About influence of the deuteron electric and magnetic plarizabities on measurement of the deuteron EDM in a storage ring

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Abstract

In the present paper influence of tensor electric and magnetic polarizabilities on spin evolution in the resonance deuteron EDM experiment is considered in details.

It is shown that besides EDM the electric and magnetic polarizabilities also contribute to the vertical spin component P_3. Moreover, the electric polarizability contributes to the P_3 component even when the deuteron EDM is supposed to be zero and thereby the electric polarizability can imitate the EDM contribution. It is shown that unlike the vertical component of the spin P_3 the component P_{33} of polarization tensor does not contain contribution from the electric polarizability, whereas contribution from the magnetic polarizability reveals only when the deuteron EDM differs from zero.

Moreover, it is also shown that when the angle ϑ between the spin direction and the vertical axis meets the condition $\sin\vartheta = \sqrt{\frac{2}{3}}$ ($\cos\vartheta = \sqrt{\frac{1}{3}}$), the initial value of P_{33} appears $P_{33}(0) = 0$. As a result, EDM contribution to the measured signal linearly growth in time starting from zero that is important for measurements.

Therefore, measurement of the P_{33} component of deuteron tensor polarization seems to be of particular interest, especially because the nonzero component P_{33} appearance on its own indicates the EDM presence (in contrast to the P_3 component, which appearance can be aroused by the tensor electric polarizability, rather than EDM).

1 INTRODUCTION

At present time the possibility to measure the electric dipole moment of a deuteron moving in a storage ring is actively discussed [1, 2]. According to [1, 2] two types of experiments are discussing now: a). deuteron energy is chosen to zeroize $(g-2)$ precession (g is the gyromagnetic ratio) b). deuteron beam velocity is modulated with the frequency Ω_f close to the $(g-2)$ precession frequency Ω, as a result, this makes possible to observe the EDM signal as growth in time of the vertical spin component [1, 2].

These methods can provide for EDM measurement the sensitivity $\sim 10^{-29}$ $e \cdot cm$. Theoretical description of the experiment [1, 2] is being done on the base of the Bargman-Myshel-Telegdy equation. But as it is shown in [3]-[5] the Bargman-Myshel-Telegdy equation can not describes deuteron spin behavior in such experiments. It turns out that the BMT equation for a deuteron should be supplemented with several additions, which describe interaction of deuteron electric and magnetic polarizabilities with the electric field in the storage ring and deuteron birefringence in matter.

Owing to the above in the experiments planned for the deuteron EDM search the contributions aroused by the tensor electric and magnetic polarizabilities of deuteron as well as the spin-dependent amplitude of forward scattering by the nuclei of a solid or gas target will be also measured. These contributions could distort the EDM signal and even bring to wrong conclusions about EDM observation. They are the systematic errors for the EDM search, which should be eliminated.

At the same time the above effects being measured in the experiments for EDM search could be even used for more reliable limits assignment for deuteron EDM.

In the present paper influence of tensor electric and magnetic polarizabilities on spin evolution in the resonance deuteron EDM experiment is considered in details.

It is shown that besides EDM the electric and magnetic polarizabilities also contribute to the vertical spin component P_3. Moreover, the electric polarizability contributes to the P_3 component even when the deuteron EDM is supposed to be zero and thereby the electric polarizability can imitate the EDM contribution. It is shown that unlike the vertical component of the spin P_3 the component P_{33} of polarization tensor does not contain contribution from the electric polarizability, whereas contribution from the magnetic polarizability reveals only when the deuteron EDM differs from zero.

Moreover, it is also shown that when the angle ϑ between the spin direction and the vertical axis meets the condition $\sin\vartheta = \sqrt{\frac{2}{3}}$ $(\cos\vartheta = \sqrt{\frac{1}{3}})$, the initial value of P_{33} appears $P_{33}(0) = 0$. As a result, EDM contribution to the measured signal linearly growth in time starting from zero that is important for measurements. Therefore, measurement of the P_{33} component of deuteron tensor polarization seems to be of particular interest, especially because the nonzero component P_{33} appearance on its own indicates the EDM presence (in contrast to the P_3 component, which appearance can be aroused by the tensor electric polarizability, rather than EDM).

2 Interactions contributing to the spin motion of a particle in a storage ring

As it is shown in [3]-[5] considering evolution of the spin of a particle in a storage ring, when measure EDM, one should take into account several interactions:

1. interactions of the magnetic and electric dipole moments with an electromagnetic field;
2. interaction of the particle with the electric field due to the tensor electric polarizability;
3. interaction of the particle with the magnetic field due to the tensor magnetic polarizability;
4. interaction of the particle with the pseudoelectric nuclear field of matter.

