Sensitivity of the neutron crystal diffraction experiment to the neutron EDM and to the nuclear P-,T-violating forces.

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We establish a link between an angle of the neutron polarization rotation in a crystal diffraction experiment and constants of the P-,T- violating interactions. The consideration applies to the energy range of thermal and resonance neutrons.

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Parity and time reversal violation in nuclear and atomic physics remains the important contemporary problem. One of the way to search for P-,T-odd interactions consists in measurement of the electric dipole moment of an elementary particle. The most precision test performed with ultra could neutrons in electric field gives restriction 10^{-26} e cm for the electric dipole moment of a neutron [1]. However others tests exist, namely, searching for the neutron dipole moment by a crystal-diffraction method was suggested [2, 3, 4, 5, 6, 7] and now is under realization in the St. Petersburg Institute for Nuclear Physics (PINP)[8, 9]. The test is based on the neutron spin rotation in the non-centrosymmetruc crystal. Correction to the neutron refractive index arises due to neutron rescattering by the crystalline planes [6, 7]. Because neutrons interact with the nuclei of a crystal by the electromagnetic and strong interactions a question arises about possible contribution of the P-,T- violating nuclear forces to the effect under consideration. In this paper we analyze some different mechanisms by which the P-,T-violating nuclear forces can contribute to the neutron spin rotation in a crystal.

According to Refs. [6, 7] angle of neutron polarization rotation ϕ_{PT} in a crystal of a length L is determined by the spin-dependent part of the refractive index:

$$\phi_{PT} = 2kL|\boldsymbol{M}_{PT}|, \tag{1}$$

where k is the neutron wave number, $|\boldsymbol{M}_{PT}|$ is absolute value of the vector forming the P-,T- violating matrix part of the neutron refractive index $n_{PT} \sim \boldsymbol{\sigma}\cdot\boldsymbol{M}_{PT}$. Vector $\boldsymbol{\sigma}$ consists of Pauli matrices.

Under condition of a weak diffraction at some particular crystalline planes corresponding to the vector $\boldsymbol{\tau}$ the refractive index can be estimated as [6, 7]

$$n_{PT} \sim \left(\frac{4\pi}{k^2 V}\right)^2 \frac{f_s\, f_{PT}(\boldsymbol{\tau})}{2\alpha_B}\, e^{-2w(\boldsymbol{\tau})} s(\boldsymbol{\tau}), \tag{2}$$

where f_s is the neutron scattering amplitude due to strong interaction, $f_{PT}(\boldsymbol{q})$ is the P-,T-violating scattering amplitude,

$$\alpha_B = \frac{\boldsymbol{\tau}(\boldsymbol{\tau}+2\boldsymbol{k})}{k^2}, \tag{3}$$

V is a volume of a unit cell of the crystal, $e^{-w(\boldsymbol{\tau})} = exp\left(-u^2\tau^2/4\right)$ is the factor of Debye-Waller and u^2 is a mean value of the square of the amplitude of the thermal motion atoms about their equilibrium positions. The multiplier $s(\boldsymbol{\tau}) = \sum_{jl}\left\{\exp(i\boldsymbol{\tau}\boldsymbol{R}_l) - \exp(-i\boldsymbol{\tau}\boldsymbol{R}_j)\right\}$ [10], where summation is carried out over atoms in a unit crystal cell, characterizes degree of violation of the central symmetry of the crystal cell and $s(\boldsymbol{\tau})$ is zero for crystal possessing center of symmetry. That is, only non central symmetric (piezoelectric) crystals to be used for the above diffraction experiments.

In the geometry, where the wave vector of the rescattered wave is parallel to the incident one (see Fig. 1) no other P-,T- even interactions are able to produce polarization rotation and the presence of the last will be a signal of the P-,T- violation.

Let us compare contribution of the different sources of P-,T- violation, namely, the P-,T- odd neutron nuclear interactions and interaction of neutron EDM with crystal electric field, to the angle of polarization rotation (or spin dichroism) which to be measured in the current PINP experiment.

