Influence of Multiple Scattering on High-energy Deuteron Quasi-optical Birefringence Effect

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Introduction

A quasi-optical macroscopic quantum phenomenon of birefringence, i.e., the effect of particle spin rotation (oscillation) and spin dichroism accompanying the passage of high-energy particles with spin $S \geq 1$ through isotropic homogeneous matter was first described in [1, 2]. This effect is analogous to the effect known in optics as the birefringence of light in an optically anisotropic medium associated with the dependence of the index of refraction on the state of light polarization, i.e., on the photon spin state. In contrast to light, whose wavelength is much longer than the distance between the atoms of matter, the de Broglie wavelength of a fast deuteron is much smaller than this distance. Nevertheless, according to [1, 2], in this case one can also introduce the spin-dependent index of refraction for particles. The deuteron birefringence effect occurs even in a homogeneous isotropic medium and is due to the intrinsic anisotropy inherent in particles with spin $S \geq 1$. Moreover, according to [1, 2], the magnitude of this effect even increases with growing particle energy, i.e., with diminishing wavelength.

It was shown in [1, 2] that the spin dichroism described by the imaginary part of the index of refraction causes tensor polarization of the initially unpolarized deuteron beam. Recently, this effect was experimentally revealed in the energy regions $5 \div 20$ MeV [3, 4] and 3 GeV [5]. According to [4, 5], the appearance of tensor polarization due to spin dichroism in particles with spin $S \geq 1$ can be used for designing the source of deuterons, which possess tensor polarization. According to [6, 7, 8], the birefringence effect should also be taken into account in experiments to search for deuteron EDM.

In theoretical description of the birefringence phenomenon, it should be taken into account that the amplitude of forward scattering of a deuteron by a nucleus, which determines the index of refraction, includes the contribution due to Coulomb interaction alongside with that due to strong interaction. The Coulomb-nuclear interference correction to the rotation angle of the polarization vector and spin dichroism of the deuteron was computed in [2, 9]. According to [9], the Coulomb-nuclear interference enables explanation of the change of sign of spin dichroism with changing deuteron energy in the range of $5 \div 20$ MeV.

It should be noted that the interaction of deuterons and matter is accompanied not only by coherent scattering, which leads to the formation of a coherent wave, but also by incoherent scattering resulting in single and multiple scattering of particles in the target. In a kinetic equation for the density matrix, the characteristics of elastically scattered particles are described by the terms proportional to the squared absolute value of the total scattering amplitude (this amplitude is the sum of the Coulomb amplitude and the nuclear scattering amplitude modified by the Coulomb interaction) and as a result, in this case the terms describing the interference of the Coulomb and nuclear interactions should also be considered. In this connection, [10, 11] give the analysis of the correction due to the stated interference terms to the rotation of the polarization vector of protons passing through polarized matter for such target thicknesses and scattering angles at which the particle angular distribution is determined by the multiple Coulomb scattering and the single nuclear scattering in matter. It was shown in [11] that in the expression for the spin rotation angle of a high energy proton beam, the contribution of the Coulomb-nuclear interference to the amplitude of nuclear scattering at zero angle is compensated by the correction from the incoherent Coulomb-nuclear scattering. This statement was also formulated for scattering of particles with spin 1 [12].

In the present paper is shown that within the domain of applicability of the first Born approximation, for the Coulomb amplitude the contributions to the rotation angle of the deuteron polarization vector coming from the Coulomb-nuclear interference are compensated only in the case when the detector provides a maximum coverage within a 4π solid angle geometry. If the detector registers the particles moving within a certain solid angle $\Delta\Omega \ll 4\pi$ with respect to the initial direction of the beam incidence, as it does in a real experiment, the interference terms stated above are not compensated. The magnitude of the contribution of the Coulomb-nuclear interference depends appreciably on the specific geometry of the experiment and is quite observable in the experiments on measuring the spin rotation angle of particles passing through the target. Multiple scattering of particles in the target leads to the fact that the magnitude of the contribution of the Coulomb-nuclear interference to the rotation angle of the beam polarization vector depends not only on the angle $\Delta\Omega$ of the detector, but also on the mean square angle of multiple scattering. It is shown that if after the beam has passed the target, the magnitude of the mean square angle of multiple scattering is of the order of magnitude or greater than the squared diffraction angle of nuclear scattering, the Coulomb-nuclear interference can be neglected for any registered angle $\Delta\Omega$ of the detector. Otherwise, the contribution of the total Coulomb-nuclear interference to the rotation angle should be taken into account.

1 Kinetic equation for the spin density matrix

Let us consider the process of particle (beam of particles) passage through a target consisting of N_t particles interacting with one another by means of a certain potential U.

Suppose that an incident particle has a rest mass m and spin S_d. The target consists of N_t bound particles with mass M and spin s.

The Hamiltonian of the scattering system is written in the form:

$$H_t = \sum_{\alpha=1}^{N_t} K_\alpha + U, \tag{1.1}$$

where K_α is the kinetic energy operator of particle α, U is the interaction energy of N_t particles of the scatterer.

The solution of the stationary Schrödinger equation

$$H_t\Psi_\gamma = W_\gamma\Psi_\gamma \tag{1.2}$$

determines the possible values of the energy W_γ of the system and the corresponding set of wave functions.

$$\Psi_\gamma = \Psi_\gamma(\vec{R}_1, s_1, \ldots, \vec{R}_N, s_N), \tag{1.3}$$

where $(\vec{R}_\alpha, s_\alpha)$ are the spatial and spin coordinates of particle α in the scatterer.

The operator of interaction between the incident particle and the scatterer is defined in terms of V: $V = \sum_{\alpha=1}^{N_t} V_\alpha$. Here V_α describes the interaction between the beam particle and the target particle α. The Hamiltonian of the whole system has a form:

$$H = K_d + H_t + V, \tag{1.4}$$

where K_d is the kinetic energy operator of the beam particle d.

