

On the Possibility of the Strong Field QED Investigation at LHC

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The interaction of high energy particles with atomic axes and planes allows to observe in crystal various effects predicted by the quantum electrodynamics of phenomena in strong electromagnetic field. In particular, the effect of electron-positron pair production by gamma-quanta in a semi-uniform field was observed for the first time in eightieth in CERN in the field of germanium crystal axes. The high energy of LHC drastically widens the possibilities of strong field QED effect investigation in crystals allowing to observe vacuum dichroism and birefringence, electron radiative self-polarization and polarized electron-positron pair production by gamma-quanta, positron (electron) anomalous magnetic moment modification and electron spin rotation in crystal field harmonics. The effect of vacuum birefringence induced by strong electric field is considered in detail.

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I. INTRODUCTION

The effects induced or strongly modified by intense electromagnetic fields greatly widen our understanding of the nature of electromagnetic field, electromagnetic interaction and particle structure. These effects play important role in dense cosmic objects as well as in cosmic ray generation.

For a long time it seemed impossible to study the effects of QED in strong fields in earth conditions. Even the unique experiments of Kapitsa and Sakharov made it possible to generate only the fields millions times lower than the critical fields

$$H_0 = m^2 c^3 / |e| \hbar = 4.4 \times 10^{13} G, \quad (1)$$

$$E_0 = m^2 c^3 / |e| \hbar = 1.32 \times 10^{16} V/m, \quad (2)$$

where m is electron mass, typical for the atmospheres of neutron stars and magnetars in which such effects of strong field QED as electron-positron pair production by gamma-quanta, gamma-quanta birefringence and splitting, electron magnetic moment modification and electron spin radiative self-polarization play important role. However in seventieth and eighties the possibility of observation of the effects of QED in strong uniform crystal fields was realized [1, 2, 3, 4, 5, 6].

A wide applicability of the uniform field approximation for description of radiation processes in the fields of crystal planes and axes is explained by the fact that a forming gamma-quantum or electron-positron pair moves inside a formation region which, on the one hand, has a length several orders exceeding the interatomic distance and, on the other, a transverse dimension which can be as small as the Compton length. The great length of formation region provides a smearing out of atomic potentials to a continuum potential remaining invariant under

the translations along crystal plane or axis. In its turn, the small transverse dimension provides nearly constant value of the field acting during formation process on particles moving at sufficiently small angles in respect to the crystal plane or axis.

The applicability of the uniform field approximation to description of process in oriented crystals allowed us to predict [7, 8] the pair production process in strong semi-uniform continuum crystal field as well as intimately connected to it strong crystal optical anisotropy in hard gamma-region, directly connected with the anisotropy of strong uniform field in vacuum. While the pair production process itself was observed and studied experimentally in CERN in middle eightieth in the field of Ge and W crystal axes, the energies 150 – 200 GeV of tertiary CERN gamma beams were that time quite far from that needed to observe the crystal optical anisotropy in hard gamma-region, namely, the effects of dichroism and birefringence as well as many important effects connected with electron spin, all of which manifest themselves in the fields of crystal planes considerably less intensive than that of crystal axes. The aim of this paper is to demonstrate that the LHC energy provides really optimal conditions to observe the effects of strong crystal optical anisotropy in hard gamma-region [1, 7] directly connected with the optical anisotropy of strong uniform field [9, 10, 11, 12, 13].

II. TYPICAL FIELDS AND ENERGIES

Let us first demonstrate that the fields of crystal axes and planes are really large. Remind [14] that nuclei positions in crystals are normally scattered by their equilibria with a mean square displacement $u \simeq 0.05 - 0.1 \text{ \AA}$. The scattering of atomic nuclei positions suppresses the continuum potential growth at distances $\Delta\rho \leq u$ from a plane formed by equilibrium atomic positions limiting, thus, the maximum electric field of a crystal axis by the value

$$E_{max}^{ax} \simeq 2Ze/ud_{ax} \simeq 10^{10} Z(V/cm) \leq 10^{12} V/cm, \quad (3)$$

