\Re \ f(0) = \Re \ f_1(0) + \Re \ f_2(0) - \frac{2k}{\pi} \Im \int t_1(\vec{\xi}) t_2(\vec{\eta}) \left| \varphi\left(\vec{\xi} - \vec{\eta}, z\right) \right|^2 \ d^2\xi \ d^2\eta \ dz \tag{72}$$

$$\Im \ f(0) = \Im \ f_1(0) + \Im \ f_2(0) + + \frac{2k}{\pi} \Re \int t_1(\vec{\xi}) t_2(\vec{\eta}) \left| \varphi\left(\vec{\xi} - \vec{\eta}, z\right) \right|^2 \ d^2\xi \ d^2\eta \ dz$$

The spin oscillation period is determined by the difference in the amplitudes $\Re \ f(m = \pm 1)$ and $\Re \ f(m = 0)$. From (69) it follows that

$$\Re \ d_1 = -\frac{2k}{\pi} \Im \int t_1(\vec{\xi}) t_2(\vec{\eta}) \left[\varphi_{\pm1}^{+}\left(\vec{\xi} - \vec{\eta}, z\right) \varphi_{\pm1}\left(\vec{\xi} - \vec{\eta}, z\right) - \varphi_0^{+}\left(\vec{\xi} - \vec{\eta}, z\right) \varphi_0\left(\vec{\xi} - \vec{\eta}, z\right) \right] \ d^2\xi \ d^2\eta \ dz$$

$$\Im \ d_1 = \frac{2k}{\pi} \Re \int t_1(\vec{\xi}) t_2(\vec{\eta}) \left[\varphi_{\pm1}^{+}\left(\vec{\xi} - \vec{\eta}, z\right) \varphi_{\pm1}\left(\vec{\xi} - \vec{\eta}, z\right) - \varphi_0^{+}\left(\vec{\xi} - \vec{\eta}, z\right) \varphi_0\left(\vec{\xi} - \vec{\eta}, z\right) \right] \ d^2\xi \ d^2\eta \ dz$$

It is well known that the characteristic radius of the deuteron is large comparing with the range of nuclear forces. For this reason, when integrating, the functions t_1 and t_2 act on φ as δ-function. Then

$$\Re \ d_1 = -\frac{4k}{\pi} \Im f_1(0) \ f_2(0) \int_0^\infty \left[\varphi_{\pm1}^{+}(0, z) \varphi_{\pm1}(0, z) - \varphi_0^{+}(0, z) \varphi_0(0, z) \right] \ dz \tag{73}$$

$$\Im \ d_1 = \frac{4k}{\pi} \Re f_1(0) \ f_2(0) \int_0^\infty \left[\varphi_{\pm1}^{+}(0, z) \varphi_{\pm1}(0, z) - \varphi_0^{+}(0, z) \varphi_0(0, z) \right] \ dz$$

The magnitude of the spin oscillation effect is determined by the difference

$$\left[\varphi_{\pm1}^{+}(0, z) \varphi_{\pm1}(0, z) - \varphi_0^{+}(0, z) \varphi_0(0, z) \right]$$

i.e. by the difference in distributions of nucleon density in the deuteron for different deuteron spin orientations. The structure of the wavefunction $\varphi_{\pm1}$ is well known:

$$\varphi_m = \frac{1}{4\pi} \left\{ \frac{u(r)}{r} + \frac{1}{\sqrt{8}} \frac{W(r)}{r} \hat{S}_{12} \right\} \chi_m \tag{74}$$

where $u(r)$ is the deuteron radial wavefunction corresponding to the S-wave; $W(r)$ is the radial function corresponding to the D-wave; the operator $\hat{S}_{12} = 6(\hat{\vec{S}} \vec{n}_r)^2 - 2\hat{\vec{S}}^2$; $\vec{n}_r = \frac{\vec{r}}{r}$; $\hat{\vec{S}} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ and $\vec{\sigma}_{1(2)}$ ate the Pauli spin matrices describing proton(neutron) spin.