To describe the process of particle d transmission through the target, find the density matrix $\hat{\rho}(t)$ of the system "incident particle + target". This density matrix satisfies the quantum Liouville equation:

$$i\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}(t)]. \tag{1.5}$$

The solution of this equation can formally be written using the evolution operator $\hat{U}(t, t_0)$

$$\hat{\rho}(t) = \hat{U}(t, t_0)\hat{\rho}(t_0)\hat{U}^{+}(t, t_0), \tag{1.6}$$

which is related to the explicitly time-independent Hamiltonian of the system as $\hat{U}(t,t_0) = e^{-\frac{i}{\hbar}\hat{H}(t-t_0)}$. The time moments t_0 and t correspond to the state of the system before and after scattering of the incident particle by the α-th scatterer.

Let us consider the target as a thermostat ($N_t \gg 1$). Then the statistical operator $\hat{\rho}$ of the system can be represented as a direct product $\hat{\rho} = \hat{\rho}_d \otimes \hat{\rho}_t$, where $\hat{\rho}_d$ is the density matrix of the incident particle, $\hat{\rho}_t$ is the equilibrium density matrix of the medium. The spin density matrix $\hat{\rho}_d$ includes the elements diagonal and nondiagonal with respect to the momenta $\vec{k}$ of the incident particle: $\hat{\rho}_d = \hat{\rho}_d(\vec{k},\vec{k}')$. However, the nondiagonal elements oscillate fast and after several collisions with target nuclei, in order to describe the process of multiple scattering, one can assume that the density matrix $\hat{\rho}_d(\vec{k},\vec{k}')$ is diagonal [10, 13, 14, 15], i.e., $\hat{\rho}_d(\vec{k},\vec{k}') = \delta_{\vec{k},\vec{k}'}\hat{\rho}_d(\vec{k})$.

The time interval Δt during which the density matrix is diagonalized satisfies the inequality $\Delta t \gg R/\bar{v}$, where R is the radius of action of the forces, $\bar{v}$ is the particle mean velocity in matter. This means that the evolution operator $U(t,t_0)$ can be replaced by the Heisenberg's S-matrix $\equiv \lim\limits_{t\to\infty, t_0\to-\infty} U(t,t_0)$, which relates the asymptotic states of the system before scattering to those after scattering [10, 13, 14, 15]. The matrix elements of the S-matrix for a scattering system consisting of N_t particles are defined as follows:

$$S_{ba} = \delta_{ba} - i(2\pi)\delta(E_b - E_a)\sum_{\alpha=1}^{N_t} (\mathcal{T}_\alpha)_{ba}\,, \tag{1.7}$$

where E_a and E_b are the total energies of the system before and after scattering, respectively; $E_a = \varepsilon_k + W_\gamma$, $E_b = \varepsilon_{k'} + W_{\gamma'}$, ε_k and $\varepsilon_{k'}$ are the energies of the incident particle before and after the collision. In the momentum approximation, choose as $\mathcal{T}_\alpha$ the scattering matrix of particle d interacting with a free particle α [16].

The limits $t_0 \to -\infty$, $t \to \infty$ are understood as the times when the incident particle is situated at distances larger than R, but the interval $\Delta t = t - t_0$ is small as compared to $l/\bar{v}$ (l is the mean free path of a particle in matter).

Denote the density matrix of the system at time $t = t_0 + \Delta t$ by $\hat{\rho}'$, accordingly before scattering by the the α-th particle of the target, the density matrix will read $\hat{\rho}$. Equation (1.6) can be rewritten for the density matrix of the incident particle, using the S-matrix defined in equation (1.7):

$$\hat{\rho}\,'_d = \mathtt{Sp}_t S\hat{\rho}S^+, \tag{1.8}$$

where $\mathtt{Sp}_t$ means taking the trace over the states of the target.

Write equality (1.8) for the diagonal momentum-space elements of the particle density matrix:

$$\begin{aligned}\hat{\rho}'_d(\vec{k}) = \hat{\rho}_d(\vec{k}) - i(2\pi)\frac{\Delta t}{2\pi}\mathtt{Sp}_t\sum_{\alpha=1}^{N_T}\langle\vec{k},\gamma|\hat{\mathcal{T}}_\alpha|\vec{k},\gamma\rangle\hat{\rho}(\vec{k},\gamma) + i(2\pi)\frac{\Delta t}{2\pi}\mathtt{Sp}_t\sum_{\alpha=1}^{N_T}\hat{\rho}(\vec{k},\gamma)\langle\vec{k},\gamma|\hat{\mathcal{T}}_\alpha^+|\vec{k},\gamma\rangle +\\ +(2\pi)^2\frac{\Delta t}{2\pi}\mathtt{Sp}_t\sum_{\alpha,\beta=1}^{N_T}\sum_{\vec{k}'\gamma'}\langle\vec{k},\gamma|\hat{\mathcal{T}}_\alpha|\vec{k}',\gamma'\rangle\hat{\rho}(\vec{k}',\gamma')\delta(\varepsilon_{k'} - \varepsilon_k + W_{\gamma'} - W_\gamma)\langle\vec{k}',\gamma'|\hat{\mathcal{T}}_\beta^+|\vec{k},\gamma\rangle.\end{aligned} \tag{1.9}$$

In the momentum space, the matrix elements for the scattering matrix operator $\hat{\mathcal{T}}$ have a form [17]:

$$\langle\vec{k}',\vec{P}'_\alpha|\hat{\mathcal{T}}_\alpha|\vec{k},\vec{P}_\alpha\rangle = (2\pi)^3\delta(\vec{k}' + \vec{P}'_\alpha - \vec{k} - \vec{P}_\alpha)\langle\vec{k}',\vec{P}'_\alpha|\hat{\mathtt{T}}_\alpha|\vec{k},\vec{P}_\alpha\rangle, \tag{1.10}$$

$\vec{P}_\alpha$ denotes the momenta of the α-th scatterer, $\hat{\mathtt{T}}_\alpha$ is the matrix of scattering by the momentum shell. Recall that $\hat{\mathcal{T}}_\alpha$ and $\hat{\mathtt{T}}_\alpha$ remain the operators with respect to spin variables.