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where Z is the monocrystal atomic number and d_{ax} in the interatomic distance on the axis. The estimate (3) was obtained by averaging a nucleus field over a straight line passing at distance u . A very similar procedure of the averaging of a nucleus field over a plane gives an order lower estimate

$$E_{max}^{pl} \simeq \pi Z e n_0 d_{pl} \leq 10^{11} V/cm \quad (4)$$

for the maximum value of the longitudinally averaged field of atomic crystal plane, where n_0 and d_{pl} are crystal atomic number density and inter-plane distance, respectively. The results of evaluation of maximum averaged fields of the crystals axes and planes most widely used in channelling studies are given in the Table. The gamma-quantum energies which are enclosed in brackets correspond to deeply cooled crystals while all the others have been obtained for the normal temperature. In their ability to deflect relativistic particles the fields (3) and (4) are equivalent, respectively, to magnetic fields of strengths $0.2 - 3GG$ and $20 - 200MG$, far exceeding that produced by cumulative magnetic generators for microsecond durations only [15].

Table

Crystal	(plane) <Axis>	E_{max} , GeV/cm	ω_0 , TeV
Diagonal	(110)	7.7	1.78
	<110>	75	0.20
Si	(110)	5.7	2.39 (1.7)
	<110>	46	0.29
Ge	(110)	9.9	1.37 (0.9)
	<110>	78	0.174
W	(110)	43	0.316
	<111>	500	0.027

Though the fields (3) and (4) are quite large, they are still far below the field (2) capable to produce electron-positron pairs in vacuum. Fortunately, such a field can be reached in the reference frame of ultra relativistic particles in which the transverse averaged crystal field is "multiplied" by the Lorentz factor equal to the particle energy measured in the units of its rest energy. Considering the pair production process which is intimately connected with dichroism and birefringence phenomena, it will be natural to use the mass $2mc^2$ as a "rest mass" of a forming electron-positron pair leading to the relativistically magnified field strength $E_{max}\omega/2m$ (below we use the system of units in which $\hbar = c = 1$) in its reference frame. This strength will reach the critical value (2) at the gamma-quantum energy

$$\omega_0 = 2m \frac{E_0}{E_{max}}, \quad (5)$$

which can be considered as a "threshold" energy of pair production in crystals. Some values of this energy given

in the Table explain why the investigations of strong field QED effects in mid eightieth were tightly restricted by the axial case, which allowed to observe the electron-positron pair production in a semi-uniform electric field, however has not made possible to observe the effects of uniform field optical anisotropy as well as the number of effects connected with electron spin. The Table also shows that observation of these effects will become possible at the LHC gamma- and electron beams the energies of which will definitely exceed $1TeV$.

III. CRYSTAL DICHROISM

From the many strong field QED effects in the uniform field the birefringence effect is naturally distinguished by its fundamental nature, simplicity of theory and by the limited energy of its optimal manifestation. This energy limit arises because of rapid growth of gamma-quantum absorption at energies $\omega > \omega_0$ due to the electron-positron pair production, accompanied by the strong dichroism. Both the dichroism and birefringence of uniform field manifest themselves much better in the field of crystal planes because the latter is parallel to the same line, the normal to crystal planes, in all the crystal volume.

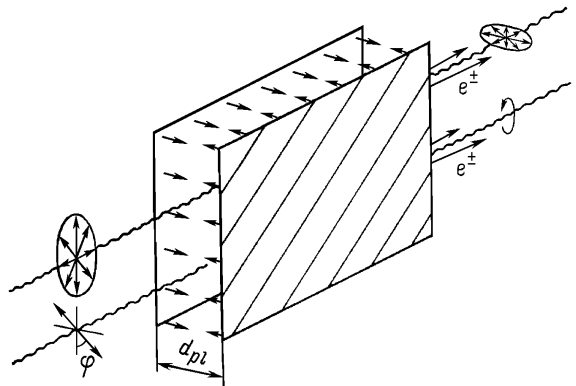


FIG. 1: Averaged field of crystal planes allowing to observe dichroism and birefringence effects inherent to strong electromagnetic field in vacuum.

