Laser in axial electric field as a tool to search for P-, T- invariance violation

V G Baryshevsky, S L Cherkas and D N Matsukevich‡

Institute for Nuclear Problems, 11 Bobruiskaya str., 220050 Minsk, Belarus

Abstract. We consider rotation of polarization plane of the laser light when a gas laser is placed in a longitudinal electric field (10 kV/cm). It is shown that residual anisotropy of the laser cavity 10^{-6} and the sensitivity to the angle of polarization plane rotation about $10^{-11} - 10^{-12}$ *rad* allows one to measure an electron EDM with the sensitivity about 10^{-30} $e \times cm$.

PACS numbers: 33.55.Ad, 11.30.Er, 42.55.Ah

‡ Present address: School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430.

1. Introduction

The standard model predicts the dipole moment of the electron at a level of about 10^{-40} $e\times cm$ while some variants of supersymmetric models forecast 10^{-30} $e\times cm$ [1, 2, 3]. Since the supersymmetry is an important ingredient of modern physics it would be desirable to achieve the sensitivity of measurements enabling us to test these predictions. The present limit on the electron EDM is $d_e < 1.6 \times 10^{-27}$ [4].

The measurement of an angle of the polarization plane rotation of light when it propagates through a gas immersed in an electric field is one of the possible ways of searching the electric dipole moment of an electron. The interaction of an electric field with the electron dipole moment leads to the splitting of the atomic levels, analogous to the Zeeman effect and, consequently, to the polarization plane rotation (similar to Faraday effect) when a photon propagates along the electric field direction.

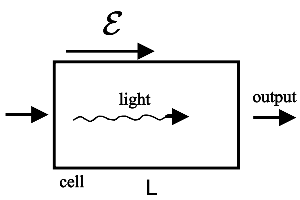

Figure 1. Scheme of the transmission experiment.

It was shown in [5, 6, 7] that in addition to the atomic level splitting one more mechanism leading to the light polarization plane rotation exists. It is the interference of the Stark and P-T- invariance violating transition amplitudes.

In a typical transmittance experiment (Fig. 1) with a gas cell the intensity of a light beam decreases when it propagates in a medium. This restricts the length available for polarization plane rotation measurements [8]. An idea to use a photon trap (resonator) with an amplifier (Fig. 2) to compensate light absorption was proposed in [7]. Because the trap contains exited medium, the amplification cancels the losses and light can stay in the resonator for a long time.

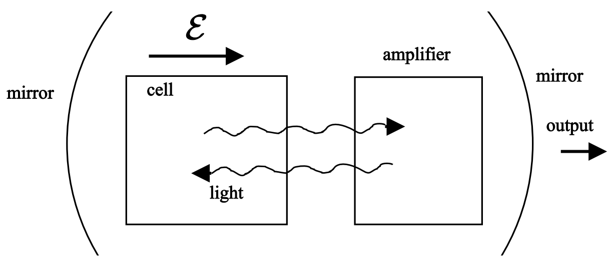

Figure 2. A gas cell with an amplifier [7] for observation of the P- T- odd polarization plane rotation.

The simplest type of the trap is a laser placed in the electric field (Fig. 3). This

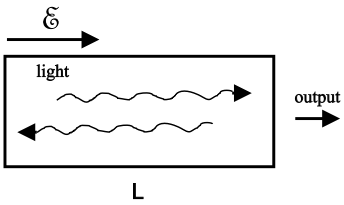

Figure 3. Laser placed in the external electric field.

system is similar to the laser in the magnetic field considered in 70th [9, 10, 11, 12, 13] and recently as a footing of the laser magnetometers [14, 15].

The main difficulty for laser magnetometery is linear anisotropy of losses of the resonator [14, 15]. The same difficulty is deferred on the measurements of P-T- noninvariant rotation in the electric field, because the linear anisotropy of losses of the resonator is much greater than the circular anisotropy created by the electric field provided that P- and T- invariance breaks. It should be remembered that in the absence of the linear anisotropy of losses an angle between the polarization plane of the laser radiation and some axis increases linearly with time, and the angular velocity of the polarization plane rotation is proportional to the P- T- odd polarizability of the atoms and strength of the electric field. Large linear anisotropy forbids polarization plane rotation. When the electric field turns on, the polarization plane rotates for a small angle and then stops. Only this angle should be measured. Below we give a detailed theoretical analysis of such a photon trap for measurements of the P-T- noninvariance.

2. The angle of polarization plane rotation in the electric field

Let us consider a stationary electromagnetic wave in a resonator containing an exited medium possessing linear and circular anisotropy. The effect of polarization plane rotation can be described by the P- T- noninvariant term $n_{PT}^{ij} \sim e^{ijk}\mathcal{E}^k$ in the tensor of the refractive index of the medium [16, 17], where e^{ijk} is the completely antisymmetric tensor and $\boldsymbol{\mathcal{E}}$ is the external electric field. Let the mirrors of the resonator be perpendicular to the z-axis, and the external electric field $\boldsymbol{\mathcal{E}}$ be directed along it.

It is convenient to choose the reference frame providing for the matrix of anisotropic losses of the resonator to be diagonal. Thus the tensor part of the generalized refractive index (see Appendix), which includes the resonator, has the form:

$$\Delta\hat{n}' = \begin{pmatrix} -d - i\chi & b + ia \\ b - ia & d + i\chi \end{pmatrix}, \qquad (1)$$

where χ, a describe linear anisotropy of losses and circular phase anisotropy correspondingly. Linear phase anisotropies are given by b and d. Prime marks the refractive index of the exited medium to distinguish it from that for the medium in the ground state. The refractive index acts as matrix in the space of the vectors $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$, where E_x and E_y are the components of the electric field of the electromagnetic wave.

Linear anisotropy of losses $\chi = \frac{1}{4}\left(\frac{1}{Q_y} - \frac{1}{Q_x}\right) \approx \frac{1}{4}\frac{\Delta Q}{Q^2}$, where $\Delta Q = Q_x - Q_y$ (see Appendix). Here Q_x and Q_y are the finesse of the resonator for the light polarized along the x and y axes, respectively. In principle, very low anisotropy of losses can be achieved. For instance, in the experiments [14, 15] the quantity $\frac{\Delta Q}{Q} = \frac{Q_x - Q_y}{Q} \sim 10^{-5}$. In our estimates we use the value of 10^{-6} in a hope that the progress in the technology of unstressed materials and the resonator design will allow it to be reached.

