Spin rotation of polarized beams in high energy storage rings

V.G. Baryshevsky
Research Institute for Nuclear Problems, Belarusian State University,
11 Bobruyskaya Str., Minsk 220050, Belarus,
e-mail: bar@inp.minsk.by

20th June 2012

Abstract

The equations for spin evolution of a particle in a storage ring are obtained considering contributions from the tensor electric and magnetic polarizabilities of the particle along with the contributions from spin rotation and birefringence effect in polarized matter of an internal target. Study of the spin rotation and birefringence effects for a particle in a high energy storage ring provides for measurement both the spin-dependent real part of the coherent elastic zero-angle scattering amplitude and tensor electric (magnetic) polarizabilities.

1 INTRODUCTION

Investigation of spin-dependent interactions of elementary particles at high energies is a very important part of programs for scientific research at storage rings [1, 2]. Such studies are being carried with the use of polarized beams and polarized targets. Dependence of scattering cross-sections on the particle spin is the subject of much studies. The experiments for measuring the spin-dependent part of the forward scattering amplitude are in preparation now [2].

It should be mentioned that it is well known in experimental particle physics how to measure a spin-dependent cross-section. However, measuring of the spin-dependent part of the forward scattering amplitude is the complicated challenge.

It was shown in [3]-[9] that there is an unambiguous method, which makes the direct measurement of the real part of the spin-dependent forward scattering amplitude in the high energy range possible. This technique is based on the effect proton (deuteron, antiproton) beam spin rotation in a polarized nuclear target and on the deuteron birefringence effect i.e. phenomenon of deuteron spin rotation and oscillation in a nonpolarized target. This technique is based on measurement of the angle of spin rotation of a high energy proton (deuteron, antiproton) in conditions of transmission experiment.

The similar phenomenon for thermal neutrons was theoretically predicted in [10] and experimentally observed in [11]-[13] (the phenomena of nuclear precession of neutron spin in a nuclear pseudomagnetic field of a polarized target).

When considering particles moving in a storage ring one should take into account influence of electromagnetic fields, which exist in the storage ring, on behavior of the particle spin.

According to [14, 15], particle spin behavior in electromagnetic fields can be described by the Bargmann-Michel-Telegdi equation. When the particle has the non-zero electric dipole moment Bargmann-Michel-Telegdi equation should be supplied with additional terms, which describe interaction of the EDM with the electromagnetic field in the storage ring. This interaction is much weaker than conventional interaction of the particle spin with the electromagnetic field. Nevertheless, the modern experimental technique provides to measure such weak interaction of EDM with the electromagnetic field [16, 17] and to carry out experimental search of the deuteron EDM at the level $d \sim 10^{-29} e \cdot cm$.

In the present paper it is shown that modern experimental technique allows to study particle spin rotation in a polarized internal target of a storage ring and, thus, to measure the spin-dependent part of the forward elastic coherent scattering amplitude. It is shown that dynamics of the particle spin in a storage ring in the experiment with an internal target can not be completely described by the BMT equation. Moreover, it is shown that dynamics of the particle spin in a storage ring in the experiment with the accuracy necessary for the EDM search for particles with the spin $S \geq 1$ (deuteron) can not be completely described by the BMT equation, too. This is due to the particle possesses the electric and magnetic tensor polarizabilities. The equations that can be applied in this case for description of spin evolution are obtained and additional contributions in these equations are analyzed.

2 Particle spin rotation in an electromagnetic field in a storage ring

Thus, let us consider now a particle with the spin S moving in the electromagnetic field of a storage ring. Interaction of the particle magnetic moment with the electromagnetic field presenting in the storage ring leads to rotation of the particle spin with respect to the momentum direction. To describe particle spin evolution it is used the Bargmann-Michel-Telegdi equation [14, 15] as follows:

$$\frac{d\vec{P}}{dt} = [\vec{P} \times \vec{\Omega}], \tag{1}$$

where t is the time in the laboratory system,

$$\vec{\Omega} = \frac{e}{mc}\left[\left(a + \frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right], \tag{2}$$

m is the mass of the particle, e is its charge, $\vec{P}$ is the spin polarization vector in the deuteron rest frame, γ is the Lorentz-factor, $\vec{\beta} = \vec{v}/c$, $\vec{v}$ is the particle velocity, $a = (g-2)/2$, g is the gyromagnetic ratio, $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields in the point of particle location.

The parameter a for protons is $a = 1.79$ and for deuterons $a = -0.14$. As a result, for the typical field in a storage ring ($B \approx 10^2 \div 10^4$ $gauss$) the spin rotation frequency is $\Omega \approx 10^5 \div 10^7$ sec^{-1}.

If a particle possesses an intrinsic dipole moment then the additional term that describes spin rotation induced by the EDM should be added to (1) [16]

$$\frac{d\vec{P}_{edm}}{dt} = \frac{d}{\hbar}\left[\vec{P}\times\left(\vec{\beta}\times\vec{B} + \vec{E}\right)\right], \tag{3}$$

where d is the electric dipole moment of a particle.

As a result, evolution of the deuteron spin due to the magnetic and electric dipole momenta can be described by the following equation:

$$\frac{d\vec{P}}{dt} = \frac{e}{mc}\left[\vec{P}\times\left\{\left(a+\frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right\}\right] + d\left[\vec{P}\times\left(c\vec{\beta}\times\vec{B} + \vec{E}\right)\right]. \tag{4}$$

The typical frequency ω_d of precession caused by particle EDM interaction with the electromagnetic field is many orders less than Ω. In conditions of the experiments [16, 17], which are planned for the deuteron EDM search at the level $d \approx 10^{-29} e\cdot cm$, the precession frequency is $\omega_d \approx 10^{-8}\cdot 10^{-9}$ sec^{-1}. Nevertheless, according to [16, 17], the change of deuteron spin direction in the storage ring can be measured even when caused by such small ω_d.

When considering particles with the spin $S \geq 1$, the more so since the accuracy of the experiments is expected to be high, the particular attention should be focused on behavior of the particle spin in the storage ring, because it is strongly influenced by several additional interactions [9, 18, 19, 20]. Note that a particle with the spin $S \geq 1$ has the electric and magnetic polarizabilities. Particle interaction with the electromagnetic field in the storage ring $\hat{V}_{\vec{E}}$ aroused by its tensor electric polarizability can be expressed as follows (suppose that $\vec{E}$ and $\vec{B}$ are orthogonal to the particle momentum - this case is realized in the storage ring):

$$\hat{V}_{\vec{E}} = -\frac{1}{2}\hat{\alpha}_{ik}(E_{eff})_i(E_{eff})_k, \tag{5}$$

where $\hat{\alpha}_{ik}$ is the electric polarizability tensor of the particle, $\vec{E}_{eff} = (\vec{E}+\vec{\beta}\times\vec{B})$ is the effective electric field in the point of particle location. The expression (5) can be rewritten as follows:

$$\hat{V}_{\vec{E}} = \alpha_S E_{eff}^2 - \alpha_T E_{eff}^2 \left(\vec{S}\vec{n}_E\right)^2, \ \vec{n}_E = \frac{\vec{E}+\vec{\beta}\times\vec{B}}{|\vec{E}+\vec{\beta}\times\vec{B}|} \tag{6}$$

where α_S is the scalar electric polarizability and α_T is the tensor electric polarizability of the particle.

The mentioned interaction results in rotation and oscillations of the spin of a particle moving in an electric field [9, 18, 19, 20]. The frequency of this rotation ω_{TE} is determined by the tensor electric polarizability α_T. Theoretical evaluations [21] give for the deuteron polarizability $\alpha_T \sim 10^{-40}$ cm^3. Therefore, for a deuteron in a storage ring the typical energy (in the frequency units) of spin interaction with the electric field due to the tensor electric polarizability is about $\omega_{TE} \sim 10^{-5}$ in the field $E_{eff} \sim 10^4$.

A particle with the spin $S \geq 1$ also has the magnetic polarizability, which is described by the magnetic polarizability tensor $\hat{\beta}_{ik}$ and interaction of the particle with the magnetic field due to the tensor magnetic polarizability is as follows:

$$\hat{V}_{\vec{B}} = -\frac{1}{2}\hat{\beta}_{ik}(B_{eff})_i(B_{eff})_k, \tag{7}$$

where $(B_{eff})_i$ are the components of the effective magnetic field $\vec{B}_{eff} = (\vec{B}-\vec{\beta}\times\vec{E})$; the energy of interaction $\hat{V}_{\vec{B}}$ (7) can be expressed as:

$$\hat{V}_{\vec{B}} = \beta_S B_{eff}^2 - \beta_T B_{eff}^2 \left(\vec{S}\vec{n}_B\right)^2, \ \vec{n}_B = \frac{\vec{B}-\vec{\beta}\times\vec{E}}{|\vec{B}-\vec{\beta}\times\vec{E}|}. \tag{8}$$

where β_S is the scalar magnetic polarizability and β_T is the tensor magnetic polarizability of the particle.

Similarly interaction $\hat{V}_{\vec{E}}$, interaction $\hat{V}_{\vec{B}}$ arouses rotation and oscillations of the particle spin with the frequency ω_T^μ. According to calculations [21] the tensor magnetic polarizability can be evaluated as $\beta_T \sim 2\cdot10^{-40}$ cm^3. Therefore, this contribution provides rotation with the frequency $\omega_T^\mu \sim 10^{-5}$ sec^{-1} in the field $B_{eff} \sim 10^4$ $gauss$.