The states with a definite value of the momentum $|\vec{k}\rangle$ are normalized according to equality $\langle\vec{k}'|\vec{k}\rangle = \frac{(2\pi)^3}{V}\delta(\vec{k}' - \vec{k})$, where V is the normalization volume. Thus, substitution of summation over all $\vec{k}$ by integration is made as follows $\sum\limits_{\vec{k}} \to \dfrac{V}{(2\pi)^3} d^3\vec{k}$; the Kronecker symbol and the Dirac function δ are related as $\delta_{\vec{k}\vec{k}'} \to \dfrac{(2\pi)^3}{V}\delta(\vec{k}-\vec{k}')$.

Scattering processes that make the contribution to (1.9) can be both elastic and inelastic. Elastic scattering occurs in the absence of any excitation in the scatterer. Inelastic scattering accompanied by a small excitation in the scatterer is called "quasi-elastic" [16]. For such scattering the momentum $\vec{q}$ transferred to the α-th scatterer is $q \ll \sqrt{2MK_\alpha}$, $q \ll \sqrt{2MU_\alpha}$, $\vec{q} = \vec{k}' - \vec{k}$; $\vec{k}$ is the momentum of the incident particle before the collision with the scatterer, $\vec{k}'$ is the momentum of the beam particle after the collision.

When the momentum transferred to the scatterer α exceeds the momentum of the target particle in the initial bound state $q \gg \sqrt{2MK_\alpha}$, $q \gg \sqrt{2MU_\alpha}$, then the so-called "quasi-free" approximation can be used. In this approximation the excitation energy of the system in one collision is exactly equal to the recoil energy $\vec{q}^2/2M$ of a free particle of the target. Note that in summation over different nuclei of the target, sums of the form given below appear in the last term in (1.10)

$$\begin{aligned} &\sum_{\alpha,\beta=1}^{N_t} \int d^3\vec{R}_1 ... d^3\vec{R}_N e^{-i\vec{q}(\vec{R}_\alpha - \vec{R}_\beta)} \hat{\rho}_t(\vec{R}_1, ...\vec{R}_N) = N_t \hat{\rho}_t + \\ &+ \sum_{\alpha \neq \beta}^{N_t} \int d^3\vec{R}_1 ... d^3\vec{R}_N e^{-i\vec{q}(\vec{R}_\alpha - \vec{R}_\beta)} \hat{\rho}_t(\vec{R}_1, ...\vec{R}_N), \end{aligned} \tag{1.11}$$

where $\hat{\rho}_t$ is the spin density matrix of the target nucleus. In deriving equation (1.11) it is assumed that the positions of target nuclei and their spin states are uncorrelated.

Upon averaging over the spin states of nuclei in the target, the second term in (1.11) can be expressed in terms of the so-called particle pair distribution function [16], and vanishes when the transferred momentum $\vec{q}$ exceeds the magnitude inverse to the correlation radius r, i.e., $q \gg r^{-1}$ [16]. For a non-crystalline target, the magnitude of the correlation radius r is of the order of the distance between nuclei. Consequently, the second term can only contribute to the kinetic equation at very small scattering angles $\vartheta_{sc} \lesssim 1/kr$, so it will be neglected in further consideration.

The resulting expression for the density matrix can be obtained from (1.9) for $\dfrac{\hat{\rho}'_d - \hat{\rho}_d}{\Delta t}$ in the form of the following integro-differential equation:

$$\begin{aligned} \frac{d\hat{\rho}_d(\vec{k},t)}{dt} &= -iVN_t \mathtt{Sp}_t \left(\hat{\mathtt{T}}(\vec{k},\vec{k})\hat{\rho}(\vec{k},t) - \hat{\rho}(\vec{k},t)\hat{\mathtt{T}}^+(\vec{k},\vec{k}) \right) + \\ &+ \frac{V^3}{(2\pi)^2} N_t \mathtt{Sp}_t \int d^3\vec{k}' \delta\left(\varepsilon_k - \varepsilon_{k'} - \frac{\vec{q}^2}{2M}\right) \hat{\mathtt{T}}(\vec{k},0;\vec{k}',-\vec{q})\hat{\rho}(\vec{k}',t)\hat{\mathtt{T}}^+(\vec{k}',-\vec{q};\vec{k},0). \end{aligned} \tag{1.12}$$

$\hat{\rho}(\vec{k})$ denotes the following dependnce: $\hat{\rho}(\vec{k}) = \hat{\rho}_d(\vec{k};\vec{S}_d) \otimes \hat{\rho}_t(\vec{S}_t)$, where $\hat{\rho}_t(\vec{S}_t)$ is the spin density matrix of the target nucleus.

Let us introduce the scattering amplitude $\hat{F}$ with matrix elements equal to [17]:

$$\hat{F}(\vec{k},\vec{k}') = -\frac{M_r}{2\pi} V^2 \hat{\mathtt{T}}(\vec{k},0;\vec{k}',-\vec{q}), \tag{1.13}$$

where $M_r = \dfrac{mM}{m+M}$ is the reduced mass.