Various important aspects of theory of electron-positron pair production in crystals and accompanying effects of dichroism and birefringence were considered in [1, 7, 8, 16, 17, 18, 19, 20] (see also [21] and [4], §15). First of all, it was shown there that the general QED formulas can be considerably simplified in the case of gamma-quantum incidence nearly parallel to crystal planes when they reduce to the formulas of the theory of electron-positron pair production by gamma-quanta in a uniform electromagnetic field [4, 9, 10] averaged over the crystal volume. Let us take the probability

$$W_{x,y}(\kappa) = \frac{\alpha m^2}{\sqrt{\pi} \omega^2}$$

$$\times \int_0^\omega d\varepsilon_+ \left[\int_\xi^\infty \Phi(y) dy + \left(2 \pm 1 - \frac{\omega^2}{\varepsilon_+ \varepsilon_-} \right) \frac{\Phi'(\xi)}{\xi} \right], \quad (6)$$

where

$$\kappa = \frac{E\omega}{E_0 m}, \quad \xi = \left(\frac{E_0 m \omega}{E \varepsilon_+ \varepsilon_-} \right)^{2/3}, \quad (7)$$

ε_- and ε_+ are the electron and positron energies, respectively.

$$\Phi(\xi) = \frac{1}{\sqrt{\pi}} \int_0^\infty \cos\left(\xi t + \frac{t^3}{3}\right) dt \quad (8)$$

is Airy function and the transverse axes x and y are, respectively, parallel and normal to the electric field strength. According to [7, 8] the probabilities of pair production by gamma-quanta propagating parallel to crystal planes and polarized parallel and normal to them can be evaluated by the simple formula

$$W_{\parallel, \perp}^{coh} = \int_0^{d_{pl}} W_{y,x}(\kappa(E(x))) \frac{dx}{d_{pl}} \quad (9)$$

in which the parameter κ in arbitrary point x should be calculated substituting the averaged electric field strength $E(x)$ of crystal planes in that point to eq. (7).

A direct averaging (9) of the local pair production probabilities (6) over an inter-plane distance d_{pl} drastically simplifies the theoretical description of pair production and related crystal optical anisotropy in hard gamma region. The limits of applicability of this simplifying procedure were also investigated. It was found [20] that at very small incidence angles $\theta \ll 1 \mu rad$ the local gamma-quantum flux attenuation near nuclear planes and axes violates an applicability of the averaging procedure (9) and leads to a new orientational effect which also can be observed at LHC. At large incidence angles $\theta \geq 100 \mu rad$ both the field non-uniformity [16] and interference of amplitudes of radiation and pair production processes in the fields of different planes (and axes) become important. An efficient numerical method based on fast Fourier transform was developed in [19] to evaluate the radiation and pair production characteristics in this case. Most important for future studies is the conclusion that the averaging procedure (9) (see also eq. (15) and (17) below) remains valid in all the incidence angle region $1 \mu rad \leq \theta \leq 100 \mu rad$ representing the main practical interest for the future investigations at LHC energies.

Since the Airy function (8) decreases like $\exp(-2\xi^{3/2}/3)$ at $\xi > 1$, the pair production is suppressed in all the crystal at any distribution of gamma-quantum energy between the electron and positron if $\xi > (3/2)^{2/3}$ at $\varepsilon_+ = \varepsilon_- = \omega/2$ and $E = E_{max}$. A "strict" estimate of the "threshold" gamma-quantum energy following from these conditions differs from the estimate (5) only by a multiplier 8/9.