At first we consider P-,T-odd neutron scattering amplitude by nucleus. The amplitude contains electromagnetic and nuclear parts. The first one is due to interaction of the neutron dipole moment with the atom electric field. The last one arises due to nuclear forces and can be deduced from the P-,T- odd nucleon-nucleon interaction, which in part can be described by the one pion exchange.

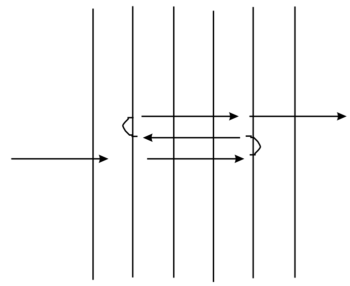

FIG. 1: Rescattering of a neutron by the crystalline planes.

Restricting only to the π-mesons Lagrangian density of the interaction can be written in the form [11]

$$\mathcal{L} = i g_\pi \bar{N} \gamma_5 (\boldsymbol{\tau}\boldsymbol{\pi}) N + \bar{g}_\pi^{(0)} \bar{N}\,(\boldsymbol{\tau}\boldsymbol{\pi})\,N + \bar{g}_\pi^{(1)} \bar{N}\,\pi_0\,N + \bar{g}_\pi^{(2)} \bar{N}(3\tau^z \pi_0 - \boldsymbol{\tau}\boldsymbol{\pi})N, \tag{4}$$

where $N(x)$ is the nucleon field and $\pi_0(x)$, $\boldsymbol{\pi}(x)$ are fields of π-mesons. Three last terms are parity and time reversal violating.

In the frame of the one pion exchange (see Fig. 2 (a)) interaction of the incident neutron with the one of nuclei nucleons can be deduced [11]:

$$V_{PT}^{(\pi)} = \frac{1}{2 m_N m_\pi^2}\Big(g_\pi \bar{g}_\pi^{(0)} (\boldsymbol{\tau}\boldsymbol{\tau}_1)(\boldsymbol{\sigma} - \boldsymbol{\sigma}_1) + g_\pi \bar{g}_\pi^{(1)} \left((\tau_1^z + \tau^z)(\boldsymbol{\sigma} - \boldsymbol{\sigma}_1) + (\tau_1^z - \tau^z)(\boldsymbol{\sigma} + \boldsymbol{\sigma}_1)\right) + g_\pi \bar{g}_\pi^{(2)} (3\tau^z \tau_1^z - \boldsymbol{\tau}\boldsymbol{\tau}_1)(\boldsymbol{\sigma} - \boldsymbol{\sigma}_1)\Big) \boldsymbol{\nabla} \delta^{(3)}(\boldsymbol{r}), \tag{5}$$

where due to low energy of the neutron the interaction is considered as having a zero range. Let us turn to the neutron-nucleus interaction suggesting that the nucleus contain equal number of non polarized proton and neutrons obeying the distribution function $\rho(\boldsymbol{r})$, which is normalized as $\int \rho(\boldsymbol{r}) d^3\boldsymbol{r} = 1$. In this suggestions the averaging gives the P-T- odd neutron-nuclei interaction:

$$V_{\rm NPT} = -\frac{A}{m_N m_\pi^2} g_\pi \bar{g}_\pi^{(1)} \boldsymbol{\sigma} \boldsymbol{\nabla} \rho(\boldsymbol{r}), \tag{6}$$

where A is the number of nucleons in nuclei, and relation $\int \rho(\boldsymbol{r}') \nabla \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}') d^3\boldsymbol{r}' = \nabla \rho(\boldsymbol{r})$ is used.

The terms proportional to the $\bar{g}_\pi^{(0)}$ and $\bar{g}_\pi^{(2)}$ vanishes because the isospin dependence leads to the opposite sign for the $n-n$ and $n-p$ interaction whereas we consider the nucleus containing equal number of the n and p. In the the general case of the nucleus with the different numbers of neutrons and protons contribution proportional to the constants $\bar{g}_\pi^{(0)}$, $\bar{g}_\pi^{(2)}$ will be present.