The equation for the particle spin wavefunction considering all these interactions is as follows:

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = \left(\hat{H}_0 + \hat{V}_{EDM} + \hat{V}_{\vec{E}} + \hat{V}_{\vec{B}} + \hat{V}_{E}^{nucl}\right)\Psi(t) \tag{1}$$

where $\Psi(t)$ is the particle spin wavefunction,

$\hat{H}_0$ is the Hamiltonian describing the spin behavior caused by interaction of the magnetic moment with the electromagnetic field (equation (1) with the only $\hat{H}_0$ summand converts to the Bargman-Myshel-Telegdy equation),

$\hat{V}_{EDM}$ describes interaction of the particle EDM d with the electric field,

$$\hat{V}_{EDM} = -d\left(\vec{\beta}\times\vec{B} + \vec{E}\right)\vec{S}, \tag{2}$$

$\vec{\beta} = \frac{\vec{v}}{c}$, $\vec{v}$ is the particle velocity, c is the speed of light.

$\hat{V}_{\vec{E}}$ describes interaction of the particle with the electric field due to the tensor electric polarizability:

$$\hat{V}_{\vec{E}} = -\frac{1}{2}\hat{\alpha}_{ik}(E_{eff})_i(E_{eff})_k, \tag{3}$$

where $\hat{\alpha}_{ik}$ is the electric polarizability tensor of the particle , $\vec{E}_{eff} = (\vec{E} + \vec{\beta}\times\vec{B})$ is the effective electric field; the expression (3) can be rewritten as follows:

$$\hat{V}_{\vec{E}} = \alpha_S E_{eff}^2 - \alpha_T E_{eff}^2\left(\vec{S}\vec{n}_E\right)^2,\ \vec{n}_E = \frac{\vec{E} + \vec{\beta}\times\vec{B}}{|\vec{E} + \vec{\beta}\times\vec{B}|} \tag{4}$$

where α_S is the scalar electric polarizability and α_T is the tensor electric polarizability of the particle.

A deuteron also has the magnetic polarizability which is described by the magnetic polarizability tensor $\hat{\beta}_{ik}$. Interaction of the particle with the magnetic field due to the tensor magnetic polarizability is as follows:

$$\hat{V}_{\vec{B}} = -\frac{1}{2}\hat{\beta}_{ik}(B_{eff})_i(B_{eff})_k, \tag{5}$$

where $(B_{eff})_i$ are the components of the effective magnetic field $\vec{B}_{eff} = (\vec{B} - \vec{\beta} \times \vec{E})$; $\hat{V}_{\vec{B}}$ (5) could be expressed as:

$$\hat{V}_{\vec{B}} = \beta_S B_{eff}^2 - \beta_T B_{eff}^2 \left(\vec{S}\vec{n}_B\right)^2, \; \vec{n}_B = \frac{\vec{B} - \vec{\beta} \times \vec{E}}{|\vec{B} - \vec{\beta} \times \vec{E}|}. \tag{6}$$

where β_S is the scalar magnetic polarizability and β_T is the tensor magnetic polarizability of the particle.

$\hat{V}_E^{nucl}$ describes the effective potential energy of particle interaction with the pseudoelectric field of the target.

3 The equations describing the spin evolution of a particle in a storage ring

Let us consider particles moving in a storage ring with low pressure of residual gas (10^{-10} Torr) and without targets inside the storage ring. In this case we can omit the effects caused by the interaction $\hat{V}_E^{nucl}$.

Let us consider a deuteron moving in a storage ring. According to the above analysis spin behavior of a deuteron can not be described by the Bargman-Myshel-Telegdy equation. The equations for particle spin motion in condition when the fields $\vec{E}$ and $\vec{B}$ are orthogonal to the particle velocity $\vec{v}$ were obtained in [3]-[5]. They can be written as follows:

$$\begin{cases} \frac{d\vec{P}}{dt} = \frac{e}{mc}\left[\vec{P} \times \left\{\left(a + \frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1}\right)\vec{\beta} \times \vec{E}\right\}\right] + \\ +\frac{d}{\hbar}\left[\vec{P} \times \left(\vec{E} + \vec{\beta} \times \vec{B}\right)\right] - \frac{2}{3}\frac{\alpha_T E_{eff}^2}{\hbar}[\vec{n}_E \times \vec{n}'_E] - \frac{2}{3}\frac{\beta_T B_{eff}^2}{\hbar}[\vec{n}_B \times \vec{n}'_B], \\ \\ \frac{dP_{ik}}{dt} = -\left(\varepsilon_{jkr}P_{ij}\Omega_r(d) + \varepsilon_{jir}P_{kj}\Omega_r(d)\right) - \\ -\frac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar}\left([\vec{n}_E \times \vec{P}]_i n_{E,k} + n_{E,i}[\vec{n}_E \times \vec{P}]_k\right) - \\ -\frac{3}{2}\frac{\beta_T B_{eff}^2}{\hbar}\left([\vec{n}_B \times \vec{P}]_i n_{B,k} + n_{B,i}[\vec{n}_B \times \vec{P}]_k\right), \end{cases} \tag{7}$$