Besides the necessity to overage the local probabilities (6) over the crystal volume (in fact, over the inter-plane

distance - see eq. (9)), there exists another complicating feature of the pair production process in crystals which does not accompany the same process in a uniform electromagnetic field in vacuum. In fact, the probabilities (9) describe only the coherent pair production process, taking place in the averaged field of crystal planes. In addition to it, the incoherent pair production process on separate nuclei also manifests itself in crystals. This process reminds the Bethe-Heitler pair production process, however is considerably modified by the influence of the averaged crystal field on the pair formation process initiated by a nucleus. The investigation of the radiation and pair production processes in the uniform field in the presence of Coulomb scatterers allowed us to combine [17] their theories in uniform electromagnetic field and in amorphous medium as well as to obtain the expressions [1, 18]

$$W_{\parallel(\perp)}^{inc} = \frac{1}{15L_{rad}\omega^3} \times \int_0^{d_{pl}} \frac{n(x)}{n_0} \frac{dx}{d_{pl}} \int_0^\omega d\varepsilon_+ \left[1 - \theta(1-\xi) \frac{\ln\xi}{2\ln(183Z^{-1/3})} \right] \times \left[\omega^2[(1 \pm 1)\xi^4 \Upsilon \mp 6\xi \Upsilon - (3 \pm 4)\xi^2 \Upsilon' - (1 \pm)\xi^3] + (\varepsilon_+^2 + \varepsilon_-^2)[(1 \mp 1)\xi^4 \Upsilon + (3 \pm 6) \times \xi \Upsilon - (5 \mp 4)\xi^2 \Upsilon' - (1 \mp 1)\xi^3] \right] \quad (10)$$

for the probabilities of incoherent pair production by polarized gamma-quanta, where

$$\Upsilon = \Upsilon(\xi) = \int_0^\infty \sin\left(\xi t + \frac{t^3}{3}\right) dt \quad (11)$$

is the epsilon function,

$$n(x) = \frac{n_0 d_{pl}}{\sqrt{2\pi u}} \exp\left(-\frac{x^2}{2u^2}\right) \quad (12)$$

is the local nuclear density and

$$\frac{1}{L_{rad}} = 4\alpha n_0 \left(\frac{Z\alpha}{m}\right)^2 \ln(183Z^{-1/3}) \quad (13)$$

is the radiation length. The energy dependence of optimal dichroic polarizer length and pair production asymmetry evaluated using the total pair production probabilities by polarized gamma-quanta

$$W_{\parallel(\perp)} = W_{\parallel(\perp)}^{coh} + W_{\parallel(\perp)}^{inc} \quad (14)$$

is given in fig. 2.

A nearly quadratic proportionality of the incoherent probability (10) to Z predetermines a higher role of incoherent processes in heavier crystals which explains the lower value of the tungsten polarizer asymmetry. In both *Si* and *Ge* crystals, possessing both lower Z and much better quality, the asymmetry value reaches its maximum at the energy close to the "threshold" energy (5), allowing to observe the effect of uniform field dichroism in optimal conditions as well as to use it for production and analysis of polarized beams in the TeV energy region.

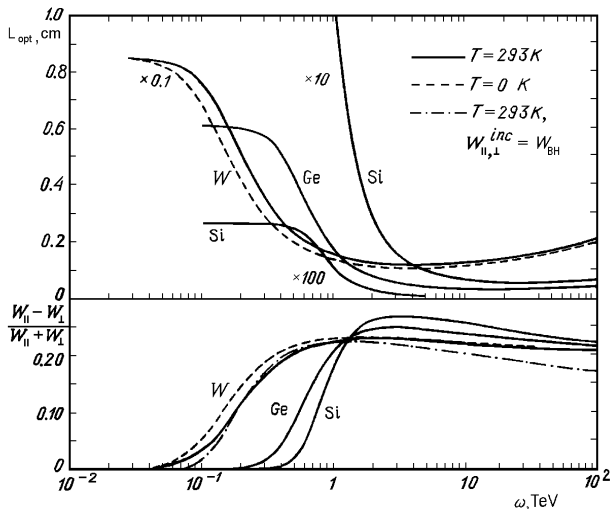


FIG. 2: Energy dependence of the "optimal" polarizator length and asymmetry coefficient of pair production by linearly polarized gamma-quanta for different crystals and temperatures.