Circular phase anisotropy $a = \frac{1}{2}\Delta n'_{PT} = \frac{1}{2}(n'_+ - n'_-)$ is produced by the electric field if P- T- invariance is violated. Linear phase anisotropies $d = \frac{1}{2}(n'_y - n'_x)$ and $b = \frac{1}{2}(n'_{135^o} - n'_{45^o})$, where n'_x, n'_y, n'_{135^o}, n'_{45^o} are the refractive indexes for the wave polarized along the x-axis and y-axis, at the angle 135^o and at the angle 45^o (relative to the x-axis), respectively. If one rotates the reference frame at 45^o, the quantity b takes up positions on the diagonal of the refractive index matrix. However more convenient is the circular basis $\begin{pmatrix} E_+ \\ E_- \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -E_x - iE_y \\ E_x - iE_y \end{pmatrix}$ in which the refractive index takes the form:

$$\hat{\Delta n'} = \begin{pmatrix} a & d - ib + i\chi \\ d + ib + i\chi & -a \end{pmatrix}. \tag{2}$$

Measurements should be performed by the analyzer placed perpendicularly to the polarization plane of the laser beam (when the electric field is turned off). The orientation of the polarization plane of the laser light is determined by residual anisotropy of losses in the resonator and coincides with one of the main axes i.e. it is directed along the vector $\boldsymbol{h} = \{\chi, 0, 0\}$.

As the electric field is turned on, the circular anisotropy a depending linearly on the electric field appears and the polarization plane rotates by the angle ϕ_{las}, which can be found from the Eq. (29) given in appendix. This equation describes stationary polarization of light. The angle ϕ_{las} is the half of the angle between vector $\boldsymbol{O}$ at $a = 0$, when there is no electric field and that at some a produced by the turned on electric field. In a typical experimental situation $a, b, d \ll \chi$. Vectors $\boldsymbol{h}$ and $\boldsymbol{r}$ in (29) are $\boldsymbol{h} = \{\chi, 0, 0\}$ (we omit circular dichroism due to an electric field for simplicity) and $\boldsymbol{r} = \{d, b, a\}$. The stationary solution for the vector $\boldsymbol{O}$ describing polarization state of the standing electromagnetic wave according to (29) has the form:

$$\boldsymbol{O}_0 = \left(1, \frac{a}{\chi} + \frac{bd}{\chi^2}, \frac{ad}{\chi^2} - \frac{b}{\chi}\right) \tag{3}$$

in the first order on a, b, d.

Eq. (3) gives the angle of polarization plane rotation

$$\phi_{las} \approx \frac{1}{2}\frac{O_{0y}}{O_{0x}} \approx \frac{a}{2\chi} \approx \Delta n'_{PT}\frac{Q^2}{\Delta Q}. \tag{4}$$

As we can see from (3) the additional parasitic angle $\frac{bd}{2\chi^2}$ appears. However, this angle can not depend linearly on the electric field. One more additional angle (so-called "base angle" [8]) arises due to inexact perpendicular orientation of the analyzer with respect to the polarization plane of the laser beam.

The conventional experimental method implies modulation of the "base angle" with the frequency Ω by the additional Faraday element placed between laser and analyzer. The signal at the output of analyzer is proportional to the squared sum of the P-T-violating angle of rotation and the "base angle". The presence of Ω component in the Fourier transform of the output signal is the signature of P-, T- invariance violation.

3. Estimates for the "trap" and "transmittance" layouts

Let us estimate the advantage of the laser experiment compared to the light transmission experiment using a cell (Fig. 3).

The P-, T- violating refractive index does not depend on the type of the atomic transition, i.e. it is approximately the same for the dipole electric, magnetic and strongly forbidden magnetic transitions [6]. In the transmittance experiments with a cell the angle of rotation is usually measured for two absorption lengths, thus, it is reasonable to choose transitions with the greatest absorption length. These are magnetic dipole and strongly forbidden magnetic transitions. The angle of polarization plane rotation in a cell for the length L is equal to (see [5, 6, 7, 8]):

$$\phi = \frac{1}{2}\Delta n_{PT} k L, \tag{5}$$

where k is the wave number. As we have mentioned, Δn_{PT} in the equation (5) differs from $\Delta n'_{PT}$ in the expression (4). The first quantity corresponds to a medium in the ground state and transitions happen from the ground level to the exited one whereas, in the case of the laser medium, the transitions happen from a top level to the bottom. These quantities are connected to each other by the relation

$$\Delta n'_{PT} = \frac{\Delta N}{N}\Delta n_{PT}, \tag{6}$$

where N is the concentration of atoms, and ΔN is the density of inversely populated atoms. Substituting $\Delta n'_{PT}$ from (6) to the equation (4) one obtaines:

$$\phi_{las} \sim \frac{\Delta N}{N}\Delta n_{PT}\frac{Q^2}{\Delta Q} \sim \Delta n_{PT} k \frac{1}{N\sigma}\frac{Q}{\Delta Q}. \tag{7}$$

In the derivation of the latter equation we have taken into account the condition of laser operation

$$\Delta N \sigma = \frac{k}{Q}, \tag{8}$$

where σ is the absorption cross section for this transition. From the equations (7) and (5) we can see that the angle of polarization rotation in the laser is equal to the angle of rotation at the absorption length $L_{abs} = \frac{1}{N\sigma}$ multiplied by $\frac{2Q}{\Delta Q}$. Thus one expect to obtain $\frac{2Q}{\Delta Q} \sim 2 \times 10^6$ enhancement in comparison with a layout, using a cell. Let us remind that the experiment with a cell uses the strongly forbidden magnetic transition (i.e. transition between shells with different main quantum numbers), therefore the real gain will only appear if the laser also operates at transitions of this type. In the laser operating at ordinary electric dipole transitions, a very low inversion of population is required to compensate absorptions in the resonator, because σ in (8) is large. The real part of the refraction index is also proportional to the inversion of population and additional suppression given by (6) arises. Thus, the most of the gas lasers, using $E1$ transitions are unsuitable as a trap for measuring P-T- violation. Although there are no lasers working at a strongly forbidden magnetic transition, lasers working at a magnetic dipole transition do exists. One of such example, namely, chemical iodine laser will be considered below.