As it is obvious the frequencies ω_{TE} and ω_T^μ are much higher than the frequency ω_d, caused by the deuteron EDM (if it is nonzero). Therefore, the influence of the tensor polarizabilities on particle spin evolution in a storage ring should be considered with particular attention.

3 The index of refraction and effective potential energy of particles in medium.

Recall now that a storage ring can contain matter inside (gas, jet target). Presence of such a target influences on spin behavior [9, 18, 19, 20] along with the influence from the electromagnetic fields. To make understanding of target influence on spin behavior clear let us recollect the following. Close connection between the coherent elastic scattering amplitude at zero angle $f(0)$ and the refraction index of medium n has been established as a result of numerous studies (see, for example, [22, 23]):

$$n = 1 + \frac{2\pi N}{k^2} f(0) \tag{9}$$

where N is the number of particles per cm^3, k is the wave number of the particle incident on the target.

The expression (9) was derived in assumption that $n-1 \ll 1$. If $k \to 0$ then $(n-1)$ grows and expression for n has the form

$$n^2 = 1 + \frac{4\pi N}{k^2} f(0)$$

Let us consider particle refraction on the vacuum-medium boundary.

The wave number of the particle in vacuum is denoted k. The wave number of the particle in medium is $k' = kn$. As it is evident the particle momentum in vacuum $p = \hbar k$ is not equal to the particle momentum in medium. Therefore, the particle energy in vacuum $E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4}$ is not equal to the particle energy in medium $E_{med} = \sqrt{\hbar^2 k^2 n^2 c^2 + m^2 c^4}$

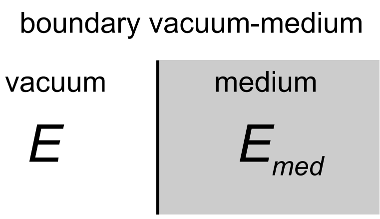

Figure 1: The particle energy in vacuum E is not equal to that in medium E_{med}.

The energy conservation law immediately necessitates to suppose that the particle in medium possesses an effective potential energy V_{eff} (see the detailed theory in [23]). This energy can be easily found from the evident equality

$$E = E_{med} + V_{eff}$$

i.e.

$$V_{eff} = E - E_{med} = -\frac{2\pi\hbar^2}{m\gamma} N f(E,0) = (2\pi)^3 N T(E), \tag{10}$$

where it is used $f(E,0) = -(2\pi)^2 \frac{E}{c^2\hbar^2} T(E) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} T(E)$, $T(E)$ is the T-matrix [23].

Above we considered the rest target. But in storage rings moving bunches can be used as a target. Therefore we should generalize the expressions (9,10) for this case. Thus, let us consider the collision of two bunches of particles. Suppose that in the rest frame of the storage ring the particles of the first beam have the energy E_1 and Lorentz-factor γ_1, whereas particles of the second beam are characterized by the energy E_2 and Lorentz-factor γ_2. Let us recollect that the phase of a wave in a medium is Lorentz-invariant. Therefore, we can find it by the following way. Let us choose the reference frame, where the second beam rests. As in this frame particles of the second beam rest, then the refraction index can be expressed in the conventional form (9):

$$n'_1 = 1 + \frac{2\pi N'_2}{{k'_1}^2} f\left(E'_1, 0\right), \tag{11}$$

where $N'_2 = \gamma_2^{-1} N_2$ is the density of the bunch 2 in its rest frame and N_2 is the density of the second bunch in the storage ring frame, k'_1, E'_1 are the wavenumber and energy of particles of the first bunch in the rest frame of the bunch 2, respectively. Suppose the length of the bunch 2 in its rest frame is L, then $L = \gamma_2\, l$, where l is the length of this bunch in the storage ring frame.

Now the change of the phase of the wave caused by the interaction of the particle 1 with the particles of bunch 2 can be found:

$$\phi = k'_1(n'_1 - 1)L = \frac{2\pi N'_2}{k'_1} f(E'_1, 0)\ L = \frac{2\pi N_2}{k'_1} f(E'_1, 0) k'_1\ l, \tag{12}$$

It is known [23]that the ratio $\frac{f(E'_1,0)}{k'_1}$ is invariant, therefore, $\frac{f(E'_1,0)}{k'_1} = \frac{f(E_1,0)}{k_1}$, where $f(E_1,0)$ is the amplitude of elastic coherent forward scattering of the particle 1 by the moving particle 2 in the rest frame of the storage ring.

As a result

$$\phi = \frac{2\pi N_2}{k_1} f(E_1,0)\ l = \frac{2\pi N_2}{k_1} f(E_1,0)\ v_{rel}\ t, \tag{13}$$

where v_{rel} is the velocity of relative motion of the particle 1 and bunch 2 (for opposing motion $v_{rel} = (v_1+v_2)(1+\frac{v_1 v_2}{c^2})^{-1}$), t is the time of interaction of the particle 1 with the bunch 2 in the rest frame of the storage ring.

The particle with the velocity $v_1 = \frac{\hbar k_1 c^2}{E_1}$ passes the distance $z = v_1\ t$ over the time t. It should be noted that the path length z differs from the length of the bunch 2, because it moves. Expression (13) can be rewritten as:

$$\phi = \frac{2\pi N_2}{k_1} f(E_1,0)\ \frac{v_{rel}}{v_1}\ z = k_1(n_1-1)z, \tag{14}$$

where the index of refraction of the particle 1 by the beam of moving particles 2 is:

$$n_1 = 1 + \frac{2\pi N_2}{{k_1}^2}\ \frac{v_{rel}}{v_1} f(E_1,0) \tag{15}$$

When $v_2 = 0$, the conventional expression (9) follows from (15).

Now the effective potential energy V_{eff} being acquired by a particle 1 when colliding with the particles of bunch 2 can be found:

$$V_{eff} = E_1 - E_{1\ med} = E_1 - \sqrt{p_1^2 c^2 n_1^2 + m_1^2 c^4} = -2\pi\hbar^2 N_2 v_{rel}\frac{f(E_1,0)}{p_1} = -2\pi\hbar^2 N_2 v_{rel}\frac{f(E'_1,0)}{p'_1}, \tag{16}$$

therefore

$$V_{eff} = -\frac{2\pi\hbar^2 N_2}{m_1\gamma_1\gamma_2} f(E'_1,0) = (2\pi)^3 N_2 T(E'_1), \tag{17}$$

where $E'_1 = m_1 c^2\gamma_1\gamma_2$ is the energy of the particle 1 in the rest frame of the bunch 2, p_1 is the particle 1 momentum in the storage ring frame, while p'_1 is the particle 1 momentum in the rest frame o the bunch 2, $p'_1 = \frac{E'_1 v'_1}{c^2}$, it is taken into account that $v'_1 = v_{rel}$.

When obtaining (16) it was used $|n_1 - 1| \ll 1$.

Let us consider now what happens when the particle possesses a spin. In this case the amplitude of the zero-angle scattering depends on the particle spin and, as a consequence, the index of refraction depends on the particle spin and can be written as:

$$\hat{n}_1 = 1 + \frac{2\pi N_2}{{k_1}^2}\ \frac{v_{rel}}{v_1}\hat{f}(E_1,0), \tag{18}$$

where $\hat{f}(E_1,0) = Tr\hat{\rho}_J\hat{F}(0)$, $\hat{\rho}_J$ is the spin density matrix of the scatterers, $\hat{F}(0)$ is the operator of the forward scattering amplitude, acting in the combined spin space of the particle and scatterer spin $\vec{J}$.

According to the above (see (10,17)) a particle possesses some effective potential energy V in matter. If the amplitude $\hat{f}(0)$ of particle scattering depends on the particle spin, then the effective energy depends on the spin orientation [3]-[9]:

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2 N_2}{m_1\gamma_1\gamma_2}\hat{f}(E'_1,0). \tag{19}$$

Thus, for example, the amplitude $\hat{f}(0)$ of the spin $S=1$ particle (for example, deuteron) scattering even in an unpolarized target depends on the particle spin and can be written as:

$$\hat{f}(E'_1,0) = d + d_1(\vec{S}\vec{n})^2. \tag{20}$$

where $\vec{S}$ is the deuteron spin operator, $\vec{n}$ is the unit vector along the deuteron momentum $\vec{k}$.

Substituting the expression for $\hat{f}(0)$ (20) in (19) one can obtain for a particle with the spin $S = 1$:

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m_1\gamma_1\gamma_2} N_2 \left(d + d_1 \left(\vec{S}\vec{n} \right)^2 \right). \tag{21}$$

Let the quantization axis z is directed along $\vec{n}$ and M denotes the magnetic quantum number. Then, for a particle in a state that is the eigenstate of the operator S_z of spin projection onto the z-axis, the effective potential energy can be written as:

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m_1\gamma_1\gamma_2} N_2 \left(d + d_1 M^2 \right). \tag{22}$$

According to (22) splitting of deuteron energy levels in matter is similar to splitting of atom energy levels in the electric field aroused from the quadratic Stark effect. Therefore, the above effect could be considered as caused by splitting of the spin levels of the particle in the pseudoelectric nuclear field of matter.

Comparison of (21) with (6) provides to conclude that interactions (21) and (6) are similar, therefore we can observe the effect of spin rotation and oscillations for a particle with $S \geq 1$ passing through nonpolarized matter (birefringence effect) [6, 7]. Henceforth, for the expression (21) we will use the notation $\hat{V}_E^{nucl} = \hat{V}_{eff}$.