IV. CRYSTAL BIREFRINGENCE

As is well known, dichroism manifestation predetermines the existence of birefringence. In particular, the polarization dependence of pair production in uniform magnetic field by photons is connected with the effect of magnetic birefringence of vacuum. This effect is described by the non-linear QED lagrangian [11, 12] and is considered [13] as "one of the most important predictions of QED which has not been verified". It also widely manifests itself in the atmospheres of neutron stars and white dwarfs, determining the linear polarization of x-ray radiation emitted from their surfaces. The effect of magnetic birefringence of vacuum causes the Cotton-Mouton effect or transformation of the linearly polarized light into the circularly polarized one (or vice versa).

For a long time people try to observe this effect measuring a small ellipticity acquired by linearly polarized light in the laboratory magnetic field. However because of the low photon frequencies and magnetic field strengths one has to observe the ellipticity as small as 10^{-9} .

An alternative method to study nonlinear effect of vacuum birefringence by observing the Cotton-Mouton effect accompanying polarized gamma-quanta propagation through crystals was proposed by Cabibbo [22]. The birefringence effect predicted by Cabibbo is connected with polarization dependence of the so-called coherent pair production and its nature considerably differs from that of the magnetic birefringence of vacuum. This effect can be observed at the energies considerably lower than the "threshold" energy (5). The preliminary results [23] of its observation are somewhat uncertain.

The LHC energy and strong fields of crystal planes provide together really optimal possibilities to observe the vacuum birefringence of semi-uniform averaged crystal field, the nature of which does not principally differ from that of the magnetic birefringence of vacuum. As is known, the relation of dichroism and birefringence effects is expressed mathematically by the dispersion relations. The latter allow to use the probabilities (9) and (10) to obtain, respectively, the coherent

$$n_{\parallel(\perp)}^{coh}(\omega) = \int_0^{d_{pl}} n_{y(x)}(\kappa(x)) \frac{dx}{d_{pl}}, \quad (15)$$

where

$$n_{x(y)}(\kappa) = \frac{1}{2} \Re \varepsilon_{x(y)} = 1 - \frac{\alpha}{3\pi} \left(\frac{E}{\kappa E_0} \right)^2 \int_0^\omega \frac{\Upsilon'(\xi)}{\xi} \left(\frac{\omega^2}{\varepsilon_+ \varepsilon_-} + \frac{1 \mp 3}{2} \right) \frac{d\varepsilon_+}{\omega}, \quad (16)$$

and incoherent

$$n_{\parallel(\perp)}^{inc} = n_{\omega \ll \omega_{\kappa=1}} - \frac{\sqrt{\pi}}{30 L_{rad} \omega^4} \int_0^{d_{pl}} \frac{n(x) dx}{n_0} \frac{dx}{d_{pl}} \times \int_0^\omega d\varepsilon_+ \left(\omega^2 [(1 \pm 1)\xi^4 \Phi \mp 6\xi \Phi - (3 \pm 4)\xi^2 \Phi'] + (\varepsilon_+^2 + \varepsilon_-^2) [(1 \mp 1)\xi^4 \Phi + (3 \pm 6)\xi \Phi - (5 \mp 4)\xi^2 \Phi'] \right) \quad (17)$$

contributions to the refractive indexes

$$n_{\parallel(\perp)} = n_{\parallel(\perp)}^{coh} + n_{\parallel(\perp)}^{inc} \quad (18)$$

of gamma-quanta polarized parallel and normal to a crystal plane being nearly parallel to the gamma-quanta momentum. As is known, the difference of refractive indexes (18) (or, more precisely, of principal values of the refraction tensor) allows to construct a quarter wave plate of thickness $L_{\lambda/4}$ which satisfies the well known relation