The P- T- odd refractive index can be expressed in terms of the P- T-odd polarizability β_{PT} of an atom:

$$\Delta n_{PT} = -4\pi N \beta_{PT}. \tag{9}$$

Two mechanisms contributing to β_{PT} were considered in [5, 6, 7]. The first one is the interference of Stark and P-T- odd transition amplitudes. The value of β_{PT} in this case can be estimated as:

$$\beta_{PT}^{mix} \sim \sum_{m,n} \frac{< g|H_T|m >< m|d^j|c >< c|d^j|n >< n|\boldsymbol{d\mathcal{E}}|g >}{(\varepsilon_m - \varepsilon_c)(\varepsilon_g - \varepsilon_c + \omega + i\Gamma/2)(\varepsilon_n - \varepsilon_g)}, \tag{10}$$

where ω is the laser working frequency, corresponding to the resonator own frequency; m, n are some intermediate atomic levels, $\varepsilon_g, \varepsilon_c, \varepsilon_n, \varepsilon_m$ are the energies of the levels, d^j are the components of the operator $\boldsymbol{d}$ of the atom dipole moment (summation on j is implied in (10) and further), H_T is the operator of P- T-violating interaction. We assume that $c \to g$, is the laser working transition, and g is the ground state.

The dependence of polarizability on frequency is given by a multiplier $\frac{1}{\omega - \omega_0 + i\Gamma/2}$, where $\omega_0 = \varepsilon_c - \varepsilon_g$ is the frequency of transition and Γ should be read as denoting the recoil line width. To take into account Doppler broadening in a gas we should average the multiplier over the Maxwell distribution of atom velocities. According to ref. [8] this reduces to:

$$\langle \frac{1}{\omega - \omega_0 + i\Gamma/2} \rangle \Rightarrow \frac{1}{\Delta_D} \left(g(u,v) - i f(u,v) \right), \tag{11}$$

where $\Delta_D = \frac{\omega}{c}\sqrt{\frac{2k_b T}{m}}$ there is the Doppler line width, c is speed of light, m is an atom mass, k_b is the Boltzman constant, T is the temperature, $v = \frac{\Gamma}{2\Delta_D}$, $u = \frac{\omega - \omega_0}{\Delta_D}$ is the detuning and

$$\left. \begin{array}{c} g(u,v) \\ f(u,v) \end{array} \right\} = \left. \begin{array}{c} \mathrm{Im} \\ \mathrm{Re} \end{array} \right\} \sqrt{\pi} e^{-w^2} (1 - \Phi(-iw)),$$

$w = u + iv$, $\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z dt e^{-t^2}$.

Using the above averaging in (10) we come to the estimate:

$$\beta_{PT}^{mix} \sim \frac{< d >^3 \mathcal{E}_z < H_T >}{(\Delta\varepsilon)^2} \frac{g(u,v)}{\Delta_D}, \tag{12}$$

where $< d >$ is the typical value of the matrix element from the operator of atom dipole moment , $\Delta\varepsilon \sim Ry$ (Ry is Rydberg constant) is the typical value of difference in atomic levels energies, $\mathcal{E}_z$ is the longitudinal component of $\boldsymbol{\mathcal{E}}$. The above estimate of β^{PT}_{mix} is valid for all kinds of atomic transitions [6]: electric dipole, magnetic dipole and strongly forbidden magnetic dipole.

The second mechanism suggests that the P-T- odd polarizability is produced by the atomic levels splitting in the electric field due to the atomic EDM. This leads to the estimates [6]:

$$\beta^{edm}_{PT} \sim \sum_{m,n} \frac{< g|\boldsymbol{d\mathcal{E}}|m >< m|d^j|c >< c|d^j|n >< n|\boldsymbol{d\mathcal{E}}|g >}{(\varepsilon_m - \varepsilon_c)(\varepsilon_n - \varepsilon_g)} \times \boldsymbol{d}_{at}\boldsymbol{\mathcal{E}} \frac{\partial}{\partial \omega} \frac{1}{\varepsilon_g - \varepsilon_c + \omega + i\Gamma/2} \quad (13)$$

for the strongly forbidden magnetic transition and

$$\beta^{edm}_{PT} \sim < g|\mu^j|c >< c|\mu^j|g > \boldsymbol{d}_{at}\boldsymbol{\mathcal{E}} \frac{\partial}{\partial \omega} \frac{1}{\varepsilon_g - \varepsilon_c + \omega + i\Gamma/2} \quad (14)$$

for the magnetic dipole transition. Here μ^j are the components of the operator of atom magnetic moment, d_{at} is the dipole moment of the atom, which can be estimated as $d_{at} \sim \frac{<d><H_T>}{\Delta\varepsilon}$. Averaging (14) we obtain

$$\beta^{edm}_{PT} \sim \frac{< d >^4 \mathcal{E}_z^3 d_{at}}{(\Delta\varepsilon)^2} \frac{1}{\Delta_D^2} \frac{\partial g(u,v)}{\partial u} \sim \frac{< d >^5 \mathcal{E}_z^3}{(\Delta\varepsilon)^3} \frac{< H_T >}{\Delta_D^2} \frac{\partial g(u,v)}{\partial u} \quad (15)$$

for the strongly forbidden transition and

$$\beta^{edm}_{PT} \sim < \mu >^2 d_{at}\mathcal{E}_z \frac{1}{\Delta_D^2} \frac{\partial g(u,v)}{\partial u} \sim \alpha^2 \frac{< d >^3 \mathcal{E}_z < H_T >}{\Delta\varepsilon\, \Delta_D^2} \frac{\partial g(u,v)}{\partial u} \quad (16)$$

for the magnetic dipole transition, where $\alpha = \frac{e^2}{\hbar c}$ [18] is the fine structure constant, $< \mu > \sim \alpha < d >$, $< d > \sim e\, a_0$, a_0 is Bohr radius.