4 Spin rotation of proton (deuteron, antiproton) in a storage ring with a polarized target

Let us consider now the experiments deals with the use of polarized beams and targets. In this case the amplitude of the elastic coherent scattering at the zero angle depends on the vector polarization $\vec{P}_t$ of the target nuclei (if the spin of the target nuclei $J \geq 1$, then the addition depending on the target tensor polarization also appears, but this addition will be omitted in below considerations, the general case is considered in [3, 7]).

For the sake of concreteness suppose the target to be rest ($\gamma_2 = 1$, $\gamma_1 = \gamma$).

The contribution to the amplitude $\hat{f}(0)$ proportional to the vector polarization of the target nuclei can be expressed as:

$$\hat{f}(\vec{P}_t, 0) = A_1 \vec{S}\vec{P}_t + A_2 (\vec{S}\vec{n})(\vec{n}\vec{P}_t). \tag{23}$$

Contributions from weak P,T-odd interactions (see [3], [18]-[20]) are omitted here.

Correspondingly, the contribution to the effective potential energy of particle interaction with matter caused by polarization of target nucleus spins looks like:

$$\hat{V}_{eff}(\vec{P}_t) = -\frac{2\pi\hbar^2}{m\gamma} N (A_1 \vec{S}\vec{P}_t + A_2 (\vec{S}\vec{n})(\vec{n}\vec{P}_t)). \tag{24}$$

The expression $\hat{V}_{eff}(\vec{P}_t)$ can be rewritten as:

$$\hat{V}_G^{nucl} \equiv \hat{V}_{eff}(\vec{P}_t) = -\vec{\mu}\vec{G} = -\frac{\mu}{S}\vec{S}\vec{G} \tag{25}$$

where μ is the particle magnetic moment,

$$\vec{G} = \frac{2\pi\hbar^2 S}{m\gamma\mu} N (A_1 \vec{P}_t + A_2 \vec{n}(\vec{n}\vec{P}_t)). \tag{26}$$

Recall now that the energy of interaction of the magnetic moment $\vec{\mu}$ with a magnetic field $\vec{B}$ is as follows:

$$V_{mag} = -\vec{\mu}\vec{B} = -\frac{\mu}{S}\vec{S}\vec{B}. \tag{27}$$

Comparing (25) and (27) one can easily find they are identical. Therefore, $\vec{G}$ can be interpreted as the effective pseudomagnetic field acting on the spin of the particle in matter with polarized nuclei. Pseudomagnetic field $\vec{G}$ is caused by nuclear interaction of particles with scatterers. Similar particle spin precession in conventional magnetic field, spin also precesses in the field $\vec{G}$ (this phenomenon is called nuclear precession of the particle spin). This phenomenon was described for the first time for slow neutrons in [10] and observed in [11]-[13].

5 The equations describing spin evolution of a particle in a storage ring

5.1 Interactions contributing to the spin motion of a particle in a storage ring

Thus, according to the above analysis, when considering evolution of the spin of a particle in a storage ring one should take into account several interactions:

1. interactions of the magnetic and electric dipole moments with electromagnetic fields;
2. interaction of the particle with electric fields due to the tensor electric polarizability;
3. interaction of the particle with magnetic fields due to the tensor magnetic polarizability;
4. interaction of the particle with the pseudoelectric nuclear field of matter;
5. interaction of the particle with the pseudomagnetic nuclear field of polarized matter.

The equation for the particle spin wavefunction considering all these interactions is as follows:

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = \left(\hat{H}_0 + \hat{V}_{EDM} + \hat{V}_{\vec{E}} + \hat{V}_{\vec{B}} + \hat{V}_E^{nucl} + \hat{V}_G^{nucl}\right)\Psi(t) \tag{28}$$

where $\Psi(t)$ is the particle spin wavefunction,

$\hat{H}_0$ is the Hamiltonian describing spin behavior caused by interaction of the magnetic moment with the electromagnetic field (equation (28) with the only $\hat{H}_0$ summand converts to the Bargmann-Michel-Telegdi equation);

$\hat{V}_{EDM}$ describes interaction of the particle EDM d with the electric field;

$\hat{V}_{\vec{E}}$ describes interaction of the particle with the electric field due to the tensor electric polarizability;

$\hat{V}_{\vec{B}}$ is the energy of interaction of the particle with the magnetic field due to the tensor magnetic polarizability;

$\hat{V}_E^{nucl}$ describes the effective potential energy of particle interaction with the pseudoelectric field of the target (21).

$\hat{V}_G^{nucl}$ describes the effective potential energy of particle interaction with the pseudomagnetic field of the target (25).

5.2 Deuteron spin rotation in electromagnetic fields of a storage ring

Let us consider a deuteron moving in a storage ring with low pressure of the residual gas (10^{-10} Torr) and without targets inside the storage ring. In this case we can omit the effects caused by the interactions $\hat{V}_E^{nucl}$ and $\hat{V}_G^{nucl}$. According to the above analysis spin behavior of a deuteron, nevertheless, can not be described by the Bargmann-Michel-Telegdi equation. According to the above interactions $\hat{V}_E$ and $\hat{V}_B$ should be considered when describing spin evolution. The corresponding equations can be written as follows [18]-[20]:

$$\begin{cases} \frac{d\vec{P}}{dt} = \left[\vec{P} \times \Omega(\vec{d})\right] - \\ -\frac{2}{3}\frac{\alpha_T E_{eff}^2}{\hbar}[\vec{n}_E \times \vec{n}'_E] - \frac{2}{3}\frac{\beta_T B_{eff}^2}{\hbar}[\vec{n}_B \times \vec{n}'_B], \\ \\ \frac{dP_{ik}}{dt} = -\left(\varepsilon_{jkr} P_{ij}\Omega_r(d) + \varepsilon_{jir} P_{kj}\Omega_r(d)\right) - \\ -\frac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar}\left([\vec{n}_E \times \vec{P}]_i n_{E,k} + n_{E,i}[\vec{n}_E \times \vec{P}]_k\right) - \\ -\frac{3}{2}\frac{\beta_T B_{eff}^2}{\hbar}\left([\vec{n}_B \times \vec{P}]_i n_{B,k} + n_{B,i}[\vec{n}_B \times \vec{P}]_k\right), \end{cases} \tag{29}$$

where

$$\begin{array}{l} \vec{\Omega}(d) = \vec{\Omega} + \vec{\Omega}_d, \\ \vec{\Omega} = \frac{e}{mc}\left\{\left(a + \frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1}\right)\vec{\beta} \times \vec{E}\right\}, \\ \vec{\Omega}_d = \frac{d}{\hbar}\left(\vec{E} + \vec{\beta} \times \vec{B}\right), \\ P_{ik} = \mathrm{Sp}\ \rho\ \hat{Q}_{ik}, \\ \hat{Q}_{ik} = \frac{3}{2}\left[S_i S_k + S_k S_i - \frac{4}{3}\delta_{ik}\right] \end{array}$$

ρ is the spin density matrix of the particle, m is the particle mass, e is its charge, $\vec{P}$ is the spin polarization vector, P_{ik} is the spin polarization tensor, $P_{xx} + P_{yy} + P_{zz} = 0$, γ is the Lorentz-factor, $\vec{\beta} = \vec{v}/c$, $\vec{v}$ is the particle velocity, $a = (g-2)/2$, g is the gyromagnetic ratio, $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields in the point of particle location, $\vec{E}_{eff} = (\vec{E} + \vec{\beta} \times \vec{B})$, $\vec{B}_{eff} = (\vec{B} - \vec{\beta} \times \vec{E})$, $\vec{n}_E = \frac{\vec{E}+\vec{\beta}\times\vec{B}}{|\vec{E}+\vec{\beta}\times\vec{B}|}$, $\vec{n}_B = \frac{\vec{B}-\vec{\beta}\times\vec{E}}{|\vec{B}-\vec{\beta}\times\vec{E}|}$, $n'_{E,i} = P_{ik} n_{E,k}$, $n'_{Bi} = P_{il} n_{Bl}$, $\Omega_r(d)$ are the components of the vector $\vec{\Omega}(d)$ ($r = 1, 2, 3$ correspond to x, y, z, respectively).

The equations for particle spin motion (29) can be rewritten as follows:

$$\frac{d\vec{P}}{dt} = [\vec{P} \times \vec{\Omega}(d)] + \Omega_T[\vec{n}_E \times \vec{n}'_E] + \Omega_T^{\mu}[\vec{n}_B \times \vec{n}'_B],$$

$$\frac{dP_{ik}}{dt} = -(\epsilon_{jkr} P_{ij}\Omega_r(d) + \epsilon_{jir} P_{kj}\Omega_r(d)) + \Omega'_T([\vec{n}_E \times \vec{P}]_i n_{Ek} + n_{Ei}[\vec{n}_E \times \vec{P}]_k) +$$

$$+\Omega'^{\mu}_T([\vec{n}_B \times \vec{P}]_i n_{Bk} + n_{Bi}[\vec{n}_B \times \vec{P}]_k) \tag{30}$$

where

$$\Omega_T = -\frac{2}{3}\frac{\alpha_T E_{eff}^2}{\hbar}, \quad \Omega'_T = -\frac{3}{2}\frac{\alpha_T E_{eff}^2}{\hbar}, \quad \Omega'_T = -\frac{2}{3}\Omega_T,$$

$$\Omega_T^{\mu} = -\frac{2}{3}\frac{\beta_T B_{eff}^2}{\hbar}, \quad \Omega'^{\mu}_T = -\frac{3}{2}\frac{\beta_T B_{eff}^2}{\hbar}, \quad \Omega'^{\mu}_T = -\frac{2}{3}\Omega_T^{\mu}.$$

Suppose the external electric field in the storage ring $\vec{E} = 0$ and the particle moves along the circle orbit.