The next step is the evaluation of the neutron scattering amplitude by nucleus in the first order on the momentum transferred $\boldsymbol{q}$

$$f_{\rm NPT}(\boldsymbol{q}) = -\frac{m_N}{2\pi} \int V_{\rm NPT}(\boldsymbol{r}) e^{-i\boldsymbol{q}\boldsymbol{r}} d^3 r = i A \frac{g_\pi \bar{g}_\pi^{(1)}}{2\pi} \frac{\boldsymbol{\sigma}\boldsymbol{q}}{m_\pi^2} F(\boldsymbol{q}), \tag{7}$$

where one can set form factor $F(\boldsymbol{q})$ to unity when the neutron wave length much greater than the radius of a nucleus.

Another source of P- T- violation is the interaction of the electric dipole moment of a neutron with the electric field of an atom leading to the following interaction:

$$V_{\rm EDM} = -d_n\, \boldsymbol{\sigma} \boldsymbol{E} = d_n\, \boldsymbol{\sigma} \boldsymbol{\nabla} \left(\frac{Ze}{r} \exp(-r/R_a) \right) = -d_n Z\, e\, \frac{\boldsymbol{\sigma}\boldsymbol{r}}{r} \left(\frac{1}{r^2} + \frac{1}{r R_a} \right) \exp(-r/R_a), \tag{8}$$

where R_a is the atom radius. Scattering amplitude for this case has the form

$$f_{\rm EDM}(\boldsymbol{q}) = \frac{m_n}{2\pi} d_n\, Z\, e \int \frac{(\boldsymbol{\sigma}\boldsymbol{r})}{r} \left(\frac{1}{r^2} + \frac{1}{r R_a} \right) \exp(-r/R_a) e^{-i\boldsymbol{q}\boldsymbol{r}} d^3\boldsymbol{r} = -2\, i\, m_n\, d_n Z e \frac{(\boldsymbol{\sigma}\boldsymbol{q})}{q^2} \left(1 - \frac{1}{1 + q^2 R_a^2} \right). \tag{9}$$

It should be noted that the neutron dipole moment can be expressed through $\bar{g}_\pi^{(0)}$ (see references in [12] and Fig. 2 (b))

$$d_n = \frac{e}{4\pi^2 m_n} g_\pi \bar{g}_\pi^{(0)} \ln \frac{m_n}{m_\pi}, \tag{10}$$

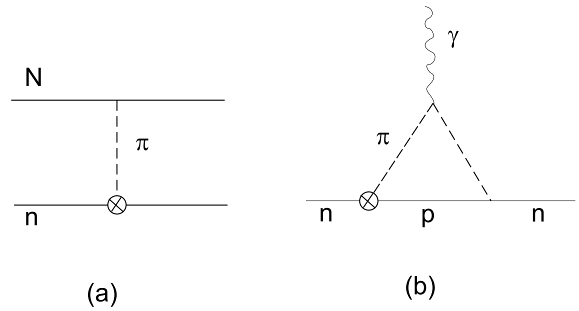

FIG. 2: (a) Diagram of one pion exchange, (b) Contribution of the pion loop to the neutron EDM. P-,T- violation vertex is denoted by a crossed circle.

which allows to compare $f_{\mathrm{EDM}}(\boldsymbol{q})$ with $f_{\mathrm{NPT}}(\boldsymbol{q})$. Finally we come to the conclusion that the contribution of the neutron EDM interacting with the electric field of the atom is much greater than that originated from the P-,T- violating nuclear forces (compare fifth and seventh columns of the Table 1), when one assumes that $\bar{g}_\pi^{(0)}$ and $\bar{g}_\pi^{(1)}$ are of the same order.

However we can consider another situation when the measurements are carried out in the vicinity of compound nucleus p-resonance. It is known that the P-odd and P- ,T- odd effects are greatly enhanced in the vicinity of the p-resonance.