Sources of P- T- violation are the electron EDM, EDM of the nucleons, and the P-T-odd electron-nucleon interaction [6]. For definiteness we consider only the first one. The matrix element of P-T- odd interaction between atomic states can be estimated as $< H_T > \sim 150 \frac{d_e}{<d>} \Delta\varepsilon$ [6]. This implies that the atom EDM is of the order of $d_{at} \sim < d > \frac{<H_T>}{\Delta\varepsilon} \sim 150 d_e$. The above estimation takes into account the Shiff theorem stating that in nonrelativistic quantum mechanic atom EDM should be zero and only relativistic effects allows it to appear [8]. Relativistic effects are given by the multiplier $\mathcal{V}^2/c^2 \sim Z^2\alpha^2$ [8], where $\mathcal{V}$ is the typical electron velocity in atom, Z is the atomic number. However, the Shiff theorem does not concern the "mixing" mechanism, because an atom EDM does not appear in it. Thus we have to use two different $< H_T >$ to describe "mixing" and "splitting" mechanisms. Unfortunately, in Ref. [6] we used the single $< H_T >$ with relativistic suppression for both the mechanisms. In this paper will take $< H_T > \sim 150 \frac{d_e}{<d>} \Delta\varepsilon$ for the "splitting" mechanism and $< H_T > \sim \frac{150}{Z^2\alpha^2} \frac{d_e}{<d>} \Delta\varepsilon$ for the "mixing" mechanism.

The absorption cross section is given by

$$\sigma = \pi A \frac{c^2}{\omega^2} \frac{f(u,v)}{\Delta_D}, \quad (17)$$

where A is the Einstein coefficient of a given transition. The angle of polarization plane rotation at one absorption length $\frac{1}{N\sigma}$ equals

$$\phi(L_{abs}) = \frac{2\pi\,\omega\beta_{PT}}{c\,\sigma}. \tag{18}$$

We are coming now to the concrete systems.

3.1. Iodin laser, operating on M1 transition

For lack of laser on strongly forbidden magnetic transition we consider chemical atomic iodine gas laser employing ${}^2P_{3/2} \to {}^2P_{1/2}$ ($\lambda = 1.315\ \mu m$) magnetic transition [19, 20], for witch $A = 7.7\ c^{-1}$. According to eq. (12) P-T-odd polarizability for "mixing" mechanism can be expressed as $\beta_{PT}^{mix} = B_{PT}^{mix} \frac{g[u,v]}{\Delta_D}$, where B_{PT}^{mix} is determined only by the atom properties and strength of external electric field but not the detuning and line broadening. For iodin atom the above estimates give $B_{PT}^{mix} = 4.5 \times 10^{-33}\ cm^3 s^{-1}$ at $\mathcal{E} = 10^4\ V/cm$. The angle of polarization plane rotation at one absorbtion length and that for the laser system $\phi_{las} = \phi(L_{abs})\frac{2\,Q}{\Delta Q}$ are given in Table 1.

Two first lines are associated with the top-table chemical iodine lasers [21, 22, 23, 24] using chemical excitation of the Iodin atoms by singlet oxygen:

$$I + O_2({}^1\Delta) \leftrightarrows I^* + O_2({}^3\Sigma),$$

where singlet oxygen is generated outside the laser and is injected into the laser cavity together with the iodine. For our case a design of reagent injecting and output have to be as possible as axially symmetric to avoid transverse anisotropy of an active medium.

Typically lasers of that type works at temperature $60 - 80\ C^o$ (this promise low thermal drifts during measurements), pressure of singlet oxygen $p \approx 1\ torr$, and pressure of the iodine $p_{[I^*]} \approx 10^{-2}p$. Almost all Iodine atoms are in the exited state, because equilibrium in the above chemical reaction is strongly accented to the right due to excess of $O_2({}^1\Delta)$. Thus, density of I^* is $N_{[I^*]} = 2.7 \times 10^{14}\ cm^{-3}$. Recoil line width at $p = 1\ torr$, and radius of the iodine atom $r_{[I]} = 0.136\ nm$ is estimated as $\Gamma/(2\pi) = 16 p\, r_{[I]}^2/\sqrt{\pi m\, k_b T} = 0.7\ MHz$, while the Doppler line width is $2\sqrt{\ln 2}\,\Delta_D/(2\pi) = 0.27\ GHz$.

Lasing condition can be satisfied, for instance, at laser cavity length $L = 50\ cm$ and two identical mirrors of reflectivity $R = \exp(-\sigma\, N_{[I^*]}\, L) = 0.9$.

Increasing of the detuning improves the signal (two last lines in a Table 1) because it increases the ratio $f(u,v)/g(u,v)$. However, the cross-section decreases. To satisfy lasing condition one have to increase exited iodine atom density (one may increases of the mirror reflectivity instead, but, as it will be discussed below, the more reflectivity the more difficult to reach small $\Delta Q/Q$). For the Iodine atom density $N_{[I^*]} = 10^{16}\ cm^{-3}$ (pressure is $p_{[I*]} = 0.37\ torr$) estimate of recoil line width give: $\Gamma/(2\pi) = 25\ MHz$ (Δ_D is the same as above). Lasing condition is satisfied with the mirror reflectivity $R = \exp(-\sigma\, N_{[I^*]}\, L) = 0.9$. This demand high pressure $p \sim 37\ torr$ generators of singlet oxygen, discussed in [25].

Table 1. P-T- odd polarizability, angle of polarization plane rotation at an absorbtion length, and that for iodine laser, calculated for the "splitting"(atom EDM) and "mixing" mechanisms at $d_e = 10^{-30}$, $\mathcal{E} = 10^4\ V/cm$ and $Q/\Delta Q = 10^6$.

mech.	u	v	$\sigma,\ cm^2$	$\beta_{PT},\ cm^3$	$\phi(L_{abs}), rad$	ϕ_{las}, rad
mix.	1	0.0021	6.7×10^{-18}	4.7×10^{-42}	2.1×10^{-19}	4.2×10^{-13}
split.	1	0.0021	6.7×10^{-18}	7.8×10^{-43}	3.4×10^{-20}	6.9×10^{-14}
mix.	2.5	0.078	2.2×10^{-19}	1.95×10^{-42}	2.7×10^{-18}	5.4×10^{-12}
split.	2.5	0.078	2.2×10^{-19}	1.2×10^{-42}	1.6×10^{-18}	3.2×10^{-12}

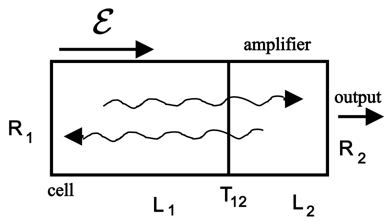

Figure 4. Simplified scheme of the two section system.