Let us now consider the equation (30) in the coordinate system that rotates with the frequency of particle velocity rotation. In such a system spin rotates with respect to the momentum with the frequency determined by $(g-2)$.The coordinate system and vectors $\vec{v}, \vec{E}, \vec{B}$ are shown in figure and denote the axes by x, y, z (or $1, 2, 3$, respectively).

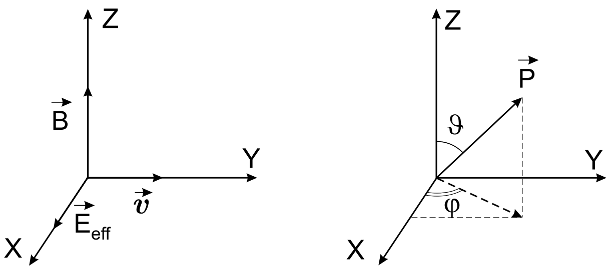

Figure 2:

As a result, the system (29) can be written as:

$$\begin{aligned} \frac{dP_1}{dt} &= \Omega P_2 - \Omega_T^{\mu} P_{23}, \\ \frac{dP_2}{dt} &= -\Omega P_1 + (\Omega_T^{\mu} - \Omega_T) P_{13} + \omega_d P_3, \\ \frac{dP_3}{dt} &= \Omega_T P_{12} - \omega_d P_2, \end{aligned} \qquad (31)$$

where $\omega_d = \frac{dE_1^{eff}}{\hbar}$,

$$\begin{aligned} \frac{dP_{11}}{dt} &= 2\Omega P_{12} + 2\omega_d P_{23}, \\ \frac{dP_{22}}{dt} &= -2\Omega P_{12}, \\ \frac{dP_{33}}{dt} &= -2\omega_d P_{23}, \end{aligned} \qquad (32)$$

$$\begin{aligned} \frac{dP_{12}}{dt} &= -\Omega\left(P_{11} - P_{22}\right) - \Omega_T' P_3 + \omega_d P_{13}, \\ \frac{dP_{13}}{dt} &= \Omega P_{23} + \Omega_T' P_2 - \Omega_T'^{\mu} P_2 - \omega_d P_{12}, \\ \frac{dP_{23}}{dt} &= -\Omega P_{13} + \Omega_T'^{\mu} P_1 - \omega_d (P_{22} - P_{33}). \end{aligned} \qquad (33)$$

Remember that $P_{11} + P_{22} + P_{33} = 0$ and $P_{ik} = P_{ki}$.

5.3 Contribution from the EDM and tensor polarizabilities to deuteron spin oscillation

Let us consider the system (31-33) more attentively.

Suppose that deuteron has neither EDM no tensor polarizabilities: in this case the system (31-33) can be expressed:

$$\frac{dP_1}{dt} = \Omega P_2, \;\; \frac{dP_2}{dt} = -\Omega P_1, \;\; \frac{dP_3}{dt} = 0, \qquad (34)$$

$$\frac{dP_{11}}{dt} = 2\Omega P_{12}, \;\; \frac{dP_{22}}{dt} = -2\Omega P_{12}, \;\; \frac{dP_{33}}{dt} = 0, \qquad (35)$$

$$\frac{dP_{12}}{dt} = -\Omega\left(P_{11} - P_{22}\right), \;\; \frac{dP_{13}}{dt} = \Omega P_{23}, \;\; \frac{dP_{23}}{dt} = -\Omega P_{13}. \qquad (36)$$

This is the conventional system of BMT equations that describes particle spin rotation with the frequency equal to $(g-2)$ precession frequency $\Omega = \frac{ea}{mc}B$. The component P_3 of vector polarization in this conditions is equal to a constant ($\frac{dP_3}{dt}=0$) along with the component P_{33} of tensor polarization ($\frac{dP_{33}}{dt}=0$).

Suppose the deuteron EDM differs from zero. Then the above system of equations converts to:

$$\frac{dP_1}{dt}=\Omega P_2,\ \frac{dP_2}{dt}=-\Omega P_1+\omega_d P_3,\ \frac{dP_3}{dt}=-\omega_d P_2, \tag{37}$$

$$\frac{dP_{11}}{dt}=2\Omega P_{12}+2\omega_d P_{23},\ \frac{dP_{22}}{dt}=-2\Omega P_{12},\ \frac{dP_{33}}{dt}=-2\omega_d P_{23}, \tag{38}$$

$$\frac{dP_{12}}{dt}=-\Omega\left(P_{11}-P_{22}\right)+\omega_d P_{13},\ \frac{dP_{13}}{dt}=\Omega P_{23}-\omega_d P_{12},\ \frac{dP_{23}}{dt}=-\Omega P_{13}-\omega_d(P_{22}-P_{33}). \tag{39}$$

From (37-39) it follows that presence of the nonzero EDM makes the vertical component P_3 of vector polarization oscillating with the frequency of $(g-2)$ precession Ω.

According to the idea [17] these oscillations can be eliminated by modulation of the deuteron velocity with the frequency Ω:

$$v=v_0+\delta v\sin\left(\Omega t+\varphi_f\right) \tag{40}$$

here φ_f is the phase of forced velocity oscillations.

As E_{eff} depends on $\vec{\beta}=\vec{v}/c$, it also appears modulated:

$$E_{eff}=E^0_{eff}+\delta E_{eff}\sin\left(\Omega t+\varphi_f\right) \tag{41}$$

Therefore $\omega_d=\frac{dE_{eff}}{\hbar}$ is also modulated with the same frequency. This makes the product $\omega_d P_2\sim\sin^2(\Omega t+\varphi_f)$. Therefore, averaging this value over the period of $(g-2)$ precession gives the result time-independent (i.e. $\frac{dP_3}{dt}=const$) and $P_3(t)=P_3(0)+const\cdot t$. For better measurement conditions it is important to make $P_3(0)=0$. This is the reason to chose particle spin laying in the horizontal plane.

All the above reasoning makes $\frac{dP_{33}}{dt}\sim const$, too. Therefore, P_{33} also linearly grows with time $P_{33}(t)=P_{33}(0)+const\cdot t$. However, when the spin lays in the horizontal plane $P_{33}(0)\neq 0$.

It is important to note (see below the section 5.5) that choosing spin orientation corresponding to $\cos^2\vartheta=\frac{1}{3}$ ($\cos\vartheta=\sqrt{\frac{1}{3}}$, $\sin\vartheta=\sqrt{\frac{2}{3}}$) one gets the component $P_{33}(0)=0$, while $P_3(0)\neq 0$.

Therefore taking ϑ making $\cos\vartheta=\sqrt{\frac{1}{3}}$ we can use the component P_{33} for EDM measurements, too.

Let us consider now the contribution from the electric and magnetic tensor polarizabilities. Then instead of the system (37-39) we should consider the system (31-33)

Some interesting implications follow from (31-33). As it was already mentioned above in the experiments for EDM search it is planned to measure growth of the vertical component of the polarization vector P_3.

According to (31) time dependence of the vertical component of the vector polarization P_3 is described by the equation

$$\frac{dP_3}{dt}=\Omega_T P_{12}-\omega_d P_2 \tag{42}$$

As it can be seen from (31) dependence of P_3 on time is determined by both the EDM and tensor polarizability of deuteron. It is interesting that the derivative of the tensor polarization component P_{33} does not contain contributions from the tensor electric polarizability and is proportional to the EDM only:

$$\frac{dP_{33}}{dt}=-2\omega_d P_{23} \tag{43}$$

Therefore, it is important to measure the component P_{33}, too. According to the above spin orientation for this case is determined by the condition $\cos^2\vartheta=\frac{1}{3}$.

5.4 Contribution from the tensor magnetic polarizability to deuteron spin oscillation

Contributions to spin rotation and oscillations from the EDM and polarizabilities are small. Therefore, they, being analyzed, could be considered as perturbations to the full system (31-33) and the role of each could be studied separately.