where m is the mass of the particle, e is its charge, $\vec{P}$ is the spin polarization vector, P_ik is the spin polarization tensor, $P_{xx} + P_{yy} + P_{zz} = 0$, γ is the Lorentz-factor, $\vec{\beta} = \vec{v}/c$, $\vec{v}$ is the particle velocity, $a = (g-2)/2$, g is the gyromagnetic ratio, $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields in the point of particle location, $\vec{E}_{eff} = (\vec{E} + \vec{\beta} \times \vec{B})$, $\vec{B}_{eff} = (\vec{B} - \vec{\beta} \times \vec{E})$, $\vec{n}_E = \frac{\vec{E}+\vec{\beta}\times\vec{B}}{|\vec{E}+\vec{\beta}\times\vec{B}|}$, $\vec{n}_B = \frac{\vec{B}-\vec{\beta}\times\vec{E}}{|\vec{B}-\vec{\beta}\times\vec{E}|}$, $n'_{E,i} = P_{ik}n_{E,k}$, $n'_{Bi} = P_{il}n_{Bl}$, $\Omega_r(d)$ are the components of the vector $\vec{\Omega}(d)$ ($r = 1, 2, 3$ correspond to x, y, z, respectively).

$$\begin{aligned} &\vec{\Omega}(d) = \vec{\Omega} + \vec{\Omega}_d, \\ &\vec{\Omega} = \frac{e}{mc}\left\{\left(a + \frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1}\right)\vec{\beta} \times \vec{E}\right\}, \\ &\vec{\Omega}_d = \frac{d}{\hbar}\left(\vec{E} + \vec{\beta} \times \vec{B}\right). \end{aligned}$$

The equations for particle spin motion (7) can be rewritten as follows:

$$\frac{d\vec{P}}{dt} = [\vec{P} \times \vec{\Omega}(d)] + \Omega_T[\vec{n}_E \times \vec{n}'_E] + \Omega_T^{\mu}[\vec{n}_B \times \vec{n}'_B],$$

$$\frac{d\vec{P_{ik}}}{dt} = -(\epsilon_{jkr}P_{ij}\Omega_r(d) + \epsilon_{jir}P_{kj}\Omega_r(d)) + \Omega'_T([\vec{n}_E \times \vec{P}]_i n_{Ek} + n_{Ei}[\vec{n}_E \times \vec{P}]_k) +$$

$$+\Omega_T'^{\mu}([\vec{n}_B \times \vec{P}]_i n_{Bk} + n_{Bi}[\vec{n}_B \times \vec{P}]_k) \tag{8}$$

where

$$\Omega_T = -\tfrac{2}{3}\frac{\alpha_T E_{eff}^2}{\hbar}, \quad \Omega'_T = -\tfrac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar}, \quad \Omega'_T = -\tfrac{2}{3}\Omega_T,$$

$$\Omega_T^{\mu} = -\tfrac{2}{3}\frac{\beta_T B_{eff}^2}{\hbar}, \quad \Omega_T'^{\mu} = -\tfrac{3}{2}\frac{\beta_T B_{eff}^2}{\hbar}, \quad \Omega_T'^{\mu} = -\tfrac{2}{3}\Omega_T^{\mu}.$$

Suppose that the external electric field in the storage ring $\vec{E} = 0$ and a particle moves along the circle orbit.

Let us now consider the equation (8) in the coordinate system that rotates with the frequency of particle velocity rotation. In such a system spin rotates with respect to the momentum with the frequency determined by $(g-2)$.The coordinate system and vectors $\vec{v}, \vec{E}, \vec{B}$ as shown in figure and denote the axes by x, y, z (or $1, 2, 3$, respectively).

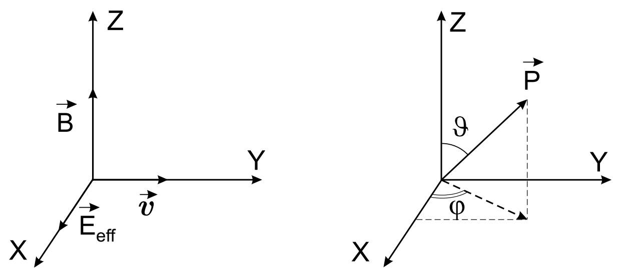

Figure 1:

Therefore, the components of the vectors are:

$$\vec{P} = (P_1, P_2, P_3)\,,$$

$$\vec{n}_E = (1, 0, 0)\,, n'_{Ei} = P_{il}n_{El} = P_{i1}$$

$$[\vec{n}_E \times \vec{n}'_E]_1 = 0,\ [\vec{n}_E \times \vec{n}'_E]_2 = -P_{31},\ [\vec{n}_E \times \vec{n}'_E]_3 = P_2, \tag{9}$$

$$[\vec{P} \times \vec{\Omega}]_1 = \Omega P_2,\ [\vec{P} \times \vec{\Omega}]_2 = -\Omega P_1,\ [\vec{P} \times \vec{\Omega}]_3 = P_2,$$

$$[\vec{n}_E \times \vec{P}]_1 = 0,\ [\vec{n}_E \times \vec{P}]_2 = -P_3,\ [\vec{n}_E \times \vec{P}]_3 = P_2,$$

$$\vec{\Omega} = \tfrac{ea}{mc}\vec{B} = (0, 0, \Omega)\,,$$

$$\vec{n}_B = (0, 0, 1)\,, n'_{Bi} = P_{il}n_{Bl} = P_{i3}$$

$$[\vec{n}_B \times \vec{n}'_B]_1 = -P_{23},\ [\vec{n}_B \times \vec{n}'_B]_2 = -P_{13},\ [\vec{n}_B \times \vec{n}'_B]_3 = 0, \tag{10}$$

$$[\vec{n}_B \times \vec{P}]_1 = -P_2,\ [\vec{n}_B \times \vec{P}]_2 = P_1,\ [\vec{n}_B \times \vec{P}]_3 = 0.$$

Substituting (9,10) to the system (7) we obtain:

$$\begin{aligned} \frac{dP_1}{dt} &= \Omega P_2 - \Omega_T^{\mu} P_{23}, \\ \frac{dP_2}{dt} &= -\Omega P_1 + (\Omega_T^{\mu} - \Omega_T) P_{13} + \omega_d P_3, \\ \frac{dP_3}{dt} &= \Omega_T P_{12} - \omega_d P_2, \end{aligned} \tag{11}$$

where $\omega_d = \frac{dE_1^{eff}}{\hbar}$.

$$\begin{aligned} \frac{dP_{11}}{dt} &= 2\Omega P_{12} + 2\omega_d P_{23}, \\ \frac{dP_{22}}{dt} &= -2\Omega P_{12}, \\ \frac{dP_{33}}{dt} &= -2\omega_d P_{23}, \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{dP_{12}}{dt} &= -\Omega\left(P_{11} - P_{22}\right) - \Omega_T' P_3 + \omega_d P_{13}, \\ \frac{dP_{13}}{dt} &= \Omega P_{23} + \Omega_T' P_2 - \Omega_T'^{\mu} P_2 - \omega_d P_{12}, \\ \frac{dP_{23}}{dt} &= -\Omega P_{13} + \Omega_T'^{\mu} P_1 - \omega_d (P_{22} - P_{33}). \end{aligned} \tag{13}$$

Remember that $P_{11} + P_{22} + P_{33} = 0$ and $P_{ik} = P_{ki}$.

4 Contribution from the EDM and tensor polarizabilities to deuteron spin oscillation

Let us consider the system (11-13) more attentively.

Suppose that deuteron has neither EDM no tensor electric polarizability: in this case the system (11-13) can be expressed:

$$\begin{aligned} \frac{dP_1}{dt} &= \Omega P_2, \\ \frac{dP_2}{dt} &= -\Omega P_1, \\ \frac{dP_3}{dt} &= 0, \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{dP_{11}}{dt} &= 2\Omega P_{12}, \\ \frac{dP_{22}}{dt} &= -2\Omega P_{12}, \\ \frac{dP_{33}}{dt} &= 0, \end{aligned} \tag{15}$$

$$\begin{aligned} \frac{dP_{12}}{dt} &= -\Omega\left(P_{11} - P_{22}\right), \\ \frac{dP_{13}}{dt} &= \Omega P_{23}, \\ \frac{dP_{23}}{dt} &= -\Omega P_{13}. \end{aligned} \tag{16}$$

This is the conventional system of BMT equations that describes particle spin rotation with the frequency equal to $(g-2)$ precession frequency $\Omega = \frac{ea}{mc}B$. The component P_3 of vector polarization in this conditions is equal to constant ($\frac{dP_3}{dt} = 0$) along with the component P_{33} of tensor polarization ($\frac{dP_{33}}{dt} = 0$).

Suppose the deuteron EDM differs from zero. Then the above system of equations converts to:

$$\begin{aligned} \frac{dP_1}{dt} &= \Omega P_2, \\ \frac{dP_2}{dt} &= -\Omega P_1 + \omega_d P_3, \\ \frac{dP_3}{dt} &= -\omega_d P_2, \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{dP_{11}}{dt} &= 2\Omega P_{12} + 2\omega_d P_{23}, \\ \frac{dP_{22}}{dt} &= -2\Omega P_{12}, \\ \frac{dP_{33}}{dt} &= -2\omega_d P_{23}, \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{dP_{12}}{dt} &= -\Omega\left(P_{11} - P_{22}\right) + \omega_d P_{13}, \\ \frac{dP_{13}}{dt} &= \Omega P_{23} - \omega_d P_{12}, \\ \frac{dP_{23}}{dt} &= -\Omega P_{13} - \omega_d (P_{22} - P_{33}). \end{aligned} \tag{19}$$

From (17-19) it follows that presence of the nonzero EDM makes the vertical component P_3 of vector polarization oscillating with the frequency of $(g-2)$ precession Ω.