3.2. Two section system with the cesium vapor cell

It does be desirable to realize an advantage of a strongly forbidden transition. For this aim one can use two section system (Fig. 4), consisting of a cell with the cesium vapor in the electric field and an amplifier. Let the length of the cell is L_1 and the length of the amplifier is L_2. Circular anisotropy takes place only in the cell, therefore the average P- T- odd refractive index of system can be written down as $\Delta n_{PT}\frac{L_1}{L_1+L_2}$. The substance of the cell is absorptive so the total absorption is written down similarly (23) (see Appendix) as:

$$\frac{1}{2Q} = -\frac{\ln(R_1 T_{12}^2 R_2 e^{-2\,L_1/L_{abs}})}{4\,k(L_1+L_2)} = -\frac{\ln(R_1 T_{12}^2 R_2)}{4\,k(L_1+L_2)} + \frac{L_1}{2\,k\,L_{abs}(L_1+L_2)}, \tag{19}$$

where T_{12} is the transmittance of the wall between the amplifier and the cell. The first item $\frac{1}{2\,Q_0} = -\frac{\ln(R_1 T_{12}^2 R_2)}{4\,k(L_1+L_2)}$ in (19) describes losses of the empty resonator.

The angle of the polarization plane rotation in such two-section system is

$$\phi_{2\,las} \sim \Delta n_{PT}\frac{L_1}{L_1+L_2}\left(\frac{1}{Q_0} + \frac{1}{k\,L_{abs}}\frac{L_1}{L_1+L_2}\right)^{-2}\frac{1}{\Delta Q_0}, \tag{20}$$

where ΔQ_0 describes the linear anisotropy of losses of the resonator. Corresponding lasing condition can be written as

$$2\kappa\,L_2 - 2L_1/L_{abs} = -\ln(R_1 R_2 T_{12}^2), \tag{21}$$

where κ is pass gain constant of the amplifier and gives $Q_0 = \frac{k(L_1+L_2)}{\kappa\,L_2 - L_1/L_{abs}}$.

Now one may use the strongly forbidden magnetic transition of the cesium atom $6S_{1/2} \to 7S_{1/2}$. Let us suggest sodium vapor containing molecules Na_2 as an amplifier. Lasing line $539.4 \pm 0.1\ nm$ of transitions $B^1\,\Pi_u \to X^1\,\Sigma_g^+$ between vibrotational levels

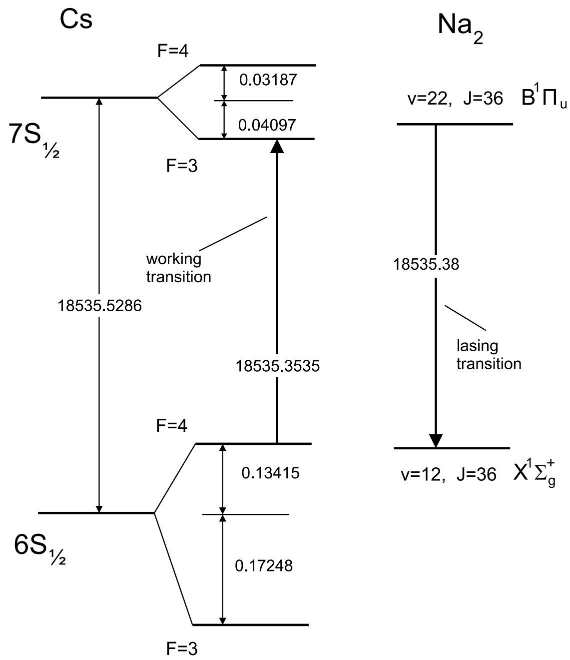

Figure 5. Energies of Cesium working transition and Na_2 lasing transition, cm^{-1}.

of Na_2 molecule were found [26] among other lasing lines under pumping 472.7 nm by Argon laser. The working cavity contained sodium vapor at 820 K, $p = 11$ $torr$ and buffer gas (Argon) with partial pressure $p_{[Ar]} = 8$ $torr$ [26]. Dimer pressure was $p_{[Na_2]} \approx 0.05\,p$, corresponding to the density of molecules $N_{[Na_2]} \approx 10^{16}$ cm^{-3}. Pass gain $\kappa = 0.1$ cm^{-1} was achieved. More accurately energy of this lasing transition were measured by sodium vapor spectroscopy [27] to be 18535.38 cm^{-1} (the same pumping was used).

The hyperfine structure of $6S_{1/2} - 7S_{1/2}$ cesium transition can be obtained by compilation data of [28, 29, 30] and is shown in Fig. 5. Nearest energy to that of lasing line has the transition between hyperfine components $F = 4$ and $F = 3$.

The pressure of cesium vapor at the temperature $T = 820$ K is 27 kPa (205 torr) and Cs atom density is $N = 2.4 \times 10^{18}$ cm^{-3}. Recoil line width estimated for cesium atom radius $r_{[Cs]} = 262$ nm is $\Gamma/(2\pi) = 0.33$ GHz. Doppler line width is $2\sqrt{\ln 2}\Delta_D/(2\pi) = 1$ GHz. Thus parameters $v = 0.28$ and detuning $u = \frac{\omega_{[Cs]} - \omega}{\Delta_D} = 1.36$, where ω is a frequency of Na_2 lasing transition and $\omega_{[Cs]}$ is that for cesium working transition (Fig. 5). Einstein coefficient of the Cs transition $6S_{1/2} \to 7S_{1/2}$ in the external electric field 10^4 kV/cm is $A = 0.034$ s^{-1}. Absorbtion length is $L_{abs} = 5$ m. Taking $L_1 = 30$ cm, $L_2 = 10$ cm, according lasing condition (21) we find $Q_0 = 5 \times 10^6$, which can be realized with the two identical mirrors of reflectivity $R = \exp(-\frac{Q_0}{k(L_1+L_2)}) = 0.34$, where T_{12} is set to unity for simplicity. According to the [6] the constant $B_{PT}^{mix} = 5.8 \times 10^{-34}$ $cm^3 s^{-1}$ at $d_e = 10^{-30}$ $e\,cm$ and external electric field $\mathcal{E} = 10$ kV/cm. After removing relativistic suppression multiplier $Z^2\alpha^2 = 0.16$ we have the quantity $B_{PT}^{mix} = 3.6 \times 10^{-33}$ $cm^3 s^{-1}$, which gives P-T- violating polarizability $\beta_{PT}^{mix} = B_{PT}^{mix}\, g(u,v)/\Delta_D = 7.1 \times 10^{-43}$ cm^3.