The system of equations taking into account the contribution from the tensor magnetic polarizability β_T is as follows:

$$\frac{dP_1}{dt} = \Omega P_2 - \Omega_T^{\mu} P_{23},\; \frac{dP_2}{dt} = -\Omega P_1 + \Omega_T^{\mu} P_{13},\; \frac{dP_{13}}{dt} = \Omega P_{23} - \Omega_T^{\prime\mu} P_2,\; \frac{dP_{23}}{dt} = -\Omega P_{13} + \Omega_T^{\prime\mu} P_1 \quad (44)$$

Introducing new variables $P_+ = P_1 + iP_2$ and $G_+ = P_{13} + iP_{23}$ and recomposing equations (44) to determine P_+ and G_+ one obtains:

$$\frac{dP_+}{dt} = -i\Omega P_+ + i\Omega_T^{\mu} G_+,$$

$$\frac{dG_+}{dt} = -i\Omega G_+ + i\Omega_T^{\prime\mu} P_+,$$

or

$$i\frac{dP_+}{dt} = \Omega P_+ - \Omega_T^{\mu} G_+,$$

$$i\frac{dG_+}{dt} = \Omega G_+ - \Omega_T^{\prime\mu} P_+,$$

Let us search $P_+, G_+ \sim e^{i\omega t}$, then (45) transforms as follows:

$$\omega \tilde{P}_+ = \Omega \tilde{P}_+ - \Omega_T^{\mu} \tilde{G}_+,$$

$$\omega \tilde{G}_+ = \Omega \tilde{G}_+ - \Omega_T^{\prime\mu} \tilde{P}_+.$$

The solution of this system can be easily found:

$$(\omega - \Omega)^2 - \Omega_T^{\mu} \Omega_T^{\prime\mu} = 0 \quad (45)$$

that finally gives

$$\omega_{1,2} = \Omega \pm \sqrt{\Omega_T^{\mu} \Omega_T^{\prime\mu}}. \quad (46)$$

The solution can be rewritten as:

$$P_+(t) = c_1 e^{-i\omega_1 t} + c_2 e^{-i\omega_2 t} = |c_1| e^{-i(\omega_1 t - \delta_1)} + |c_2| e^{-i(\omega_2 t - \delta_2)} \quad (47)$$

Therefore,

$$P_1(t) = |c_1| \cos(\omega_1 t - \delta_1) + |c_2| \cos(\omega_2 t - \delta_2) \quad (48)$$

$$P_2(t) = -|c_1| \sin(\omega_1 t - \delta_1) - |c_2| \sin(\omega_2 t - \delta_2) \quad (49)$$

According to (48,49) the nonzero deuteron tensor magnetic polarizability makes the spin rotating with two frequencies ω_1 and ω_2 instead of Ω and, therefore, experiences beating with the frequency $\Delta\omega = \omega_1 - \omega_2 = 2\sqrt{\Omega_T^{\mu} \Omega_T^{\prime\mu}} = \frac{\beta_T B_{eff}^2}{\hbar}$.

Let us recall now that EDM interaction with the electric field makes the deuteron spin rotating around the direction of this field and leads to appearance of P_3 component proportional to $P_2(t)$:

$$\frac{dP_3}{dt} \sim -\omega_d P_2. \quad (50)$$

Therefore,

$$\frac{dP_3}{dt} = \omega_d \left(|c_1| \sin(\omega_1 t - \delta_1) + |c_2| \sin(\omega_2 t - \delta_2) \right). \tag{51}$$

According to the idea [17] to measure the EDM one should modulate the particle velocity ($v = v_0 + \delta v \sin(\Omega_f t + \varphi_f)$) with the frequency Ω_f close to the frequency Ω of $(g-2)$ precession.

If the magnetic polarizability is equal to zero, then $\omega_1 = \omega_2 = \Omega$ and spin rotates in the horizontal plane with the frequency Ω. In this case velocity modulation with the same frequency $\Omega_f = \Omega$ gives:

$$\frac{dP_3}{dt} \sim \sin^2(\Omega t) \tag{52}$$

and the vertical component $P_3 = \int_0^t \frac{dP_3}{dt'} dt'$ linearly grows with time.

However, $\omega_1 \neq \omega_2$ and velocity modulation, for example, with the frequency $\Omega = \omega_1$ provides for slow spin oscillation with the frequency $\omega_1 - \omega_2$ instead of linear growth.

According to evaluation [21] the tensor magnetic polarizability $\beta_T \sim 2 \cdot 10^{-40}$, therefore in the field $B \sim 10^4$ gauss the beating frequency $\Delta\omega \sim 10^{-5}$.

Measurement of the frequency of this beating makes possible to measure the tensor magnetic polarizability of the deuteron (nuclei).

Thus, due to presence of the tensor magnetic polarizability the horizontal component of spin rotates around $\vec{B}$ with two frequencies ω_1, ω_2 instead of expected rotation with the frequency Ω.

This is the reason for the component P_3 to experience the similar oscillations caused by the EDM. Therefore, particle velocity modulation with the frequency Ω provides for eliminating of oscillation with the frequency Ω, but P_3 oscillations with the frequency $\Delta\omega$ rest (similarly P_{33}). Study of these oscillations is necessary because they can distort the EDM measurements.

5.5 Contribution from the tensor electric polarizability to deuteron spin oscillation

Let us consider now contribution caused by the tensor electric polarizability. From the system (32) it follows

$$\begin{aligned} &\tfrac{d(P_{11}-P_{22})}{dt} = 4\Omega P_{12}, \\ &\tfrac{d^2 P_{12}}{dt^2} = -\Omega \tfrac{d(P_{11}-P_{22})}{dt} - \Omega'_T \tfrac{dP_3}{dt} = -(4\Omega^2 + \Omega_T \Omega'_T) P_{12}. \end{aligned} \tag{53}$$

Thus we have the equation

$$\frac{d^2 P_{12}}{dt^2} + \omega_{12}^2 P_{12} = 0 \tag{54}$$

where $\omega_{12} = \sqrt{4\Omega^2 + \Omega_T \Omega'_T} \approx 2\Omega$, because $\Omega_T \Omega'_T \ll \Omega^2$.

The solution for this equation can be found in the form:

$$P_{12} = c_1 \cos \omega_{12} t + c_2 \sin \omega_{12} t \tag{55}$$

Let us find coefficients c_1 and c_2: when $t = 0$ the equation (55) gives $c_1 = P_{12}(0)$. The coefficient c_2 can be found from

$$\frac{d(P_{12})}{dt}(t \to 0) = \omega_{12} c_2, \tag{56}$$

therefore

$$c_2 = \frac{1}{\omega_{12}} \frac{d(P_{12})}{dt}(t \to 0), \tag{57}$$

From the equation (33)

$$\frac{dP_{12}}{dt}(t \to 0) = -\Omega \left(P_{11}(t \to 0) - P_{22}(t \to 0)\right), \quad (58)$$

it follows that

$$c_2 = -\frac{P_{11} - P_{22}}{2}, \quad (59)$$

and

$$P_{12} = P_{12}(0)\cos\omega_{12}t - \frac{P_{11} - P_{22}}{2}\sin\omega_{12}t \quad (60)$$

As a result, one can write the following equation for the vertical component of the spin P_3:

$$\frac{dP_3}{dt} = \Omega_T P_{12}(t) = \Omega_T[P_{12}(0)\cos 2\Omega t - \frac{P_{11}(0) - P_{22}(0)}{2}\sin 2\Omega t] \quad (61)$$

As it can be seen the vertical component of the spin oscillates with the frequency 2Ω.

The equation describing contribution from the tensor electric polarizability and EDM to P_3 looks like (31):

$$\frac{dP_3}{dt} = \Omega_T P_{12} - \omega_d P_2. \quad (62)$$

As P_2 oscillates with the frequency Ω, then the product $\omega_d P_2$ contains the non-oscillating terms (see section 5.3) and contribution to P_3, which is caused by the EDM, linearly grows with time, when $\Omega_f = \Omega$. If $\Omega_f \neq \Omega$, then contribution to P_3 caused by EDM slowly oscillates with the frequency $\Omega_f - \Omega$.

It is important that modulation of the velocity $v = v_0 + \delta v \sin(\Omega_f t + \varphi_f)$ results in E_{eff} oscillation (see (41)) and, therefore, E_{eff}^2 also oscillates with time and appears proportional to $\sin^2(\Omega_f t + \varphi_f)$. As a result, the contribution to P_3 caused by the tensor electric polarizability can be expressed as:

$$\frac{dP_3(\alpha_T)}{dt} \sim \Delta\Omega_T \sin^2(\Omega_f t + \varphi_f)[P_{12}(0)\cos 2\Omega t - \frac{P_{11}(0) - P_{22}(0)}{2}\sin 2\Omega t], \quad (63)$$

where $\Delta\Omega_T = -\frac{2}{3}\frac{\alpha_T}{\hbar}(\delta E_{eff})^2$ i.e.

$$\frac{dP_3(\alpha_T)}{dt} \sim -\frac{1}{2}\Delta\Omega_T \cos(2\Omega_f t + 2\varphi_f)[P_{12}(0)\cos 2\Omega t - \frac{P_{11}(0) - P_{22}(0)}{2}\sin 2\Omega t] \quad (64)$$

According to (64) for a partially polarized deuteron beam the derivative $\frac{dP_3}{dt}$ depends on the deuteron polarization components P_{12} and $\frac{P_{11}(0)-P_{22}}{2}$.

For simplicity let us consider a deuteron beam in the pure polarization state. In this case the components P_{12} and $\frac{P_{11}(0)-P_{22}}{2}$ can be written using the explicit expression for the spin wavefunctions. Suppose $\vec{n}(\vartheta, \varphi)$ is the unit vector directed along the deuteron spin (ϑ and φ are the polar and azimuth angles (see Fig.2)). So the spin wavefunction that describes the deuteron spin state with the magnetic quantum number $m = 1$ can be expressed as follows (in the Cartesian basis):

$$\chi_1(\vartheta, \varphi) = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = -\frac{1}{\sqrt{2}} \begin{pmatrix} \cos\vartheta\cos\varphi - i\sin\varphi \\ \cos\vartheta\sin\varphi + i\cos\varphi \\ -\sin\vartheta \end{pmatrix} \quad (65)$$

Polarization vector can be written as:

$$\vec{P} = \langle\hat{S}\rangle = \chi_1^+ \hat{S}\chi_1 = i[\vec{a} \times \vec{a}^*] \quad (66)$$

and components of polarization tensor

$$\langle P_{ik}\rangle = \chi_1^+ \hat{Q}_{ik}\chi_1 = -\frac{3}{2}\{a_i a_k^* + a_k a_i^* - \frac{2}{3}\}, \quad (67)$$

where $\hat{Q}_{ik}$ is the spin tensor of rank two ($\hat{Q}_{ik} = \frac{1}{2}(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i - \frac{4}{3}\delta_{ik}S)$). Therefore,

$$P_{12} = \frac{3}{4}\sin 2\varphi \sin^2\vartheta, \qquad (68)$$

$$\frac{P_{11} - P_{22}}{2} = \frac{3}{4}\cos 2\varphi \sin^2\vartheta, \qquad (69)$$

$$P_{33} = -\frac{3}{2}\left(\sin^2\vartheta - \frac{2}{3}\right). \qquad (70)$$

Using (49,53,54) one can obtain:

$$\frac{dP_3(\alpha_T)}{dt} \sim -\frac{3}{8}\Delta\Omega_T \sin^2\vartheta \cos(2\Omega_f t + 2\varphi_f) \times [\sin 2\varphi \cos 2\Omega t - \cos 2\varphi \sin 2\Omega t] \qquad (71)$$

From (71) it follows that $\frac{dP_3}{dt}$ slowly oscillates with the frequency $(\Omega_f - \Omega)$.