$$\omega(n_{\parallel} - n_{\perp})L_{\lambda/4} = \frac{\pi}{2}. \quad (19)$$

The LHC energies provide really optimal conditions allowing to observe the nonlinear vacuum birefringence effect by a complete transformation on a linearly polarized gamma-beam to a circularly polarized one in the quarter wave plate of modest thickness with acceptable losses of beam intensity. The energy dependence of the quarter wave plate thickness and the coefficient of linearly polarized gamma-beam attenuation at this thickness is given in fig. 3. One can easily see that the gamma-beam attenuation by a quarter wave plate grows fast both at low and high energies due to the increase of relative values of probabilities of, respectively, incoherent (10) and coherent (6) pair production. Due to this growth an optimal energy region for the observation of the birefringence effect arises. However especially large

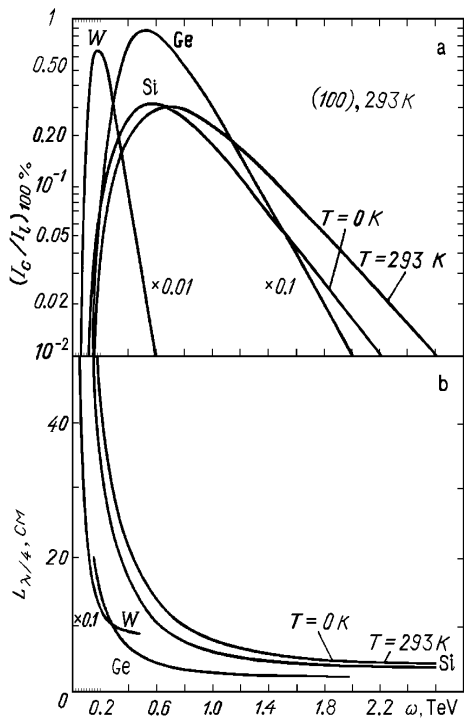


FIG. 3: Energy dependence of the attenuation coefficient and quarter wave thickness for different crystals and temperatures. The dashed lines correspond to the low energy limits [1] of eq. (10) and (16)

role of incoherent processes makes the use of tungsten quarter wave plate not very effective even in its optimal region $\omega \sim 0.1 - 0.2 \text{ TeV}$. On the other hand both *Ge* and *Si* quarter wave plates possess modest lengths and beam attenuation coefficients in the energy region $\omega \sim 0.5 - 1 \text{ TeV}$, which, thus, turns out to be the optimal region for study of the vacuum birefringence using gamma-quantum beams which can readily be generated at LHC.

V. SPIN EFFECTS

It should be pointed out that the effects of dichroism and birefringence inherent to the uniform electromag-

netic field can be observed in the field of crystal planes (see fig. 1) which has opposite directions on the opposite sides of crystal planes only because the signs of these effects do not depend on the sign of the field projection on the plane normal. Unfortunately, all the effects connected with the electron spin behavior in electromagnetic field do not possess this property and can not be so directly observed in usual crystals. A method making it possible to observe these effects was suggested in [24] and consists in using *bent crystals* in which channelling particles move in the regions in which the transverse field of particular direction dominates. Remind that bent crystals can be also used both to clean the halo of the LHC beam and to extract it for experiments on a fixed target.

Bent crystals also allow to observe electron self-polarization [25, 26] (see also [27, 28]) and polarized electron-positron pair production by gamma-quanta [8, 29] (see also [30]), positron (electron) drastic magnetic moment decrease [31, 32], and electron spin rotation in a circularly polarized electromagnetic wave [33, 34, 35] in crystal field harmonics [36]. Comparing with the crystal optical anisotropy in the gamma-region, the theory of effects connected with the electron (positron) spin will be additionally complicated by the necessity to consider both the charged particle motion in crystal field and its disturbances caused by multiple scattering and radiative cooling [37, 38].

Note also that another mechanism of dichroism and birefringence connected with electron and positron scattering asymmetry by crystal axes [39] was predicted in [40, 41]. Special attempts to study the manifestation on these effects at LHC energies should be undertaken.

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