According to the idea [1] these oscillations can be eliminated if the deuteron velocity is modulated with the frequency Ω:

$$v = v_0 + \delta v \sin\left(\Omega t + \varphi_f\right) \tag{20}$$

here φ_f is the phase of forced oscillations of the velocity.

As E_{eff} depends on $\vec{\beta} = \vec{v}/c$ it also appears modulated:

$$E_{eff} = E^0_{eff} + \delta E_{eff} \sin\left(\Omega t + \varphi_f\right) \tag{21}$$

Therefore $\omega_d = \frac{dE_{eff}}{\hbar}$ is also modulated with the same frequency. This makes the product $\omega_d P_2 \sim \sin^2(\Omega t + \varphi_f)$. Therefore averaging this value over the period of $(g-2)$ precession gives the result time-independent (i.e. $\frac{dP_3}{dt} = const$) and $P_3(t) = P_3(0) + const \cdot t$. For better measurement conditions it is important to make $P_3(0) = 0$. This is the reason to chose particle spin to be in the horizontal plane.

All the above reasoning makes $\frac{dP_{33}}{dt} \sim const$, too. Therefore, P_{33} also linearly grows with time $P_{33}(t) = P_{33}(0) + const \cdot t$. However, if the spin lays in the horizontal plane $P_{33}(0) \neq 0$.

It is important to note (see below the section 4.2) that if the spin orientation corresponds to $\cos^2\vartheta = \frac{1}{3}$ ($\cos\vartheta = \sqrt{\frac{1}{3}}$, $\sin\vartheta = \sqrt{\frac{2}{3}}$) then the component $P_{33}(0) = 0$, while $P_3(0) \neq 0$.

Therefore taking ϑ corresponding to $\cos\vartheta = \sqrt{\frac{1}{3}}$ we can use the component P_{33} for EDM measurements, too.

Let us consider the contribution from the electric and magnetic tensor polarizabilities. Then instead the system (17-19) we should consider the system (11-13)

$$\frac{dP_1}{dt} = \Omega P_2 - \Omega_T^{\mu} P_{23},$$
$$\frac{dP_2}{dt} = -\Omega P_1 + (\Omega_T^{\mu} - \Omega_T) P_{13} + \omega_d P_3, \qquad (22)$$
$$\frac{dP_3}{dt} = \Omega_T P_{12} - \omega_d P_2,$$

$$\frac{dP_{11}}{dt} = 2\Omega P_{12} + 2\omega_d P_{23},$$
$$\frac{dP_{22}}{dt} = -2\Omega P_{12}, \qquad (23)$$
$$\frac{dP_{33}}{dt} = -2\omega_d P_{23},$$

$$\frac{dP_{12}}{dt} = -\Omega\,(P_{11} - P_{22}) - \Omega_T' P_3 + \omega_d P_{13},$$
$$\frac{dP_{13}}{dt} = \Omega P_{23} + \Omega_T' P_2 - \Omega_T'^{\mu} P_2 - \omega_d P_{12}, \qquad (24)$$
$$\frac{dP_{23}}{dt} = -\Omega P_{13} + \Omega_T'^{\mu} P_1 - \omega_d (P_{22} - P_{33}).$$

Some interesting implications follow from (22-24). As it was already mentioned above in the experiments for EDM search it is planned to measure growth of the vertical component of the polarization vector P_3.

According to (22) time dependence of the vertical component of the vector polarization P_3 is described by the equation

$$\frac{dP_3}{dt} = \Omega_T P_{12} - \omega_d P_2 \qquad (25)$$

As it can be seen the time dependence of P_3 is determined by both the EDM and tensor polarizability of deuteron. It is interesting that the derivative of the tensor polarization component P_{33} does not contain contributions from tensor electric polarizability and is proportional to the EDM only:

$$\frac{dP_{33}}{dt} = -2\omega_d P_{23} \qquad (26)$$

Therefore, it is important to measure the component P_{33}, too. According to the above spin orientation for this case is determined by the condition $\cos^2\vartheta = \frac{1}{3}$.

4.1 Contribution from the tensor magnetic polarizability to deuteron spin oscillation

Contributions to spin rotation and oscillations from EDM and polarizabilities are small. Therefore, they, being analyzed, could be considered as perturbations to the full system (22-24) and the role of each could be studied separately.