Use of (74) yields

$$\Re \ d_1 = -\frac{3}{2\pi k} \Im \{f_1(0) f_2(0)\} \, \mathrm{D} = -\frac{\sigma}{\pi k} \Im \{f_p(0) f_n(0)\} \, \mathrm{D} \tag{75}$$

$$\Im \ d_1 = \frac{3}{2\pi k} \Re \{f_1(0) f_2(0)\} \, \mathrm{D} = \frac{\sigma}{\pi k} \Re \{f_p(0) f_n(0)\} \, \mathrm{D}$$

where $\mathrm{D} = \int_0^\infty \left(\frac{1}{\sqrt{2}} \frac{u(r) W(r)}{r^2} - \frac{1}{4} \frac{W^2(r)}{r^2} \right) dr$, $r^2 = \xi^2 + z^2$. Applying the optical theorem $\Im \ f = \frac{k}{4\pi} \ \sigma$, where σ is the total scattering cross-section, one can obtain

$$\Re \ d_1 = -\frac{3}{2\pi^2} \left(\Re \ f_p(0) \sigma_n + \Re \ f_n(0) \sigma_p \right) \mathrm{D} \tag{76}$$

$$\Im \ d_1 = \frac{3}{2\pi k} \left(\Re \ f_1 \ \Re \ f_2 - \frac{k^2}{(4\pi)^2} \sigma_1 \sigma_2 \right) \mathrm{D} = \left(\frac{6}{\pi k} \Re \ f_p \ \Re \ f_n - \frac{3k}{(2\pi)^3} \sigma_p \sigma_n \right) \mathrm{D} \tag{77}$$

where $\sigma_{p(n)}$ is the total cross-section of the proton-proton (neutron-proton) nuclear scattering.