Then equation (1.12) can be transformed into a form:

$$\begin{aligned} \frac{d\hat{\rho}_d(\vec{k},t)}{dt} &= \frac{2\pi i}{M_r} N \mathtt{Sp}_t \left(\hat{F}(\vec{k},\vec{k})\hat{\rho}(\vec{k},t) - \hat{\rho}(\vec{k},t)\hat{F}^+(\vec{k},\vec{k}) \right) + \\ &+ N \mathtt{Sp}_t \int d\Omega_{\vec{k}'} \frac{k'^2}{M_r^2 \left(\frac{k'}{m} - \frac{(\vec{k}-\vec{k}')\vec{n}'}{M}\right)} \hat{F}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',t)\hat{F}^+(\vec{k}',\vec{k}), \end{aligned} \tag{1.14}$$

where N denotes the number of particles in the target per unit volume, $\vec{n}'$ is the unit vector in the direction of the momentum $\vec{k}'$. The absolute value of vector $\vec{k}'$ is determined from the equation:

$$\varepsilon_k = \varepsilon_{k'} + \frac{(\vec{k}-\vec{k}')^2}{2M}. \tag{1.15}$$

Note that when the condition $m \geq M$ for the masses of the incident particles is fulfilled, the denominator in the integrand of equation (1.14) vanishes for the value of the scattering angle of the incident particle equal to $\cos\theta = \frac{\sqrt{m^2-M^2}}{m}$, with the absolute value of vector $\vec{k}'$ being equal to $k\sqrt{\frac{m-M}{m+M}}$.

Equation (1.14) simplifies when a particle (proton, deuteron, antiproton) passes through a target with nuclei whose mass is much larger than the mass of the incoming particle. In this case we can neglect the effect of the energy loss of the incident particle through scattering. So we can neglect the recoil energy $\vec{q}^2/2M$ in the δ-function (1.12), (1.15). As a result, one obtains a simple kinetic equation describing the time and spin evolution of the incident particle as it passes through the target [10]:

$$\frac{d\hat{\rho}_d(\vec{k},t)}{dt} = \frac{2\pi N}{m}\mathtt{Sp}_t\left[\hat{F}(\vec{k},\vec{k})\hat{\rho}_d(\vec{k},z) - \hat{\rho}_d(\vec{k},z)\hat{F}^+(\vec{k},\vec{k})\right] + N\frac{k}{m}\mathtt{Sp}_t\int d\Omega_{\vec{k}'}\hat{F}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',t)\hat{F}^+(\vec{k}',\vec{k}), \tag{1.16}$$

where $|\vec{k}| = |\vec{k}'|$.

The first term on the right-hand side of (1.16), which describes refraction of particle in the target, can be represented as follows:

$$\hat{F}(0)\hat{\rho}(\vec{k},t) - \hat{\rho}(\vec{k},t)\hat{F}^+(0) = \left[\tfrac{1}{2}\left(\hat{F}(0)+\hat{F}^+(0)\right), \hat{\rho}(\vec{k},t)\right] + \left\{\tfrac{1}{2}\left(\hat{F}(0)-\hat{F}^+(0)\right), \hat{\rho}(\vec{k},t)\right\}, \tag{1.17}$$

where $[,]$ is the commutator, $\{\,,\}$ is the anticommutator.

The part proportional to the commutator leads to the rotation of the polarization vector due to elastic coherent scattering (as a result of the refraction effect [10]); the anticommutator describes the reduction in the intensity and depolarization of the beam which has passed through the target. The last term in (1.16) determines the effect of incoherent scattering on the change of $\hat{\rho}_d$ (in the general case, single and multiple scattering).

As stated above, equation (1.16) is not applicable to the description of the process of proton (deuteron) transmission through the target containing light nuclei (protons (deuterons)). To describe multiple scattering in this case, a more general equation (1.14) should be solved.

Further we shall concern ourselves with the deuteron passage through a carbon target, and so we shall use equation (1.16).

For the sake of concreteness, let us consider the process of deuteron passage through the target with spinless nuclei. In this case the density matrix $\hat{\rho}(\vec{k})$, as well as the amplitude $\hat{F}(\vec{k},\vec{k}')$ contain only spin variables of the scattered beam: $\hat{\rho}(\vec{k}) = \hat{\rho}_d(\vec{k})$. For the amplitude $\hat{F}(\vec{k},\vec{k}')$, we shall introduce the notation $\hat{F}(\vec{k},\vec{k}') = \hat{f}(\vec{k},\vec{k}')$, where $\hat{f}(\vec{k},\vec{k}') \equiv \hat{f}(\vec{k},\vec{k}';\hat{\vec{S}}_d)$.

As a result, equation (1.16) can be written as follows:

$$\begin{aligned}\frac{d\hat{\rho}_d}{dz} &= \frac{\pi i}{k}N\left[(\hat{f}(\vec{k},\vec{k})+\hat{f}^+(\vec{k},\vec{k})), \hat{\rho}_d(\vec{k})\right] + \frac{\pi i}{k}N\left\{(\hat{f}(\vec{k},\vec{k})-\hat{f}^+(\vec{k},\vec{k})), \hat{\rho}_d(\vec{k})\right\} + \\ &+N\int d\Omega'_{\vec{k}}\hat{f}(\vec{k},\vec{k}')\hat{\rho}_d(\vec{k}')\hat{f}^+(\vec{k}',\vec{k}),\end{aligned} \tag{1.18}$$

where $z = vt$ (v is the particle velocity) is the distance traveled by the incident particle in matter. Hereinafter, the subscript d of the density matrix will be dropped.