In the ideal case, when $\Omega_f = \Omega$ (as it is proposed in [17] for EDM measurement) (71) converts to:

$$\frac{dP_3(\alpha_T)}{dt} = -\frac{3}{8}\Delta\Omega_T \sin^2\vartheta \cos(2\Omega t + 2\varphi_f)\sin(2\Omega t - 2\varphi) \qquad (72)$$

In the general case, when the phases φ_f and φ are arbitrary, (72) contains terms that do not depend on time and, therefore, P_3 linearly grows with time, like the signal from the EDM does.

It is interesting that making $\varphi_f = -\varphi$ provides:

$$\frac{dP_3(\alpha_T)}{dt} \sim \cos(2\Omega t - 2\varphi)\sin(2\Omega t - 2\varphi) = \frac{1}{2}\sin(4\Omega t - 4\varphi) \qquad (73)$$

that makes this contribution to P_3 quickly oscillating and depressed. But even in this ideal case it rests the contribution caused by the tensor magnetic polarizability (7) (in real situation $\Omega \neq \Omega_f$, though).

Measurement of these contribution provides to measure the tensor electric polarizability.

According to evaluations [21] $\alpha_T \sim 10^{-40}$ cm^3, therefore, for the field $E_{eff} = B \sim 10^4$ gauss the frequency $\Omega_T \sim 10^{-5}$ sec^{-1}. When considering modulation one should estimate $\Delta\Omega_T \sim \Omega_T(\frac{\delta v}{v_0})^2$, then suppose $(\frac{\delta v}{v_0})^2 \sim 10^{-2} - 10^{-3}$ we obtain $\Delta\Omega_T \sim 10^{-7} - 10^{-8}$ sec^{-1}, that exceeds the magnitude of ω_d for the deuteron EDM $d = 10^{-29}$ $e \cdot cm$.

6 Particle spin rotation in a storage ring with an internal target

Let now a gas target be placed inside the storage ring. According to the section 5.1, spin evolution in this case is also influenced by the nuclear pseudoelectric and pseudomagnetic fields.

Time evolution of the spin and tensor polarization can be found by the equations:

$$\vec{P} = \frac{\langle\Psi(t)|\frac{\vec{S}}{S}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle}; P_{ik} = \frac{\langle\Psi(t)|\hat{Q}_{ik}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle} \qquad (74)$$

$\hat{Q}_{ik}$ is the tensor of rank two (the tensor polarization), for $S = 1$ the tensor $\hat{Q}_{ik} = \frac{3}{2}(S_iS_k + S_kS_i - \frac{4}{3}\delta_{ik})$.

But it should be taken into account that multiple scattering, which occurs in the target, brings to both change in the angular spread and depolarization of the particle beam.

So the spin density matrix formalism should be used to derive equations for evolution of the particle spin.

The density matrix of the system "deuteron+target" is

$$\rho = \rho_p \otimes \rho_t, \qquad (75)$$

where ρ_d is the density matrix of the particle beam and ρ_t is the density matrix of the target.

The equation for the particle density matrix can be written as [5, 18]:

$$\frac{d\rho_p}{dt} = -\frac{i}{\hbar}\left[\hat{H}_p, \rho_p\right] + \left(\frac{\partial \rho_p}{\partial t}\right)_{col}, \tag{76}$$

where $\hat{H}_p$ is the Hamiltonian describing particle motion in external electromagnetic fields, $\left(\frac{\partial \rho_p}{\partial t}\right)_{col}$ describes evolution of the density matrix due to collisions with the target atoms (nuclei).

The collision term $\left(\frac{\partial \rho_d}{\partial t}\right)_{col}$ can be expressed as follows:

$$\left(\frac{\partial \rho_d}{\partial t}\right)_{col} = vN\mathrm{Sp}_t\left[\frac{2\pi i}{k}\left[F(\theta=0)\rho - \rho F^{+}(\theta=0)\right] + \int d\Omega F(\vec{k}')\rho(\vec{k}')F^{+}(\vec{k}')\right], \tag{77}$$

where $\vec{k}' = \vec{k} + \vec{q}$, $\vec{q}$ is the momentum transferred to a nucleus of matter from the incident particle, v is the speed of the incident particles, N is the atom density in matter, F is the scattering amplitude depending on the spin operators of the deuteron and the matter nucleus (atom), F^{+} is the Hermitian conjugate for the operator F. The first term in (77) describes coherent elastic scattering of a particle by matter nuclei, while the second term is for multiple scattering. Let us consider the first term in (77). It can be written as:

$$\left(\frac{\partial \rho_p}{\partial t}\right)_{col}^{(1)} = vN\frac{2\pi i}{k}\left[\hat{f}(0)\rho_p - \rho_p\hat{f}(0)^{+}\right]. \tag{78}$$

The amplitude $\hat{f}(0)$ of particle scattering in the target at the zero angle is

$$\hat{f}(0) = \mathrm{Sp}_t\hat{F}(0)\rho_t. \tag{79}$$

As a result one can obtain:

$$\left(\frac{\partial \rho_p}{\partial t}\right)_{col}^{(1)} = -\frac{i}{\hbar}\left(\hat{V}_{eff}\rho_p - \rho_p\hat{V}_{eff}^{+}\right). \tag{80}$$

where

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}N\hat{f}(0) = -\frac{2\pi\hbar^2}{m\gamma}N(d + A_1\vec{S}\vec{P}_t + A_2(\vec{S}\vec{n})(\vec{n}\vec{P}_t) + d_1(\vec{S}\vec{n})^2). \tag{81}$$

The latter expression can be rewritten as:

$$\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}Nd - \frac{\mu}{S}\vec{S}\vec{G} - \frac{2\pi\hbar^2}{m\gamma}Nd_1(\vec{S}\vec{n})^2). \tag{82}$$

where $\vec{G} = \frac{2\pi\hbar^2 S}{m\gamma\mu}N(A_1\vec{P}_t + A_2\vec{n}(\vec{n}\vec{P}_t))$ (see (26)). For particles with the spin $\frac{1}{2}$ the term proportional to d_1 is absent.

Finally, the expression (76) reads:

$$\frac{d\rho_p}{dt} = -\frac{i}{\hbar}\left[\hat{H}_p, \rho_d\right] - \frac{i}{\hbar}\left(\hat{V_{eff}}\rho_p - \rho_p\hat{V}_{eff}^{+}\right) + vN\mathrm{Sp}_t\int d\Omega F(\vec{k}')\rho(\vec{k}')F^{+}(\vec{k}'). \tag{83}$$

The last term in the above formula, which is proportional to Sp_t, describes the multiple scattering process and spin depolarization aroused from it.

When studying interaction of particles with a target inside a storage ring the density of the target is chosen to make multiple scattering in it small for the observation time; depolarization of the beam also appears small.

For high-energy particles scattering at small angles is important. Therefore the large number of scattered particles remain on their orbits. For this case the latter term should be considered.

Further analysis of equations (83) in the present paper will be done for such time of the experiment that provides the last term in (83) can be cast out.

As a result we have:

$$\frac{d\rho_p}{dt} = -\frac{i}{\hbar}\left[\hat{H}_p, \rho_d\right] - \frac{i}{\hbar}\left(\hat{V_{eff}}\rho_p - \rho_p\hat{V}_{eff}^{+}\right). \tag{84}$$

Now the equation for evolution of spin properties of a particle in a storage ring can be obtained. The polarization vector is as follows:

$$\vec{P} = \frac{\text{Sp } \rho_p(t) \ \frac{\vec{S}}{S}}{\text{Sp } \rho_p(t)} = \frac{\text{Sp } \rho_p(t) \ \vec{S}}{I(t) \ S}, \tag{85}$$

where $I(t) = \text{Sp } \rho_p(t)$ is the beam intensity.

From (85) one can get the differential equation providing to find the beam polarization:

$$\frac{d\vec{P}}{dt} = \frac{\text{Sp } \frac{d\rho_p(t)}{dt} \ \vec{S}}{SI(t)} - \vec{P}\frac{1}{I(t)}\frac{dI(t)}{dt} \tag{86}$$

When the particle spin is $S = 1$ then the polarization tensor should also be found:

$$P_{ik} = \frac{\text{Sp } \rho_p \ \hat{Q_{ik}}}{I(t)}, \tag{87}$$

Change in the tensor polarization with time can be written as:

$$\frac{dP_{ik}}{dt} = \frac{1}{I(t)}\text{Sp}\left(\frac{d\rho_p}{dt}\hat{Q_{ik}}\right) - P_{ik}\frac{1}{I(t)}\frac{dI(t)}{dt}. \tag{88}$$

Using the equation for the density matrix (83) and the expression (86,88) one can obtain the equation that summarizes BMT equation (30) for the case when spin evolution is influenced by the pseudoelectric and pseudomagnetic nuclear fields.