The system of equations considering contribution from the tensor magnetic polarizability β_T is as follows:

$$\frac{dP_1}{dt} = \Omega P_2 - \Omega_T^{\mu} P_{23},$$
$$\frac{dP_2}{dt} = -\Omega P_1 + \Omega_T^{\mu} P_{13},$$
$$\frac{dP_{13}}{dt} = \Omega P_{23} - \Omega_T'^{\mu} P_2, \qquad (27)$$
$$\frac{dP_{23}}{dt} = -\Omega P_{13} + \Omega_T'^{\mu} P_1$$

Introducing new variables $P_+ = P_1 + iP_2$ and $G_+ = P_{13} + iP_{23}$ and recomposing equations (27) to determine P_+ and G_+ we obtain:

$$\frac{dP_+}{dt} = -i\Omega P_+ + i\Omega_T^\mu G_+,$$

$$\frac{dG_+}{dt} = -i\Omega G_+ + i\Omega_T'^\mu P_+,$$

or

$$i\frac{dP_+}{dt} = \Omega P_+ - \Omega_T^\mu G_+,$$

$$i\frac{dG_+}{dt} = \Omega G_+ - \Omega_T'^\mu P_+,$$

Let us search $P_+, G_+ \sim e^{i\omega t}$ then (28) transforms as follows:

$$\omega\tilde{P}_+ = \Omega\tilde{P}_+ - \Omega_T^\mu \tilde{G}_+,$$

$$\omega\tilde{G}_+ = \Omega\tilde{G}_+ - \Omega_T'^\mu \tilde{P}_+.$$

The solution of this system can be easily find:

$$(\omega - \Omega)^2 - \Omega_T^\mu \Omega_T'^\mu = 0 \tag{28}$$

that finally gives

$$\omega_{1,2} = \Omega \pm \sqrt{\Omega_T^\mu \Omega_T'^\mu} \tag{29}$$

Rewriting the solution

$$P_+(t) = c_1 e^{-i\omega_1 t} + c_2 e^{-i\omega_2 t} = |c_1| e^{-i(\omega_1 t - \delta_1)} + |c_2| e^{-i(\omega_2 t - \delta_2)} \tag{30}$$

Therefore,

$$P_1(t) = |c_1|\cos(\omega_1 t - \delta_1) + |c_2|\cos(\omega_2 t - \delta_2) \tag{31}$$

$$P_2(t) = -|c_1|\sin(\omega_1 t - \delta_1) - |c_2|\sin(\omega_2 t - \delta_2) \tag{32}$$

According to (31,32)the nonzero deuteron tensor magnetic polarizability makes the spin rotating with two frequencies ω_1 and ω_2 instead of Ω and, therefore, experiences beating with the frequency $\Delta\omega = \omega_1 - \omega_2 = 2\sqrt{\Omega_T^\mu \Omega_T'^\mu} = \frac{\beta_T B_{eff}^2}{\hbar}$.

Let us recall now that EDM interaction with the electric field makes the deuteron spin rotating around the direction of this field and leads to appearance of P_3 component proportional to $P_2(t)$

$$\frac{dP_3}{dt} \sim -\omega_d P_2 \tag{33}$$

Therefore,

$$\frac{dP_3}{dt} = \omega_d\left(|c_1|\sin(\omega_1 t - \delta_1) + |c_2|\sin(\omega_2 t - \delta_2)\right) \tag{34}$$

According to the idea [1] to measure the EDM the particle velocity ($v = v_0 + \delta v \sin(\Omega_f t + \varphi_f)$) should be modulated with the frequency Ω_f close to the frequency Ω of $(g-2)$ precession.

If the magnetic polarizability is equal to zero, then $\omega_1 = \omega_2 = \Omega$ and spin rotates in the horizontal plane with the frequency Ω. In this case velocity modulation with the same frequency $\Omega_f = \Omega$ gives

$$\frac{dP_3}{dt} \sim \sin^2(\Omega t) \tag{35}$$

and the vertical component P_3 linearly grows with time.

However, $\omega_1 \neq \omega_2$ and velocity modulation, for example, with the frequency $\Omega = \omega_1$ provides for slow spin oscillation with the frequency $\omega_1 - \omega_2$ instead of linear growth.

According to the evaluation [6] the tensor magnetic polarizability $\beta_T \sim 2 \cdot 10^{-40}$, therefore the beating frequency $\Delta\omega \sim 10^{-5}$ in the field $B \sim 10^4$ gauss.

Measurement of the frequency of this beating makes possible to measure the tensor magnetic polarizability of the deuteron (nuclei).

Thus, due to the presence of tensor magnetic polarizability the the horizontal component of spin rotates around $\vec{B}$ with two frequencies ω_1, ω_2 instead of expected rotation with the frequency Ω.

This is the reason for the component P_3 caused by the EDM to experience the similar oscillations. Therefore, particle velocity modulation with the frequency Ω provides for eliminating oscillation with Ω frequency, but P_3 oscillations with the frequency $\Delta\omega$ rest (similarly P_{33}). Study of these oscillations is necessary because they can distort the EDM measurements.