2 Scattering Amplitude

For particles with spin 1 (deuterons), the amplitude $\hat{f}(\vec{k},\vec{k}')$ can be expressed in terms of the deuteron spin operator $\hat{\vec{S}}$, quadrupolarization tensor $\hat{Q}_{ik}$ and the combination of vectors $\vec{k}$ and $\vec{k}'$:

$$\hat{f}(\vec{k},\vec{k}') = A\hat{I} + B(\hat{\vec{S}}\vec{\nu}) + C_1\hat{Q}_{ik}\mu_i\mu_k + C_2\hat{Q}_{ik}\mu_{1i}\mu_{1k}, \tag{2.1}$$

where A, B, C_1 and C_2 are the parameters depending on θ, $\vec{\nu} = [\vec{k}\times\vec{k}']/|[\vec{k}\times\vec{k}']|$, $\vec{\mu} = (\vec{k}-\vec{k}')/|\vec{k}-\vec{k}'|$, $\vec{\mu}_1 = (\vec{k}+\vec{k}')/|\vec{k}+\vec{k}'|$, the components of the tensor $\hat{Q}_{ik}$ are defined as: $\hat{Q}_{ik} = \frac{3}{2}\left(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i - \frac{4}{3}\delta_{ik}\hat{I}\right)$, $\hat{I}$ is the 3×3 identity matrix.

Write the explicit form of the zero-angle scattering amplitude $\hat{f}(\vec{k},\vec{k})$. In view of (2.1), we have:

$$\hat{f}(\vec{k},\vec{k}) = f_0(0) + f_1(0)(\vec{S}\vec{n})^2, \tag{2.2}$$

where $\vec{n} = \vec{k}/k$ is the unit vector in the direction $\vec{k}$ and the following notations are introduced: $f_0 = A - 2C_1 - 2C_2$, $f_1 = 3C_2$.

In the general case f_0 and f_1 are the complex functions, and according to the optical theorem, the imaginary parts of f_0 and f_1 can be expressed in terms of the corresponding total cross-sections:

$$\mathrm{Im} f_0(0) = \frac{k}{4\pi}\,\sigma^0_{tot}, \quad \mathrm{Im} f_1(0) = \frac{k}{4\pi}\left[\sigma^{\pm 1}_{tot} - \sigma^0_{tot}\right], \tag{2.3}$$

where σ^0_{tot}, $\sigma^{\pm 1}_{tot}$ are the total scattering cross-sections for the initial spin state of the deuteron with a magnetic quantum number $M = 0$ and $M = \pm 1$, respectively (the quantization axis z is directed along $\vec{n}$).

The deuteron interacts with the target nuclei via nuclear and Coulomb interactions. The amplitude $\hat{f}(\vec{k},\vec{k}')$ of the deuteron scattering by a target nuclei in this case can be represented in the form:

$$\hat{f}(\vec{k},\vec{k}') = \hat{f}_{coul}(\vec{k},\vec{k}') + \hat{f}_{nucl,coul}(\vec{k},\vec{k}'), \tag{2.4}$$

where $\hat{f}_{coul}$ is the amplitude of the Coulomb scattering of the deuteron by a nucleus in the absence of nuclear interaction, $\hat{f}_{nucl,coul}$ is the amplitude of scattering of Coulomb-distorted waves by a nuclear potential.

The matrix elements of the amplitude f_{ba} in the case of scattering by a fixed center are related to the elements of the operator $\mathcal{T}$ as [17]:

$$f_{ba} = -\frac{m}{2\pi}V\mathcal{T}_{ba}. \tag{2.5}$$

In the case of two interactions, the matrix elements of the operator $\mathcal{T}$ are defined in a standard manner

$$\mathcal{T}_{ba} = \langle\Phi_b|V_{coul} + V_{nucl}|\psi^+_a\rangle. \tag{2.7}$$

$\Phi_{a(b)}$ describes the initial (final) state of the system "particle–nucleus" in the area, where the interaction is absent, the wave function ψ^+_a satisfies the integral equation $\psi^+_a = \Phi_a + (\varepsilon_a - K_d + i\eta)^{-1}(V_{coul} + V_{nucl})\psi^+_a$, where K_d is the kinetic energy operator of the incident particle (deuteron).

Upon introducing a wave function φ^-_b, which describes a converging wave in scattering by a Coulomb potential alone and corresponds to a final state Φ_b: $\varphi^-_b = \Phi_b + (\varepsilon_a - K_d - i\eta)^{-1}V_{coul}\varphi^-_b$, the matrix elements of operator $\mathcal{T}$ can be represented as a sum of two terms [18]:

$$\mathcal{T}_{ba} = \mathcal{T}^{coul}_{ba} + \mathcal{T}^{nucl,coul}_{ba}, \tag{2.8}$$

where the matrix elements $\mathcal{T}^{coul}_{ba}$ correspond to the Coulomb interaction $\mathcal{T}^{coul}_{ba} = \langle\Phi_b|\,V_{coul}\,|\varphi^+_a\rangle$, the part $\mathcal{T}^{nucl,coul}_{ba} \equiv \langle\varphi^-_b\,|\,V_{nucl}\,|\psi^+_a\rangle$ determines the amplitude of scattering by a nuclear potential of waves that have been scattered by the potential V_{coul}.

Operator $\mathcal{T}_{nucl,coul}$ can be expressed in terms of the operators of the Coulomb scattering and nuclear scattering as the following infinite series:

$$\begin{aligned}\mathcal{T}_{nucl,coul} &= \mathcal{T}_{nucl} + \mathcal{T}_{nucl}G_0\mathcal{T}_{coul} + \mathcal{T}_{coul}G_0\mathcal{T}_{nucl} + \\ &+ \mathcal{T}_{coul}G_0\mathcal{T}_{nucl}G_0\mathcal{T}_{coul} + \mathcal{T}_{nucl}G_0\mathcal{T}_{coul}G_0\mathcal{T}_{nucl} + \mathcal{T}_{nucl}G_0\mathcal{T}_{coul}G_0\mathcal{T}_{nucl}G_0\mathcal{T}_{coul} + \\ &+ \mathcal{T}_{coul}G_0\mathcal{T}_{nucl}G_0\mathcal{T}_{coul}G_0\mathcal{T}_{nucl} + \ldots,\end{aligned} \tag{2.9}$$

where G_0 is the stationary Green's function. By definition $G_0 = \dfrac{1}{\varepsilon_a - K_d + i\eta}$.