In the case of p-resonance neutron scattering amplitude has the form [13, 14]

$$f_{PT}^{res}(\boldsymbol{q}) \sim \frac{(\boldsymbol{\sigma q})}{k^2} \frac{\gamma_p W_{ps} \gamma_s}{(E - E_p + i\Gamma_p)(E_s - E_p)}, \tag{11}$$

where γ_p is the capture amplitude of the p-resonance, γ_s is that of nearest s-resonance, W_{sp} is the matrix element of the P-,T- violating interaction between compound $s-$ and $p-$ states of the compound nuclei. Capture amplitude to the $p-$resonance is suppressed compared to the $s-$resonance one: $\gamma_p \sim (kR)\gamma_s$, where R is the radius of the nuclei. According to the theory of the compound state reactions the wave function of the compound state can be represented as a sum of the shell-model functions: $\Psi_p = \sum_m^N C_{pm}\psi_m$, where ψ_m is the basic shell model functions and $N \sim 10^6$ is the number of the principal components [13, 14]. The coefficients C_{pm} as well as C_{sm} are random giving the following estimate for the matrix element between compound states $W_{sp} \sim W/\sqrt{N}$ where W is the typical value of the single particle P-T- odd interaction, which according to the equation (6) can be estimated as $W \sim \frac{g_\pi \bar{g}_\pi^{(1)} A}{m_n m_\pi^2 R^4}$. Note, that not only the single particle interaction (6) give rise to the contribution to the matrix element, but also original two-particle interaction. Thus, the terms proportional $\bar{g}_\pi^{(0)}$ and $\bar{g}_\pi^{(2)}$ can also give contribution of the same order to the matrix element W. Taking into account that the typical energy interval between compound states $E_s - E_p \sim \Delta E/N$, where $\Delta E \sim 7\ MeV$ is the typical energy distance between shell model states we come to the estimate

$$f_{PT}^{res}(\boldsymbol{q}) \sim \frac{(\boldsymbol{\sigma q})}{k^2} \frac{1}{kR} \frac{\sqrt{N}\,W}{\Delta E} \frac{\gamma_p^2}{\Gamma_p} \tag{12}$$

in the vicinity of p-resonance $E - E_p \sim \Gamma_p$.

Another way to estimate P-,T- odd resonance amplitude is to use results of the measurement of P-violating, T-conserving spin rotation in ^{139}La [15, 16, 17, 18]. In these experiments it was found the relation of the weak matrix element $\widetilde{W}_{sp}$ between compound states to the energy difference of the p-resonance with the nearest s-resonance

$$\frac{\widetilde{W}_{sp}}{E_s - E_p} = \frac{1.7 \times 10^{-3}}{38} = 4.5 \times 10^{-5}. \tag{13}$$

In assumption that this matrix element originates from the π -meson Lagrangian including strong and P- odd, T-even interaction [19]

$$\mathcal{L} = i g_\pi \bar{N} \gamma_5 (\boldsymbol{\tau \pi}) N + h_\pi \bar{N} (\boldsymbol{\tau} \times \boldsymbol{\pi})_3 N, \tag{14}$$

leading to the P-odd, T-even two particle interaction

$$V_{\mathrm{P}}(\boldsymbol{r}) = i \frac{g_\pi h_\pi}{2\sqrt{2} m_N m_\pi^2} (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_3 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) [(\boldsymbol{p}_1 - \boldsymbol{p}_2), \delta^{(3)}(\boldsymbol{r})], \tag{15}$$

TABLE I: Angle of neutron spin rotation due to interaction of the neutron EDM with the crystal electric field — ϕ_{EDM}, due P-,T- violating nuclear forces under potential scattering — ϕ_{NPT} and under resonance scattering — ϕ_{res}. The quantities marked by the letter $^{a)}$ corresponds to the estimate (12), whereas the case marked with the letter $^{b)}$ corresponds to the Eqs. (11), (16).

E, eV	$\Delta\theta$	$\Delta k/k$	$\frac{4\pi}{k^2V}f_s$	ϕ_{EDM}	ϕ_{res}	ϕ_{NPT}
0.003	10^{-3}	10^{-6}	5.6×10^{-6}	4.7×10^{-5}	—	2.8×10^{-11}
0.1	10^{-3}	10^{-6}	1.7×10^{-7}	3.5×10^{-8}	—	6.7×10^{-13}
0.73	10^{-3}	10^{-6}	2.4×10^{-8}	1.4×10^{-10}	$^{a)}\,8.3\times10^{-9}$	2.0×10^{-14}
					$^{b)}\,5.6\times10^{-10}$	
0.73	10^{-4}	10^{-8}	2.4×10^{-8}	1.4×10^{-8}	$^{a)}\,8.3\times10^{-7}$	2.0×10^{-12}
					$^{b)}\,5.6\times10^{-8}$	