4.2 Contribution from the tensor electric polarizability to deuteron spin oscillation

Let us consider now contribution caused by the tensor electric polarizability. From the system (12) it follows

$$\begin{aligned} &\tfrac{d(P_{11}-P_{22})}{dt} = 4\Omega P_{12}, \\ &\tfrac{d^2P_{12}}{dt^2} = -\Omega\tfrac{d(P_{11}-P_{22})}{dt} - \Omega_T' \tfrac{dP_3}{dt} = -(4\Omega^2 + \Omega_T\Omega_T')P_{12}. \end{aligned} \tag{36}$$

Thus we have the equation

$$\frac{d^2P_{12}}{dt^2} + \omega_{12}^2 P_{12} = 0 \tag{37}$$

where $\omega_{12} = \sqrt{4\Omega^2 + \Omega_T\Omega_T'} \approx 2\Omega$, because $\Omega_T\Omega_T' \ll \Omega^2$.

The solution for this equation can be found in the form:

$$P_{12} = c_1 \cos\omega_{12}t + c_2 \sin\omega_{12}t \tag{38}$$

Let us find coefficients c_1 and c_2: when $t = 0$ the equation (38) gives $c_1 = P_{12}(0)$. The coefficient c_2 can be found from

$$\frac{d(P_{12})}{dt}(t \to 0) = \omega_{12}c_2, \tag{39}$$

therefore

$$c_2 = \frac{1}{\omega_{12}}\frac{d(P_{12})}{dt}(t \to 0), \tag{40}$$

From the equation (13)

$$\frac{dP_{12}}{dt}(t \to 0) = -\Omega\left(P_{11}(t \to 0) - P_{22}(t \to 0)\right), \tag{41}$$

that

$$c_2 = -\frac{P_{11} - P_{22}}{2}, \tag{42}$$

and

$$P_{12} = P_{12}(0)\cos\omega_{12}t - \frac{P_{11} - P_{22}}{2}\sin\omega_{12}t \tag{43}$$

As a result we can write the following equation for the vertical component of the spin P_3:

$$\frac{dP_3}{dt} = \Omega_T P_{12}(t) = \Omega_T[P_{12}(0)\cos 2\Omega t - \frac{P_{11}(0) - P_{22}(0)}{2}\sin 2\Omega t] \tag{44}$$

As it can be seen the vertical component of the spin oscillates with the frequency 2Ω.

But it should be reminded that according to the equations (7) interaction of the EDM with an electric field causes oscillations of the vertical component of the spin with the frequency Ω. According to the idea [1] these oscillations can be eliminated if the deuteron velocity is modulated with the frequency Ω_f that should be taken as close to the frequency Ω as possible:

$$v = v_0 + \delta v \sin(\Omega_f t + \varphi_f) \tag{45}$$

As a result E_{eff} depends on $\vec{\beta} = \vec{v}/c$ it also appears modulated:

$$E_{eff} = E^0_{eff} + \delta E_{eff}\sin(\Omega_f t + \varphi_f). \tag{46}$$

The equation describing contribution from the tensor electric polarizability and EDM to P_3 looks like (11)

$$\frac{dP_3}{dt} = \Omega_T P_{12} - \omega_d P_2. \tag{47}$$

As P_2 oscillates with the frequency Ω, then the product $\omega_d P_2$ contains the non-oscillating terms and contribution to P_3 caused by EDM linearly grows with time, if $\Omega_f = \Omega$. If $\Omega_f \neq \Omega$ contribution to P_3 caused by EDM slowly oscillates with the frequency $\Omega_f - \Omega$.

It is important that modulation of the velocity $v = v_0 + \delta v \sin(\Omega_f t + \varphi_f)$ results in oscillation of E^2_{eff} also oscillates with time and appears proportional to $\sin^2(\Omega_f t + \varphi_f)$. As a result the contribution to P_3 caused by the tensor electric polarizability can be expressed as:

$$\frac{dP_3}{dt} \sim \Delta\Omega_T \sin^2(\Omega_f t + \varphi_f)[P_{12}(0)\cos 2\Omega t - \frac{P_{11}(0) - P_{22}(0)}{2}\sin 2\Omega t] \tag{48}$$

i.e.

$$\frac{dP_3}{dt} \sim -\frac{1}{2}\Delta\Omega_T \cos(2\Omega_f t + 2\varphi_f)[P_{12}(0)\cos 2\Omega t - \frac{P_{11}(0) - P_{22}(0)}{2}\sin 2\Omega t] \tag{49}$$

According to (49) for a partially polarized deuteron beam the derivative $\frac{dP_3}{dt}$ depends on the deuteron polarization components P_{12} and $\frac{P_{11}(0)-P_{22}}{2}$

For simplicity let us consider a deuteron beam in pure polarization state. In this case the components P_{12} and $\frac{P_{11}(0)-P_{22}}{2}$ can be written using the explicit expression for the spin wavefunctions. Suppose $\vec{n}(\vartheta, \varphi)$ is the unit vector directed along the deuteron spin (ϑ and φ are the polar and azimuth angles (see Fig.1)). So the spin wavefunction that describes the deuteron spin state with the magnetic quantum number $m = 1$ can be expressed as follows (in the Cartesian basis):

$$\chi_1(\vartheta, \varphi) = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = -\frac{1}{\sqrt{2}}\begin{pmatrix} \cos\vartheta\cos\varphi - i\sin\varphi \\ \cos\vartheta\sin\varphi + i\cos\varphi \\ -\sin\vartheta \end{pmatrix} \tag{50}$$