In view of the definition (2.5), equation (2.9) can be written, using the corresponding scattering amplitudes:

$$\begin{aligned}
\hat{f}_{nucl,coul}(\vec{k},\vec{k}') = \hat{f}_{nucl}(\vec{k},\vec{k}') - \frac{1}{(2\pi)^2 m}\int \frac{\hat{f}_{nucl}(\vec{k},\vec{k}'')\hat{f}_{coul}(\vec{k}'',\vec{k}')}{\varepsilon_k - \varepsilon_{k''} + i\eta} d^3\vec{k}'' - \\
-\frac{1}{(2\pi)^2 m}\int \frac{\hat{f}_{coul}(\vec{k},\vec{k}'')\hat{f}_{nucl}(\vec{k}'',\vec{k}')}{\varepsilon_k - \varepsilon_{k''} + i\eta} d^3\vec{k}'' + \\
+\frac{1}{(2\pi)^4 m^2}\iint \frac{\hat{f}_{nucl}(\vec{k},\vec{k}'')\hat{f}_{coul}(\vec{k}'',\vec{k}''')\hat{f}_{nucl}(\vec{k}''',\vec{k}')}{(\varepsilon_k - \varepsilon_{k''} + i\eta)(\varepsilon_k - \varepsilon_{k'''} + i\eta)} d^3\vec{k}'' d^3\vec{k}''' + \\
+\frac{1}{(2\pi)^4 m^2}\iint \frac{\hat{f}_{coul}(\vec{k},\vec{k}'')\hat{f}_{nucl}(\vec{k}'',\vec{k}''')\hat{f}_{coul}(\vec{k}''',\vec{k}')}{(\varepsilon_k - \varepsilon_{k'''} + i\eta)(\varepsilon_k - \varepsilon_{k'''} + i\eta)} d^3\vec{k}'' d^3\vec{k}''' - \ldots.
\end{aligned} \tag{2.10}$$

3 Polarization characteristics of the deuterons registered by the detector during the beam's passage through a thin target

3.1 Scattering in a Thin Target

Due to a long-range character of Coulomb interaction, Coulomb scattering of deuterons by target nuclei occurs at small angles and the Coulomb amplitude for $\theta \ll 1$ is much larger than the nuclear scattering amplitude. In this case, for solving equation (1.18), one can apply the perturbation theory [11] using as a zero approximation the solution of kinetic equation (1.18), where the collision term is determined only by the Coulomb interaction between the incident particle and the nuclei of matter. The contribution to the evolution of the spin density matrix $\hat{\rho}(\vec{k})$ coming from nuclear scattering and Coulomb-nuclear interference is considered as a correction. It should be noted that taking account of nuclear and interference factors as perturbations is valid only for such target thicknesses z for which multiple nuclear scattering can be neglected [10]. In this part, in order to illustrate the principal patterns of relationships and simplify the form of obtained relations, we shall further analyze the characteristics of a deuteron beam for the case of a very thin target, where the change in the deuteron polarization state occurs only due to single scattering events in matter in addition to coherent scattering.

Let us consider the process of particle transmission through the target, whose thickness z is much smaller than the mean free path of the deuteron in matter, i.e., $z < 1/N\sigma$, σ is the total cross-section of the deuteron scattering by a nucleus. As a consequence, in the first order perturbation theory, the solution of (1.19) can be represented in a form:

$$\begin{aligned}
\hat{\rho}(\vec{k},z) = \hat{\rho}(\vec{k},0) + \frac{\pi i}{k} N\left[(\hat{f}(\vec{k},\vec{k}) + \hat{f}^{+}(\vec{k},\vec{k})), \hat{\rho}(\vec{k},0)\right] z + \frac{\pi i}{k} N\left\{(\hat{f}(\vec{k},\vec{k}) - \hat{f}^{+}(\vec{k},\vec{k})), \hat{\rho}(\vec{k},0)\right\} z + \\
+ N\hat{f}(\vec{k},\vec{k}_0)\hat{\rho}(0)\hat{f}^{+}(\vec{k}_0,\vec{k}) z,
\end{aligned} \tag{3.1}$$

where $\hat{\rho}(\vec{k},0)$ is the density matrix of the beam when it enters the target, i.e, when $z = 0$. It describes the distribution over the momenta of the particles entering the target relative to the direction $\vec{k}_0$.

In obtaining relation (3.1) it was also assumed that the initial angular distribution of the beam is much smaller that the characteristic angular width of the differential scattering cross-section. In this case the amplitude $\hat{f}(\vec{k},\vec{k}')$ in the term $\int d\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',0)\hat{f}^{+}(\vec{k}',\vec{k})$ can be removed from the integrand at point $\vec{k}' = \vec{k}_0$. As a result, we have $\int d\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',0)\hat{f}^{+}(\vec{k}',\vec{k}) \simeq \hat{f}(\vec{k},\vec{k}_0)\hat{\rho}(0)\hat{f}^{+}(\vec{k}_0,\vec{k})$, where $\hat{\rho}(0) = \int d\Omega_{\vec{k}'}\hat{\rho}(\vec{k}',0)$ is the spin part of the beam's density matrix $\hat{\rho}(\vec{k},0)$. The term $N\int d\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',0)\hat{f}^{+}(\vec{k}',\vec{k})$ or $N\hat{f}(\vec{k},\vec{k}_0)\hat{\rho}(0)\hat{f}^{+}(\vec{k}_0,\vec{k})$ is the contribution to the evolution of the density matrix, which describes single scattering of particles in the direction of $\vec{k}$. It is known, in particular, that $\mathrm{Sp}\hat{f}(\vec{k},\vec{k}_0)\hat{\rho}(0)\hat{f}^{+}(\vec{k}_0,\vec{k})$ is the probability for a particle to undergo a single elastic collision with a nucleus and get displaced by the angle corresponding to the momentum direction $\vec{k}$ ($\mathtt{Sp}$ is taking the trace over the spin variables of the deuteron).