Let us consider first particles with the spin $S = \frac{1}{2}$. The density matrix for such a particle can be expressed as follows:

$$\rho_{p\frac{1}{2}} = I_{\frac{1}{2}}(t)(\frac{1}{2}\hat{I} + \vec{P}\vec{S}), \tag{89}$$

where $\hat{I}$ is the unit matrix in the spin space.

With the help of (85) it is possible to find that

$$\frac{dI_{\frac{1}{2}}(t)}{dt} = -\frac{i}{\hbar}\text{Sp}(\hat{V}_{eff}\rho_{p\frac{1}{2}} - \rho_{p\frac{1}{2}}\hat{V}_{eff}^{+}) = -(\varkappa + \frac{2\mu}{\hbar}\vec{G}_2\vec{P})I_{\frac{1}{2}}(t), \tag{90}$$

where $\varkappa = vN\sigma_{tot}$, v is the particle speed, σ_{tot} is the total cross-section of particle scattering by a nonpolarized nucleus and $\vec{G}_2$ is the imaginary part of the pseudomagnetic nuclear field. The expression (90) can be rewritten as:

$$\frac{dI_{\frac{1}{2}}(t)}{dt} = -\varkappa_{abs}(\vec{P})I_{\frac{1}{2}}(t), \tag{91}$$

where the absorption coefficient $\varkappa_{abs} = vN\left[\sigma_{tot} + \frac{1}{2}\sigma_1(\vec{P}\vec{P}_t) + \frac{1}{2}\sigma_2(\vec{P}\vec{n})(\vec{n}\vec{P}_t)\right] = \varkappa + \vec{P}\vec{g}_t$, $\vec{g}_t = \frac{1}{2}vN(\sigma_1\vec{P}_t + \sigma_2\vec{n})(\vec{n}\vec{P}_t)$.

The cross-section $\sigma_1 = \sigma_{tot}^{\uparrow\uparrow}(\vec{n} \perp \vec{P}_t) - \sigma_{tot}^{\uparrow\downarrow}(\vec{n} \perp \vec{P}_t)$ is the difference between the cross-sections of particle scattering by a polarized nucleus with the $\vec{P}$ parallel ($\vec{P} \uparrow\uparrow \vec{P}_t$) and antiparallel ($\vec{P} \uparrow\downarrow \vec{P}_t$) to the target polarization vector $\vec{P}_t$ in conditions when the particle momentum is orthogonal to the target polarization vector ($\vec{n} \perp \vec{P}_t$). The cross-section $\sigma_2 = \sigma_{tot}^{\uparrow\uparrow}(\vec{n}||\vec{P}_t) - \sigma_{tot}^{\uparrow\downarrow}(\vec{n}||\vec{P}_t)$ is the difference between the cross-sections of particle scattering by a polarized nucleus with the $\vec{P}$ parallel ($\vec{P} \uparrow\uparrow \vec{P}_t$) and antiparallel ($\vec{P} \uparrow\downarrow \vec{P}_t$) to the target polarization vector $\vec{P}_t$ in conditions when the particle momentum is parallel to the target polarization vector ($\vec{n}||\vec{P}_t$).

The equation (91) describes the well known fact that the coefficient of particle beam absorption in a polarized target depends on the respective orientations of the beam and target polarization vectors.

Let us obtain now the equation that describes evolution of the particle vector polarization under action of pseudomagnetic nuclear field $\vec{G}$.

According to (85,86) one can obtain:

$$\frac{d\vec{P}}{dt} = \frac{1}{SI_{\frac{1}{2}}(t)}\mathrm{Sp}\frac{d\rho_p}{dt}\vec{S} - \vec{P}\frac{1}{I_{\frac{1}{2}}(t)}\frac{dI_{\frac{1}{2}}(t)}{dt} = -\frac{2\mu}{\hbar}\left[\vec{G}_1 \times \vec{P}\right] - \frac{2\mu}{\hbar}(\vec{G}_2 - \vec{P}(\vec{G}_2\vec{P})), \tag{92}$$

where $vecG_1 = \mathrm{Re}\vec{G}$, $\vec{G}_2 = \mathrm{Im}\vec{G}$.

The equation (92) can be rewritten as:

$$\frac{d\vec{P}}{dt} = \left[\vec{P} \times \vec{\Omega}_{nuc}\right] - (\vec{g}_t - \vec{P}(\vec{P}\vec{g}_t)), \tag{93}$$

where $\vec{\Omega}_{nuc}$ is the frequency of spin precession in the pseudomagnetic nuclear field

$$\vec{\Omega}_{nuc} = \frac{2\mu}{\hbar}\vec{G}_1 = \frac{2\pi\hbar}{m\gamma}N(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t)). \tag{94}$$

Adding the contribution from the pseudomagnetic nuclear field (93) to the BMT equation (30) one can finally obtain the equation describing spin evolution for particles moving in a storage ring with a polarized target inside:

$$\frac{d\vec{P}}{dt} = \left[\vec{P} \times (\vec{\Omega}(d) + \vec{\Omega}_{nuc})\right] - (\vec{g}_t - \vec{P}(\vec{P}\vec{g}_t)), \tag{95}$$

$$\frac{dI_{\frac{1}{2}}(t)}{dt} = -(\varkappa + \vec{P}\vec{g}_t)I_{\frac{1}{2}}(t), \tag{96}$$

where $\vec{g}_t = \frac{1}{2}vN(\sigma_1\vec{P}_t + \sigma_2\vec{n}(\vec{n}\vec{P}_t))$.

The contribution from the pseudomagnetic nuclear field to spin evolution of the $S = 1$ particle (for example, deuteron) can be obtained in the similar way. The spin density matrix for the spin 1 particle is expressed as follows:

$$\rho_1 = I_1(t)(\frac{1}{3}\hat{I} + \frac{1}{2}(\vec{P}\vec{S}) + \frac{1}{9}P_{ik}\hat{Q}_{ik}) \tag{97}$$

The effective potential energy of deuteron interaction with a polarized target is expressed as $\hat{V}_{eff} = -\frac{2\pi\hbar^2}{m\gamma}Nd - \frac{\mu}{S}\vec{S}\vec{G} - \frac{2\pi\hbar^2}{m\gamma}Nd_1(\vec{S}\vec{n})^2$ (see (82)).

The energy $\hat{V}_{eff}$ for a deuteron (in contrast to a particle with the spin $S = \frac{1}{2}$) contains both deuteron interaction with the pseudomagnetic field and pseudoelectric field (the term proportional to d_1). The equations describing deuteron spin rotation (birefringence effect) in the pseudoelectric field of the nonpolarized internal target of a storage ring were obtained in [18].

If the internal target in a storage ring is polarized, then using (82,84,97) one can obtain the change in the beam intensity and polarization vector as follows:

$$\frac{dI_1(t)}{dt} = -\left[(\varkappa + 2\vec{g}_t\vec{P}) - \frac{\chi}{3}(2 + P_{ik}n_in_k)\right]I_1(t) \tag{98}$$

where $\chi = -\frac{4\pi v N}{k}\mathrm{Im}d_1 = -vN(\sigma_{M=1} - \sigma_{M=0})$, $\varkappa = vN\sigma_{M=0}$, $\sigma_{M=1}$ and $\sigma_{M=0}$ are the total cross-sections of deuteron scattering by a nonpolarized nucleus for the deuteron state with the magnetic quantum number $M = 1$ and $M = 0$, respectively (the quantization axis is directed along $\vec{n}$).

The polarization vector can be expressed as:

$$\begin{array}{c} \frac{d\vec{P}}{dt} = \left[\vec{P} \times \vec{\Omega}_{nuc}\right] - \frac{2}{3}P_{ik}g_{tk} - \frac{4}{3}\vec{g}_t + 2\vec{P}(\vec{P}\vec{g}_t) + \frac{\eta}{3}[\vec{n} \times \vec{n}']+ \\ +\frac{\chi}{2}(\vec{n}(\vec{n}\cdot\vec{P}) + \vec{P}) - \frac{2\chi}{3}\vec{P} - \frac{\chi}{3}(\vec{n}\cdot\vec{n}')\vec{P} \end{array} \tag{99}$$

where $\eta = -\frac{4\pi N}{k}\mathrm{Re}\ d_1$, $n_i' = P_{ik}n_k$.

According to (99) the nuclear pseudomagnetic field causes deuteron spin precession with the frequency $\vec{\Omega}_{nuc}$.

Contribution (99) should be added to the equation (30) to make them describing evolution of the deuteron spin in a storage ring at presence of target.

The similar approach provides to find the addition to the tensor polarization evolution $\frac{dP_{ik}}{dt}$ (see expression (88)). But it is too bulky to adduce it here.

However, when particle beam absorption in the target can be neglected, one can make all changes in the equation, which describes deuteron spin behavior, by replacement of $\vec{\Omega}(d)$ with new value $\vec{\Omega}(d) + \vec{\Omega}_{nuc}$.