one may suggests the same coefficient of proportionality between the coupling $g_\pi h_\pi$ and matrix element $\widetilde{W}_{sp}$, and between the P-,T- violating coupling $g_\pi\bar{g}_\pi^{(i)}$ and matrix element W_{sp}. This allows to estimate W_{sp} directly:

$$W_{sp}=\frac{\bar{g}_\pi^{(i)}}{h_\pi}\widetilde{W}_{sp}, \tag{16}$$

where h_π can be estimated as $h_\pi=1.9\times10^{-7}$ [19] (it can be compared with the value $g_\pi\approx13$). Other quantities are total neutron width of p-resonance $\Gamma_p=0.045\ eV$, neutron width $\Gamma_p^n=3.6\times10^{-8}\ eV$ [20] and that for s-resonance $\Gamma_s^n=0.1\ eV$. Energy distance to the s-resonance state is $E_s-E_p\sim38\ eV$ [15].

Let us come to the estimates of the polarization rotation angle in the different range of energies. For the parameter α_B (3) describing deflection from the exact diffraction condition in the backscattering geometry (Bragg angle equals 180^o) we have $\tau=2k$ and

$$\alpha_B=4\left(\Delta k/k+\Delta\theta^2\right). \tag{17}$$

The quantity $\Delta\theta\sim10^{-3}$ describes typical angle spread around the Bragg direction and $\Delta k/k\sim10^{-6}$ describes wave number spread. Other parameters are $L=0.5\ m$, $u=0.1$ Å, $V^{1/3}=5$ Å, $R_a=2$ Å, $R=1.45A^{1/3}\ fm$, where $A=139$ is the mass number of nuclei and $Z=57$ is charge number. The value of the factor $s(\boldsymbol{\tau})$ describing the absence of the center of symmetry was set to unity in the above calculations because the in the most of the piezoelectric crystals central symmetry is violated strongly. Value of the neutron EDM is taken $10^{-26}\ e\ cm$ and corresponding product of the constants $g_\pi\bar{g}_\pi^{(i)}$ is determined from the equation (10) in suggestion that all the P-,T- violating constants $\bar{g}_\pi^{(1)}$, $\bar{g}_\pi^{(2)}$, $\bar{g}_\pi^{(3)}$ are of the same order.

It should be noted that from the one hand the angle $\Delta\theta$ should be greater then the mosaism of a crystal (typical value $10^{-3}-10^{-4}$) and from the other hand it should satisfy the condition of a weak diffraction: $\alpha_B=4(\Delta\theta^2+\Delta k/k)>>\frac{4\pi}{k^2V}f_s$.

To summarize we have compared direct neutron EDM contribution and that of the P-,T-violating nuclear forces to the angle of neutron polarization rotation in the crystal diffraction experiment. It turns out to be that at energies near 0.003 eV only the first one is considerable, (see first line of the Table 1) and the total number of neutrons needed is $N_{tot}=1/\phi_{EDM}^2\sim4.5\times10^8$. Corresponding accumulation time is $T=N_{tot}/(N_0S\frac{\Delta\theta}{\pi}2\frac{\Delta k}{k})\sim0.8\ sec$ under the neutron flux $N_0=10^{15}\ \ neutrons/cm^2/sec$ at the reactor zone (i.e. at the bottom of the reactor channel), area of the crystal $S=30\times30\ cm^2$. It should be noted that we have considered the best conditions, that is, the greatest possible neutron flux and the large area crystal containing relatively heavy La atoms (scattering length $f_s=8.2$ fm), whereas at present time only large perfect crystals of quartz containing light elements of periodic table are available.

At higher energies, under resonance scattering, contribution of the P-,T-violating nuclear forces begin to dominate, however, at the energy 0.73 eV of the ^{139}La resonance one needs $N_{tot}=1.5\times10^{12}$ (see last line of table 1, case $^{a)}$) and accumulation time $T=30$ days under the same flux and crystal area. Hence the experiment in the resonance range should be done at the reactors having much excess of the neutrons with energies near 0.73 eV.

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