Let us also consider the fact that scattering at high energies chiefly occurs at small angles $\theta \ll 1$. The analysis shows that in this case, the terms in the amplitude (2.1), which are proportional to B and C_1, lead to insignificant depolarization of the detected beam [10] and will be dropped hereinafter. As a result, the amplitude $\hat{f}$ in (3.1) has a from:

$$\hat{f}(\vec{k},\vec{k}') = f_0(\theta) + f_1(\theta)(\vec{S}\vec{n})^2, \tag{3.2}$$

where $\vec{n} = \vec{k}/k$ is the unit vector in the direction of $\vec{k}$, $f_0 = A - 2C_1 - 2C_2$, $f_1 = 3C_2$.

Using the solution (3.1) and the explicit form of the spin structure of the amplitude $\hat{f}(\vec{k},\vec{k}')$ (3.2), it is possible to find the dependence of the intensity and the polarization characteristics of the beam on the direction of the particle scattering and on the distance z traveled by the deuteron in matter. In a real experiment, the scattered particles are registered within a certain interval of finite momenta because the collimator of the detector has a finite angular width. We shall therefore consider further in this paper the characteristics of the beam transmitted through the target in the interval of solid angles $\Delta\Omega$ with respect to the initial direction of the beam propagation. In fact, due to the axial symmetry of the collimator, $\Delta\Omega$ is determined by the angular width of the detector collimator $2\vartheta_{det}$ (for further calculations it is also assumed that ϑ_{det} is much larger than the initial angular distribution of the beam).

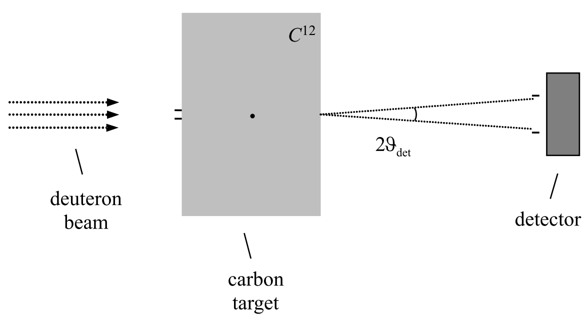

Fig. 1: Scheme of scattered beam detection by the central detector.

In the case of deuterons (particles with spin 1), the polarization state is characterized by the polarization vector $\vec{P}(\vec{k}) = \mathtt{Sp}\hat{\rho}(\vec{k})\hat{\vec{S}}$ and the quadrupolarization tensor $\mathbf{Q}$, whose components are defined as $Q_{ik}(\vec{k}) = \mathtt{Sp}\hat{\rho}(\vec{k})\hat{Q}_{ik}$. The spin density matrix $\hat{\rho}(\vec{k})$ can be written in the following general form:

$$\hat{\rho}(\vec{k}) = \frac{1}{3}I(\vec{k})\hat{\mathrm{I}} + \frac{1}{2}\vec{P}(\vec{k})\hat{\vec{S}} + \frac{1}{9}Q_{ik}(\vec{k})\hat{Q}_{ik}, \tag{3.3}$$

where $I(\vec{k}) = \mathtt{Sp}\hat{\rho}(\vec{k})$. Note that alongside with the quantities $\vec{P}$ and $\mathbf{Q}$, normalized spin characteristics of the beam are also used to describe the beam polarization.

Denote the intensity, the polarization vector, and the quadrupolarization tensor of the beam in the area occupied by the detector with angular width $\Delta\Omega$ by $\mathcal{I} \equiv \int_{\Delta\Omega} \mathrm{d}\Omega I(\vec{k},z)$, $\vec{\mathcal{P}} \equiv \int_{\Delta\Omega} \mathrm{d}\Omega \vec{P}(\vec{k},z)$, $\mathcal{Q} \equiv \int_{\Delta\Omega} \mathrm{d}\Omega \mathbf{Q}(\vec{k},z)$, respectively. On the basis of the solution (3.1) of kinetic equation (1.18), using the explicit form of the density matrix (3.3) and the scattering amplitude (3.2), one can obtain the expression for the integral characteristics of the deuteron beam:

$$\mathcal{I}(z) = I_0 + N\left(-\sigma^0_{tot} + \int_{\Delta\Omega} |f_0|^2 \mathrm{d}\Omega\right) z I_0 +$$
$$N\left(-(\sigma^{\pm 1}_{tot} - \sigma^0_{tot}) + 2\int_{\Delta\Omega} \mathrm{Re}(f_0 f_1^*)\mathrm{d}\Omega + \int_{\Delta\Omega} |f_1|^2 \mathrm{d}\Omega\right)\left[\frac{2}{3} I_0 + \frac{1}{3}(\mathbf{Q}_0\vec{n})\vec{n}\right] z,$$

$$\vec{\mathcal{P}}(z) = \vec{P}_0 + N\left(-\sigma^0_{tot} + \int_{\Delta\Omega} |f_0|^2 \mathrm{d}\Omega\right) z\vec{P}_0 + N\left(-\frac{1}{2}(\sigma^{\pm 1}_{tot} - \sigma^0_{tot}) + \int_{\Delta\Omega} \mathrm{Re}(f_0 f_1^*)\mathrm{d}\Omega\right)\left[\vec{P}_0 + \vec{n}(\vec{P}_0\vec{n})\right] z +$$
$$\left(\int_{\Delta\Omega} |f_1|^2 d\Omega\right)\vec{n}(\vec{P}_0\vec{n})z - \frac{2}{3}\frac{2\pi N}{k}\mathrm{Re}\left[f_1(0) - \frac{ik}{2\pi}\int_{\Delta\Omega} f_0^* f_1 \mathrm{d}\Omega\right][\vec{n}\times(\mathbf{Q}_0\vec{n})]z,$$