Accordingly, the angle of polarization plane rotation in the two-section system is $\phi_{2\,las} = 7 \times 10^{-11}\ rad$ at $\Delta Q_0 = 10^{-6} Q_0$. However, the more is the number of borders the more difficult is to achieve low residual anisotropy of the system. How to avoid an additional border is discussed below.

3.3. Mixture of two gases

In principle one may use mixture of gases§ in a single laser cavity. Concerning to the previous example this means that the sodium laser have to work with the 205 *torr* cesium vapor instead of 8 *torr* Argon buffer gas. This can occur if the quenching of the exited state of Na_2 molecule by cesium will not be strong and should be checked experimentally (certainly one may diminish cesium density to make laser functional). For the pass gain $\kappa = 0.1\ cm^{-1}$, and $L = 40\ cm$ the lasing condition is satisfied with the mirrors reflectivity $R = \exp(-\kappa\, L) = 0.02$ showing that we have a reserve if the gain will be smaller due to quenching.

According to the eq. (4), (9), and previous subsection estimates for the cesium atom density and linewidthes we have $\phi_{las} = 2.5 \times 10^{-11}\ rad$.

Note that at present time sensitivity $10^{-8} rad/\sqrt{Hz}$ for measurements of polarization rotation angle is achieved [31]. At accumulation time 10^6 s (11.5 days) this gives 10^{-11} rad.

Let us to do some remarks about residual anisotropy of resonator. Suppose that the resonator consists of two identical mirrors and anisotropy is created by the anisotropy of the mirror reflectivity ΔR. Thus we have $\frac{\Delta Q}{Q} = -\frac{1}{\ln R}\frac{\Delta R}{R}$. This expression tells us that the more reflectivity of the mirrors, the more $\Delta Q/Q$ at the same $\Delta R/R$. According to the expression $\phi_{las} = \Delta n_{PT} Q^2/\Delta Q$ polarization plane rotation for the laser with mixture of a gases can be rewritten in terms of $\Delta R/R$ as $\phi_{las} = \Delta n_{PT}\, k\, L \frac{R}{\Delta R}$, and for the case of two section system we have form (20): $\phi_{2las} = \Delta n_{PT}\, k\, L_1 \frac{R}{\Delta R}$ when $k\, L_{abs} >> Q_0$. The above consideration shows, that because the working size of the resonator L and L_1 are not be increased considerably due to presence of a strong electric field the only possibility to increase the effect is to lower anisotropy of loses ΔR of the mirrors. Let us remind that the design of pumping should be done axially symmetric.

4. Conclusion

We have considered P-T- odd rotation of the polarization plane of the laser in the axial electric field to obtain new constraint to the P-T- odd interactions. The main problems have to be solved are the measurement of the tiny angle of polarization plane rotation and producing the resonators with small linear anisotropy of losses. Let us lay down some ways to this aim.

Concerning to the resonator, it can be made with movable mirrors included to the self-consistent scheme of measurements to suppress linear anisotropy of losses by fine

§ The idea was suggested one of the referees of the article.

tuning.

Then, to lower mean anisotropy of the mirror the technology can be developed of producing the mirror itself as consisting from the small sub-mirrors with chaotic orientation.

Then, it is desirable to modulate the external electric field to avoid possible systematic errors quadratic on the strength of the field.

At last, as it has be done in the laser magnetometery [15], the small external axial magnetic field can be applied to compel the polarization plane to rotate. In such a way the problem of the measurement of the small angles turns to the problem of measuring of the frequency difference of polarization plane rotation with and without electric field.

To summarize, use of photon traps with an amplifier for measurements of P-T-invariance violation with the modern high technological level of the equipment (residual anisotropy $\frac{\Delta Q}{Q} \sim 10^{-6}$, external electric field 10 kV/cm and ability to measure angles of polarization rotation $10^{-11} - 10^{-12}$ rad allows one to achieve the sensitivity for electron dipole moment measurements at the level of 10^{-30} $e \times cm$ and, thus, to test the predictions of some supersymmetric models. We hope that progress in technology of resonators and precision polarization measurements makes such experiments possible.

5. Appendix

One can obtain condition for stationary oscillation in a resonator supposing that the amplitude of the running wave after light travels back and forth in the resonator is equal to the initial amplitude. If, for example, the resonator is filled with a medium with a constant refractive index n then the amplitude of a running wave departed from some point near the mirror (Fig. 3) and returned to the original point will be equal to $\boldsymbol{E} = e^{-2iknL}\boldsymbol{E}_0$, where k is the wave number of the wave in vacuum, L is the length of the resonator. Since the initial and final amplitudes are equal to each other, $knL = m\,\pi$, where m is an integer number. Difference of the refractive index from that given by the above condition by Δn results in change of the amplitude $\Delta\boldsymbol{E} = -2i\Delta n\, k\, L\, \boldsymbol{E}$ after the full passage. Dividing the last equation by the propagation time $T \approx 2L/c$ we find:

$$\frac{d\boldsymbol{E}}{dt} = -i\Delta n\, \omega \boldsymbol{E}. \tag{22}$$

This equation may be derived also in a less heuristic way [9, 10, 11, 12, 13]. Losses corresponding to the reflectance of the resonator mirrors can be “smeared out” over the volume of the resonator through the addition of quantity:

$$\frac{i}{2\,Q} = -\frac{i}{2}\frac{\ln(R_1 R_2)}{2k\,L} \tag{23}$$

to Δn, where R_1, R_2 are the reflectance of mirrors. It is easy to see that after the full pass the amplitude of the wave is multiplied by $\sqrt{R_1 R_2}$. If there are some areas with small variations of the refractive index then we should use the average index of refraction $\Delta n = \frac{L_1\Delta n_1 + L_2\Delta n_2 + \ldots}{L_1 + L_2 + \ldots}$.

If the wave propagates along z axis then the amplitude of the electromagnetic wave contains two components $\boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$. It is convenient to use the circular basis $E_+ = -\frac{1}{\sqrt{2}}(E_x + iE_y)$, $E_- = \frac{1}{\sqrt{2}}(E_x - iE_y)$. In the general case of an anisotropic medium and resonator, Δn is a complex 2×2 matrix which can be written in the form $\hat{\Delta} n = (\mathcal{N}_0 + \boldsymbol{\sigma}\boldsymbol{\mathcal{N}})$, where $\mathcal{N}_0 = r_0 + ih_0$ is a complex number, $\boldsymbol{\mathcal{N}} = \boldsymbol{r} + i\boldsymbol{h}$ is a complex vector, $\boldsymbol{\sigma} \equiv \{\sigma_x, \sigma_y, \sigma_z\}$ are the Pauli matrices.