Note that as the target inside the storage ring is of the finite size then the particle moving in the storage ring interacts with the target at times (the expressions for $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ contain the density N, which means the density in the point of particle location i.e. $N = N(t)$). This is the reason for $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ oscillating with the frequency of particle rotation in the storage ring Ω_0.

For further analysis let us expand $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ into Fourier series and be confined with the zero harmonics. In this case $\vec{\Omega}_{nuc}$ and $\vec{g}_t$ appears to be constants and can be written as:

$$\begin{array}{l} \vec{\Omega}_{nuc} = \frac{2\pi\hbar}{m\gamma}N(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t))\frac{l}{vT} = \\ = \frac{2\pi\hbar}{m\gamma}\frac{j_t}{L}(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t)) = \\ = \frac{2\pi\hbar}{m\gamma}\frac{j_t\nu}{v}(\mathrm{Re}A_1\vec{P}_t + \mathrm{Re}A_2\vec{n}(\vec{n}\vec{P}_t)), \end{array} \tag{100}$$

where l is the target length, $j_t = N\cdot l$ is the target density in cm^{-2} (usually for a polarized gas target $j_t \approx 10^{14}\ \mathrm{cm}^{-2}$), T is the period of beam rotation in the storage ring, ν is the frequency of particle rotation in the storage ring, L is the particle orbit length inside the storage ring.

The zero harmonics for $\vec{g}_t$ is as follows:

$$\begin{array}{l} \vec{g}_t = \frac{1}{2}vN(\sigma_1\vec{P}_t + \sigma_2\vec{n}(\vec{n}\vec{P}_t))\frac{l}{vT} = \\ = \frac{1}{2}j_t\nu(\sigma_1\vec{P}_t + \sigma_2\vec{n}(\vec{n}\vec{P}_t)). \end{array} \tag{101}$$

Let us evaluate now the effect magnitude. For protons, antiprotons and deuterons with the energy from MeV to GeV $\mathrm{Re}A_1 \sim 10^{-12} \div 10^{-13}$ cm. Considering $\nu \sim 10^6\ \mathrm{s}^{-1}$ one gets for the nuclear precession frequency $\Omega_{nuc} = 10^{-4} \div 10^{-5}\ \mathrm{s}^{-1}$.

Note (see (16,17)) that the ratio $\frac{1}{\gamma}\mathrm{Re}A_{1(2)}$ is proportional to the T-matrix and, as it follows, Ω_{nuc} depends on the particle energy only due to possible dependence of the T-matrix on energy.

The obtained estimation for Ω_{nuc} allows to expect, for example for COSY, to observe the spin rotation angle $\vartheta = \Omega_{nuc}t \approx 10^{-1} \div 10^{-2}$ rad in the observation time $t \sim 10^3$ s. This value is quite observable.

To prevent suppression of the the spin precession in the pseudomagnetic nuclear field by the storage ring magnetic field $\vec{B}$ the polarized target should be placed in the straight section of the storage ring, where the field $\vec{B} = 0$ and the particle moves along the straight trajectory.

Let us now consider the particular example. Suppose the axis OZ is orthogonal to the orbit plane $(OS||\vec{B})$ (see Fig.2). The pseudomagnetic field is directed along the axis OX. In this case for the particle in the straight section the storage ring the vertical spin component rotates around the direction of the pseudomagnetic field in the ZOY plane. Just the change of the vertical spin component should be observed.

As the typical spin rotation angle $\vartheta = \Omega_{nuc} t \ll 1$, then change of the vertical spin component with time can be expressed as:

$$P_3(t) = P_3(0) - \frac{1}{2}\vartheta^2 t^2 = P_3(0) - \Omega_{nuc}^2 t^2. \qquad (102)$$

The effect can be strengthened when adding a magnetic field $\vec{b} \gg \vec{G}_1$ directed along the pseudomagnetic field $\vec{G}_1$ (along the axis OX in the case under consideration). In this case, the angle of rotation is expressed as $\vartheta = \vartheta_{mag} + \vartheta_{nuc} = (\Omega_{mag}(\vec{b}) + \Omega_{nuc})t$ and the vertical component $P_3(t) = P_3(0) - \frac{1}{2}\vartheta_{mag}^2 t^2 - \vartheta_{mag}\vartheta_{nuc}$ (the terms $\sim \vartheta_{nuc}^2$ are neglected comparing with the previous terms).

When make the direction of the field $\vec{b}$ changing during the experiment (i.e. the sign of ϑ_{mag} changing) one gets the possibility to measure the difference $P_3(\vec{b} \uparrow\uparrow \vec{G}_1, t) - P_3(\vec{b} \uparrow\downarrow \vec{G}_1, t) = 2\vartheta_{mag}\vartheta_{nuc}$. This provides to study the possibility of effect observation and measurement of ϑ_{nuc} and, therefore, to measure the spin-dependent part of the amplitude of elastic coherent zero-angle scattering.

7 Conclusion

In the present paper the equations for spin evolution of a particle in a storage ring are obtained considering contributions from the tensor electric and magnetic polarizabilities of the particle along with the contributions from spin rotation and birefringence effect in polarized matter of a gas target.

Influence of the tensor electric and magnetic polarizabilities on spin evolution in the resonance deuteron EDM experiment is considered in details.

It is shown that, besides the EDM, the electric and magnetic polarizabilities also contribute to the vertical spin component P_3. Moreover, the electric polarizability contributes to the P_3 component even when the deuteron EDM is supposed to be zero and thereby the electric polarizability can imitate the EDM contribution. It is shown that unlike the vertical component of the spin P_3 the component P_{33} of polarization tensor does not contain contribution from the electric polarizability, whereas contribution from the magnetic polarizability reveals only when the deuteron EDM differs from zero.

It is also shown that when the angle ϑ between the spin direction and the vertical axis meets the condition $\sin\vartheta = \sqrt{\frac{2}{3}}$ ($\cos\vartheta = \sqrt{\frac{1}{3}}$), the initial value of P_{33} appears $P_{33}(0) = 0$. As a result, the EDM contribution to the measured signal linearly growth in time starting from zero that is important for measurements.

Therefore, measurement of the P_{33} component of the deuteron tensor polarization seems to be of particular interest, especially because appearance of the nonzero component P_{33} on its own indicates the EDM presence (in contrast to the P_3 component, which appearance can be aroused by the tensor electric polarizability, rather than EDM).

It is also shown that study of spin rotation and the birefringence effect for a particle in a high energy storage ring provides for measurement of the spin-dependent real part of the coherent elastic zero-angle scattering amplitude.

References

[1] C.Lechanoine-Lelue and F.Lehar Rev.Mod.Phys. **65** (1993) 47.

[2] R.Engels, K.Grigoriev, A.Gussen et al. Proceedings of the 6th International Conference on Nuclear Physics at Storage Rings STORI'05, Schriften des Forschungszentrums Julich, Matter and Materials, Volume 30, 2005, p. 381.

[3] V.G. Baryshevsky, Sov. J. Nucl.Phys. **38** (1983) 569; V.G. Baryshevsky, Phys. Lett. B 120 (1983) 267.

[4] V.G. Baryshevsky, I.Ya.Dubovskaya, Phys. Lett. B **256** (1989) 529.

[5] V.G. Baryshevsky, A.G. Shekhtman Phys. Rev. C **53**, n.1 (1996) 267.

[6] V. G. Baryshevsky, Phys. Lett. **171A** (1992) 431.

[7] V. G. Baryshevsky, J. Phys.G **19** (1993) 273.

[8] V. G. Baryshevsky, K. G. Batrakov and S. Cherkas J. Phys.G **24** (1998) 2049.

[9] V. Baryshevsky, Proceedings of the 6th International Conference on Nuclear Physics at Storage Rings STORI'05, Schriften des Forschungszentrums Julich, Matter and Materials, Volume 30, 2005, p.277.

[10] V.G. Baryshevsky and M.I. Podgoretsky, Zh. Eksp. Teor. Fiz. **47** (1964) 1050.

[11] A. Abragam et al., C.R. Acad. Sci. **274** (1972) 423.

[12] M. Forte, Nuovo Cimento A **18** (1973) 727.

[13] A. Abragam and M. Goldman, Nuclear magnetism: order and disorder (Oxford Univ. Press, Oxford, 1982).

[14] W. Bryan, Montague Phys. Reports **113**, no.1 (1984) 1-96.

[15] S.R. Mane, Yu.M. Shatunov and K. Yokoya, Rep.Prog.Phys. 68 (2005) p.1997-2265.

[16] F. Farley et al., Phys. Rev. Lett., 93 (2004) N 5.

[17] Yu. Orlov, Proceedings of the 6th International Conference on Nuclear Physics at Storage Rings STORI'05, Schriften des Forschungszentrums Julich, Matter and Materials, Volume 30, 2005, p. 223.

[18] V. Baryshevsky, A. Shirvel, LANL e-print archive: hep-ph/0503214.

[19] V. Baryshevsky, LANL e-print archive: hep-ph/0504064.

[20] V. Baryshevsky, A. Gurinovich LANL e-print archive, hep-ph/0506135.

[21] Jiunn-Wei Chen, Harald W. Grießhammer, Martin J. Savage and Roxanne P. Springer, Nucl.Phys. A644 (1998) 221-234 (LANL e-print arXiv:nucl-th/9806080).

[22] M.Lax, Rev.Mod.Phys. **23** (1951) 287.

[23] M.L. Goldberger and K.M. Watson, Collision Theory: John Wiley, New York, 1964.