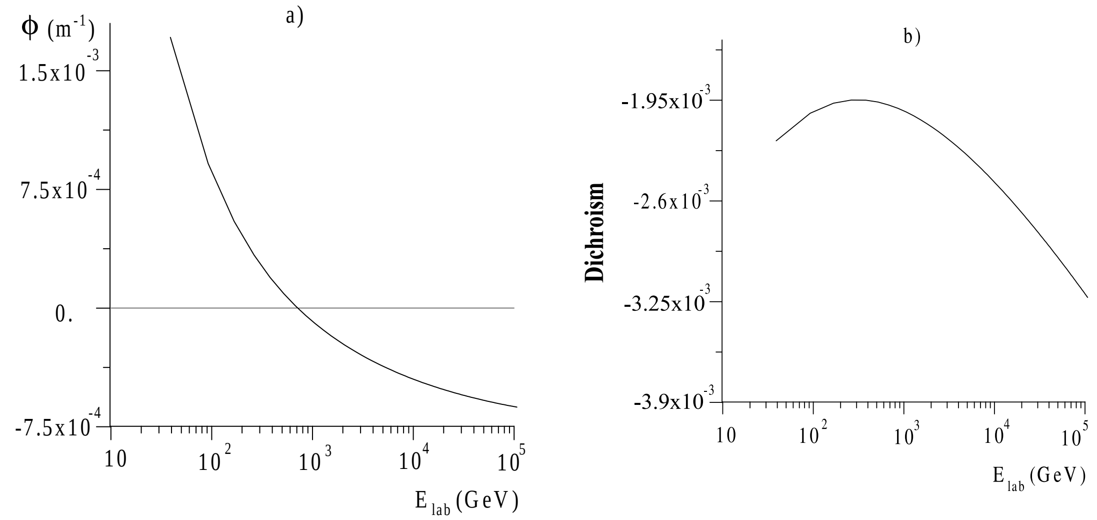

Figure 13: Rotation angle and dichroism in hydrogen target at high energies

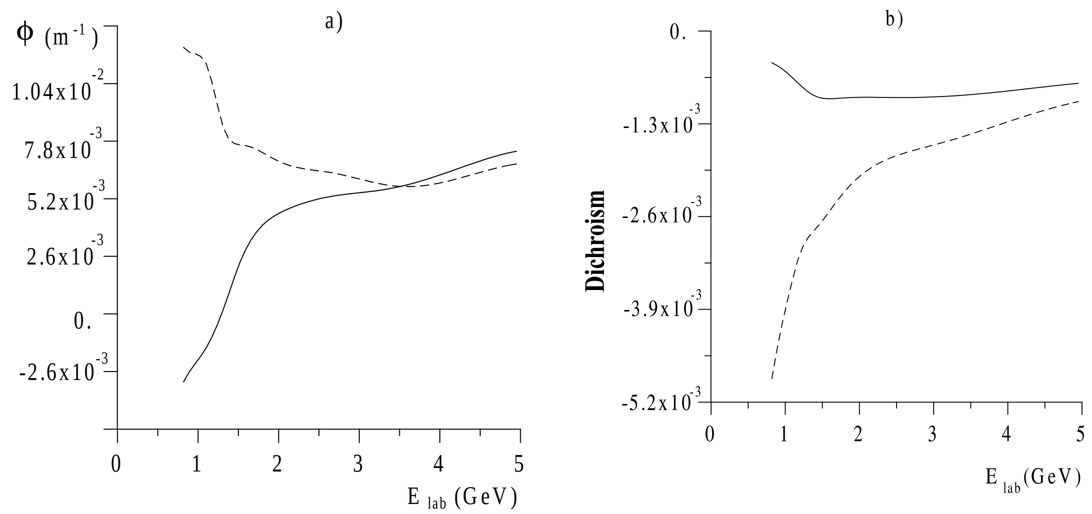

Figure 14: Spin rotation angle and dichroism in hydrogen target at deuteron energies lower than 5 Gev. Solid curve corresponds to the calculation with the spinless $N - N$ amplitudes

References

[1] V. Baryshevsky, Phys. Lett. 171 A (1992) 431.

[2] V. Baryshevsky, J. Phys. G 19 (1993) 273.

[3] V. Baryshevsky, A. Rouba, R. Engels, F. Rathmann, H. Seyfarth, H. Ströher, T. Ullrich, C. Düweke, R. Emmerich, A. Imig, J. Ley, H. Paetz gen. Schieck, R. Schulze, G. Tenckhoff, C. Weske, M. Mikirtytchiants, A. Vassiliev, LANL e-print archive: hep-ph/0501045.

[4] V. Baryshevsky, LANL e-print archive, hep-ph/0109099v1, hep-ph/0201202.

[5] V. Baryshevsky, F. Rathmann, Oscillation and rotation of deuteron spin in polarized and unpolarized targets, COSY summer school, Vol.1 (2002).

[6] F. Farley et al., Phys. Rev. Lett., 93 (2004) N 5.

[7] Deuteron EDM Proposal, http://www.iucf.indiana.edu/ $\sim dedm/deuteron\_proposal\_040816.pdf$.

[8] V. Baryshevsky, D. Baryshevsky, J. Phys. B 27 (1994) 3277.

[9] M. Lax, Rev.Mod.Phys. **23** (1951) 287.

[10] M.L. Goldberger and K.M. Watson, Collision Theory: John Wiley, New York, 1964.

[11] Berestetsky V., Liphshits Y., Pitaevsky Y., Relativistic quantum theory, P-1, M., 1968.

[12] Baryshevsky V., Shekhtman G., Phys. Rev., C 53 (1996) 267.

[13] W. Cryz, L.C. Maximon Ann.Phys., NY **52** (1969) 59.

[14] Handbuch der Physik **39** (Berlin: Springer, 1957) 112.

[15] V. Baryshevsky, A. Shirvel, LANL e-print archive: hep-ph/0503214.

[16] V. Baryshevsky, K. Batrakov, S. Cherkas, LANL e-print archive: hep-ph/9907464.