Let us define the density matrix $\rho_{ij}(t) = E_i(t)E_j^*(t)$ and parameterize it with the help of ξ_0 and $\boldsymbol{\xi}$ as $\rho = (\xi_0 + \boldsymbol{\sigma\xi})/2$. Taking derivatives of the density matrix and replacing derivatives of $\frac{dE_j}{dt}$ using (22) we find $i\frac{1}{\omega}\frac{d\rho}{dt} = \Delta\hat{n}\rho - \rho\Delta\hat{n}^+$, which results in

$$\frac{1}{2\omega}\frac{d\boldsymbol{\xi}}{dt} = h_0\boldsymbol{\xi} + \xi_0\boldsymbol{h} + \boldsymbol{r} \times \boldsymbol{\xi},$$
$$\frac{1}{2\omega}\frac{d\xi_0}{dt} = h_0\xi_0 + (\boldsymbol{h} \cdot \boldsymbol{\xi}). \qquad (24)$$

From the definition of a density matrix we see, that

$$\xi_x =< \sigma_x >= E_+^*E_- + E_-^*E_+,$$
$$\xi_y =< \sigma_y >= -i(E_+^*E_- - E_-^*E_+),$$
$$\xi_z =< \sigma_z >= E_+^*E_+ - E_-^*E_-,$$
$$\xi_0 =< I >= E_+^*E_+ + E_-^*E_-, \qquad (25)$$

where I denotes unit matrix. Instead of $\boldsymbol{\xi}$ and ξ_0 let's define the unit vector $\boldsymbol{O} = \frac{\boldsymbol{\xi}}{\xi_0}$. Equations (24) then take the form

$$\frac{1}{2\omega}\frac{d\boldsymbol{O}}{dt} = \boldsymbol{r} \times \boldsymbol{O} + \boldsymbol{h} - \boldsymbol{O}(\boldsymbol{h} \cdot \boldsymbol{O}). \qquad (26)$$

For entirely polarized light $|\boldsymbol{O}| = 1$ the Eq.(26) reduces to

$$\frac{1}{2\omega}\frac{d\boldsymbol{O}}{dt} = \boldsymbol{r} \times \boldsymbol{O} + \boldsymbol{O} \times (\boldsymbol{h} \times \boldsymbol{O}). \qquad (27)$$

Orientation of the polarization ellipse is described by the angle ϕ between the major axis of polarization ellipse and x [32]:

$$\mathrm{tg}\, 2\phi = \frac{E_yE_x^* + E_xE_y^*}{E_xE_x^* - E_yE_y^*} = -i\frac{E_+E_-^* - E_-E_+^*}{E_+E_-^* + E_-E_+^*} = -\frac{O_y}{O_x}.$$

Thus the angle of polarization rotation is equal to one half of the angle of the vector $\boldsymbol{O}_\perp$ rotation in a plane xy, taken with the opposite sign. For example, if $\boldsymbol{O}_\perp$ rotates on the angle $\left(-\frac{\pi}{2}\right)$, then the ellipse of polarization rotates on the angle $\frac{\pi}{4}$. Rotation of $\boldsymbol{O}_\perp$ by the angle (-2π) means rotation of the polarization ellipse on the angle π and, after this rotation the ellipse coincides with itself.

The z-component of the vector $\boldsymbol{O}$ is equal to the ellipticity of laser radiations.

In the general case $(\boldsymbol{h} \cdot \boldsymbol{r}) \neq 0$ polarization always tends to the stationary solution. The typical picture of this is shown in Fig. 6. Under constant $\boldsymbol{h}$ and $\boldsymbol{r}$ the stationary

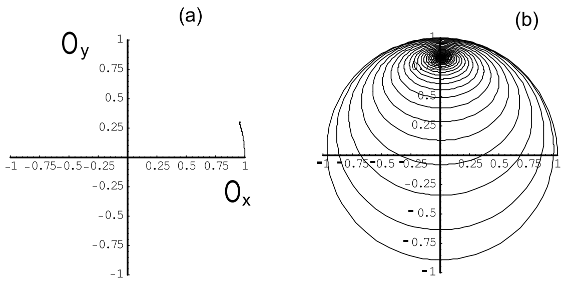

Figure 6. Evolution of polarization toward stationary solution in the case $\boldsymbol{r} = \{0.01, 0, 0.3\}, \boldsymbol{h} = \{1, 0, 0\}$,(a) and when $\boldsymbol{r} = \{0.01, 0, 1.2\}, \boldsymbol{h} = \{1, 0, 0\}$ (b)

solution of the equation (26) is written down as

$$\boldsymbol{O}_0 = \alpha\,\boldsymbol{r} + \beta\,\boldsymbol{h} + \gamma\,\boldsymbol{r} \times \boldsymbol{h},$$

$$\gamma = \frac{h^2 + r^2 - \sqrt{(h^2 + r^2)^2 - 4 \mid \boldsymbol{h} \times \boldsymbol{r} \mid^2}}{2 \mid \boldsymbol{h} \times \boldsymbol{r} \mid^2},$$

$$\alpha = \pm\sqrt{\gamma(1 - \gamma h^2)}, \quad \beta = \pm\frac{\gamma^{3/2}(\boldsymbol{r} \cdot \boldsymbol{h})}{\sqrt{1 - \gamma h^2}}. \tag{28}$$

Signs $\pm$ correspond to two various solutions. One of them is stable. In the particular case $(\boldsymbol{h} \cdot \boldsymbol{r}) = 0$ the solution (28) reduces to

$$\boldsymbol{O}_0 = \pm\frac{\sqrt{h^2 - r^2}}{h^2}\boldsymbol{h} + \frac{\boldsymbol{r} \times \boldsymbol{h}}{h^2}, \quad \mid \boldsymbol{h} \mid > \mid \boldsymbol{r} \mid; \;\; \boldsymbol{O}_0 = \pm\frac{\sqrt{r^2 - h^2}}{r^2}\boldsymbol{r} + \frac{\boldsymbol{r} \times \boldsymbol{h}}{r^2}, \quad \mid \boldsymbol{r} \mid > \mid \boldsymbol{h} \mid . \tag{29}$$

Nonlinear properties of the laser medium results in dependence of the factor h_0 on $\mid E \mid^2 = \xi_0$. In this case in addition to the equation (26) we should consider equation

$$\frac{1}{2\omega}\frac{d \ln \xi_0}{dt} = h_0 + (\boldsymbol{h} \cdot \boldsymbol{O}). \tag{30}$$

However, nonlinearity of this type has no influence on the polarization evolution.