Polarization vector can be written as

$$\vec{P} = \langle\hat{S}\rangle = \chi_1^+\hat{S}\chi_1 = i[\vec{a}\times\vec{a}^*] \tag{51}$$

and components of polarization tensor

$$\langle P_{ik} \rangle = \chi_1^+ \hat{Q}_{ik} \chi_1 = -\frac{3}{2}\{a_i a_k^* + a_k a_i^* - \frac{2}{3}\}, \tag{52}$$

where $\hat{Q}_{ik}$ is the spin tensor of rank two. Therefore,

$$P_{12} = \frac{3}{4} \sin 2\varphi \sin^2 \vartheta, \tag{53}$$

$$\frac{P_{11} - P_{22}}{2} = \frac{3}{4} \cos 2\varphi \sin^2 \vartheta, \tag{54}$$

$$P_{33} = -\frac{3}{2}\left(\sin^2 \vartheta - \frac{2}{3}\right). \tag{55}$$

Using (49,53,54) one can obtain:

$$\frac{dP_3}{dt} \sim -\frac{3}{8}\Delta\Omega_T \sin^2 \vartheta \cos(2\Omega_f t + 2\varphi_f) \times [\sin 2\varphi \cos 2\Omega t - \cos 2\varphi \sin 2\Omega t] \tag{56}$$

From (56) it follows that $\frac{dP_3}{dt}$ slowly oscillates with the frequency $(\Omega_f - \Omega)$

In the ideal case, when $\Omega_f = \Omega$ (as it is proposed in [1] for EDM measurement) (56) converts to

$$\frac{dP_3}{dt} = -\frac{3}{8}\Delta\Omega_T \sin^2 \vartheta \cos(2\Omega t + 2\varphi_f) \sin(2\Omega t - 2\varphi) \tag{57}$$

In the general case, when the phases φ_f and φ are arbitrary, (57) contains terms that do not depend on time and, therefore, P_3 linearly grows with time like the signal from the EDM does.

It is interesting that making $\varphi_f = -\varphi$ gives

$$\frac{dP_3}{dt} \sim \cos(2\Omega t - 2\varphi) \sin(2\Omega t - 2\varphi) = \frac{1}{2} \sin(4\Omega t - 4\varphi) \tag{58}$$

that makes this contribution to P_3 quickly oscillating and depressed. But even in this ideal case it rests the contribution caused by the tensor magnetic polarizability (5), in real situation $\Omega \neq \Omega_f$, though.

Measurement of these contribution provides to measure the tensor electric polarizability.

According to the evaluations [6] $\alpha_T \sim 10^{-40}$ cm^3 , therefore for the field $E_{eff} = B \sim 10^4$ gauss the frequency $\Omega_T \sim 10^{-5}$ sec^{-1}. When considering modulation we should estimate $\Delta\Omega_T \sim \Omega_T(\frac{\delta}{v_0})^2$, then suppose $(\frac{\delta}{v_0})^2 \sim 10^{-2} - 10^{-3}$ we obtain $\Delta\Omega_T \sim 10^{-7} - 10^{-8}$ sec^{-1}, that exceeds the magnitude of ω_d for the deuteron EDM $d = 10^{-29}$ $e \cdot cm$.

5 Conclusion

In the present paper influence of tensor electric and magnetic polarizabilities on spin evolution in the resonance deuteron EDM experiment is considered in details.

It is shown that besides EDM the electric and magnetic polarizabilities also contribute to the vertical spin component P_3. Moreover, the electric polarizability contributes to the P_3 component even when the deuteron EDM is supposed to be zero and thereby the electric polarizability can imitate the EDM contribution. It is shown that unlike the vertical component of the spin P_3 the component P_{33} of polarization tensor does not contain contribution from the electric polarizability, whereas contribution from the magnetic polarizability reveals only when the deuteron EDM differs from zero.

Moreover, it is also shown that when the angle ϑ between the spin direction and the vertical axis meets the condition $\sin\vartheta = \sqrt{\frac{2}{3}}$ ($\cos\vartheta = \sqrt{\frac{1}{3}}$), the initial value of P_{33} appears $P_{33}(0) = 0$. As a result, EDM contribution to the measured signal linearly growth in time starting from zero that is important for measurements.

Therefore, measurement of the P_{33} component of deuteron tensor polarization seems to be of particular interest, especially because the nonzero component P_{33} appearance on its own indicates the EDM presence (in contrast to the P_3 component, which appearance can be aroused by the tensor electric polarizability, rather than EDM).

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