One more manifestation of nonlinearity is so-called "self-rotation" [12] resulting in possibility of dependence:

$$\frac{i}{\omega}\frac{d\boldsymbol{E}}{dt} \sim a_{sf}(\boldsymbol{E} \cdot \boldsymbol{E})\boldsymbol{E}^*. \tag{31}$$

For E_- and E_+ we have:

$$\frac{i}{\omega}\frac{dE_-}{dt} \sim \frac{i}{\omega\sqrt{2}}\left(\frac{dE_x}{dt} - i\frac{E_y}{dt}\right) \sim \frac{a_{sf}}{\sqrt{2}}E^2(E_x^* - iE_y^*) \sim -a_{sf}E^2E_+^* \sim 2a_{sf} \mid E_+ \mid^2 E_-,$$

$$\frac{i}{\omega}\frac{dE_+}{dt} \sim -\frac{i}{\omega\sqrt{2}}\left(\frac{dE_x}{dt} + i\frac{E_y}{dt}\right) \sim -\frac{a_{sf}}{\sqrt{2}}E^2(E_x^* + iE_y^*) \sim -a_{sf}E^2E_-^* \sim 2a_{sf} \mid E_- \mid^2 E_+,$$

(we use here the expression $E^2 = -2E_-E_+$). Taking into account that $\mid E_+ \mid^2 = \frac{\xi_0}{2}(1 + O_z)$ and $\mid E_- \mid^2 = \frac{\xi_0}{2}(1 - O_z)$ we can see the terms $r_z \sim -\mathrm{Re}\, a_{sf}\xi_0 O_z$, $h_z \sim -\mathrm{Im}\, a_{sf}\xi_0 O_z$ appearing in the z-components of $\boldsymbol{r}$ and $\boldsymbol{h}$. This results in rotation of the plane of polarization even in the absence of circular anisotropy. However, rotation does not depends on external electric field and can be distinguished from the P-T-noninvariant effects by modulation of the external electric field, moreover, the ellipticity O_z of the laser light is extremely low.

References

[1] Ellis J R, Hisano J, Raidal M and Shimizu Y 2002 *Phys. Lett.* B **528** 86
[2] Masina I 2003 *Nucl. Phys.* B **671** 432
[3] Farzan Y and Peskin M E 2004 *Phys. Rev.* D **70** 095001
[4] B. C. Regan *et al.* 2002 *Phys. Rev. Lett.* **88** 071805-4
[5] Baryshevsky V G 1999 *Phys. Lett.* A **260** 24
[6] Baryshevsky V G and Matsukevich D N 2002, *Phys. Rev.* A **66** 062110
[7] Baryshevsky V G 1999 *Actual Problems of Particle Physics, proceedings of International School-Seminar (Gomel)* vol 2 (Dubna: JINR) p 93
[8] Khriplovich I B 1991 *Parity nonconservation in atomic phenomena* (London: Gordon & Breach)
[9] Lamb W E 1964 *Phys. Rev.* A **134** 1429
[10] D'Yakonov M I 1966 *Zh. Eksp. Theor. Fiz* **49** 1169 [*Sov. Phys.–JETP* **22** 812]
[11] Haeringen W V 1967 *Phys. Rev.* **158** 256
[12] Alekseev A I and Galitskii V M 1969 *Zh. Eksp. Theor. Fiz.* **57** 1002 [*Sov. Phys.–JETP* **30** 548 (1970)]
[13] Voitovich A P 1984 *Magnetooptics of gas lasers*, (Minsk: Nauka i Technika) [in Russian]
[14] Bretenaker F, Lepine B, Cotteverte J-C and Le Floch A 1992 *Phys. Rev. Lett.* **69** 909
[15] Emil O, Poison J, Bretenaker F and Le Floch A 1998 *J. Appl. Phys.* **83** 4994
[16] Baranova N B, Bogdanov Yu V and Zel'dovich B Ya 1977 *Usp. Fiz. Nauk* **123** 349 [*Sov. Phys. Usp.* **20** 870]
[17] Sushkov O P and Flambaum V V 1978 *Zh. Eksp. Teor. Fiz.* **72** 1208 [*Sov. Phys.–JETP* **48** 608]
[18] Berestetskii V B, Lifshitz E M and Pitaevskii L P 1982 *Quantum electrodynamics* (Oxford: Pergamon Press, Oxford)
[19] Hohla K and Kompa K L 1976 *Handbook of chemical lasers* ed R W F Gross and J F Bott (New York: Wiley)
[20] Brederlow G, Fill E and Witte K J 1983 *The high-power iodine laser* (Heidelberg: Springer-Verlafg)
[21] Bernard D J, McDermott W E, Pchelkin N R and Bousek R R 1979 *Appl.Phys.Lett.* **34** 40
[22] Richardson R J and Wiswall C E 1979 *Appl.Phys.Lett.* **35** 138
[23] Vagin N P *et al* 1984 *Kvantovaya Elektronika* **11** 1688 [*Sov. J. Quantum Electr.* **14** 1138]
[24] Churassy S, Bouvier A J, Bouvier A, Erba B and Setra M 1994 *J. Physique III* **4** 2013
[25] Zagidullin M V and Nikolaev V D 1999 *Proc. SPIE vol 3688, p 54, 6th International Conference on Industrial Lasers and Laser Applications '98*
[26] Wellegehausen B, Shahdin S, Friede D and Welling H 1977 *Appl. Phys.* **13** 97
[27] Camacho J J et al 1998 *J. Mol. Spectrosc.* **191** 248
[28] Arditi M and Carver T R 1958 *Phys. Rev.* **112** 449
[29] Gilbert S L, Watts R N and Wieman C E 1983 *Phys. Rev.* A **27** 581
[30] Weber K-H and Sansonetti C J 1987 *Phys. Rev.* A **35** 4650
[31] Cameron R *et al.* 1993 *Phys. Rev.* D **47** 3707
[32] Landay L D and Lifshits E M 1982 *Field Theory*, (Oxford: Pergamon Press)