$$\mathcal{Q}(z) = \mathbf{Q}_0 + N\left(-\sigma^0_{tot} + \int_{\Delta\Omega} |f_0|^2 \mathrm{d}\Omega\right) z\mathbf{Q}_0 + N\left(-(\sigma^{\pm 1}_{tot} - \sigma^0_{tot}) + 2\int_{\Delta\Omega} \mathrm{Re}(f_0 f_1^*)\mathrm{d}\Omega\right)\Big[\,\mathbf{Q}_0 +$$
$$\frac{1}{3}(3\vec{n}\otimes\vec{n} - \mathbf{I})I_0 - \frac{1}{2}\left((\mathbf{Q}_0\vec{n})\otimes\vec{n} + \vec{n}\otimes(\mathbf{Q}_0\vec{n})\right) + \frac{1}{3}\mathbf{I}(\mathbf{Q}_0\vec{n})\vec{n}\Big] z + \left(\int_{\Delta\Omega} |f_1|^2 \mathrm{d}\Omega\right)\Big[\frac{1}{3}(3\vec{n}\otimes\vec{n} - \mathbf{I})I_0 -$$
$$\frac{1}{2}\left((\mathbf{Q}_0\vec{n})\otimes\vec{n} + \vec{n}\otimes(\mathbf{Q}_0\vec{n})\right) + + \frac{1}{2}\vec{n}^{\times}\mathbf{Q}_0\vec{n}^{\times} + \vec{n}\otimes\vec{n}\ (\mathbf{Q}_0\vec{n})\vec{n} - \frac{1}{6}\mathbf{I}(\mathbf{Q}_0\vec{n})\vec{n} + \frac{1}{2}\mathbf{Q}_0\Big] z -$$
$$\frac{3}{2}\frac{2\pi N}{k}\mathrm{Re}\left[f_1(0) - \frac{ik}{2\pi}\int_{\Delta\Omega} f_0^* f_1 d\Omega\right]\left([\vec{n}\times\vec{P}_0]\otimes\vec{n} + \vec{n}\otimes[\vec{n}\times\vec{P}_0]\right) z, \tag{3.4}$$

where $\vec{P}_0$ and $\mathbf{Q}_0$ are the polarization vector and the quadrupolarization tensor of the deuteron beam at entering the target, respectively: $\vec{P}_0 = \int \mathrm{d}\Omega\vec{P}(\vec{k},0)$, $\mathbf{Q}_0 = \int \mathrm{d}\Omega\mathbf{Q}(\vec{k},0)$; $\vec{n} = \vec{k}_0/k_0$ is the unit vector in the direction of the deuteron momentum $\vec{k}_0$ (the quantization axis), $\otimes$ is the dyadic product of vectors: $(\vec{n}\otimes\vec{n})_{ij} = n_i n_j$, $\vec{n}^{\times}$ is the tensor dual to vector $\vec{n}$: $(\vec{n}^{\times})_{ij} = \varepsilon_{ijk}n_k$, $(\mathbf{Q}_0\vec{n})$ is the vector having the components $(\mathbf{Q}_0\vec{n})_l = Q_{0\,lk}n_k$.

Now let us analyze the obtained solutions. First of all, pay attention to the fact that according to (3.4), the intensity and the polarization characteristics of the beam depend on the magnitude of the interval $\Delta\Omega$ of the solid angle. As is seen, at $\Delta\Omega \to 0$, the contribution due to single scattering disappears in expressions for $\mathcal{I}$, $\vec{\mathcal{P}}$, and $\mathcal{Q}$.

Let us give a more detailed treatment of the change in the polarization vector $\vec{\mathcal{P}}$ depending on the target thickness z. Let $\vec{P}_0$ be an arbitrary, not equal to $\pi/2$, angle with the direction $\vec{n}$. Choose a coordinate system so that the z-axis in it coincides with the direction $\vec{n}$ of the deuteron incidence onto the target, while the axes x and y are located in such a way that the initial polarization vector $\vec{P}_0$ lies in the xz plane.

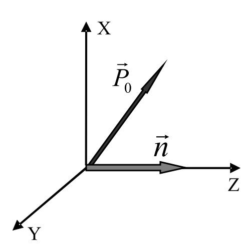

Fig. 2: Coordinate frame

In this case $Q_{0\,yz} = 0$, hence the polarization vector $\mathbf{Q}_0\vec{n}$ also lies in the plane defined by vectors $\vec{P}_0$ and $\vec{n}$. Then from the expression for $\vec{\mathcal{P}}$ follows that the first three terms in it describe the change in the component of the polarization vector, which lies in the plane $(\vec{P}_0, \vec{n})$, i.e., these terms lead to the change in the magnitude of the polarization vector (depolarization). The last term, which is defined by vector $[\vec{n}\times(\mathbf{Q}_0\vec{n})]$, leads to the appearance in the initial polarization vector of a component perpendicular to the plane $(\vec{P}_0, \vec{n})$, which grows with growing z, i.e., this term describes the rotation of vector $\vec{\mathcal{P}}$ with respect to the direction of $\vec{n}$. The rotation angle φ_{eff} is defined as follows:

$$\varphi_{eff} = \frac{2\pi N}{k}\mathrm{Re}\left[f_1(0) - \frac{ik}{2\pi}\int_{\Delta\Omega} f_0^* f_1 \mathrm{d}\Omega\right] z. \tag{3.5}$$

As it was shown in [1, 2], the part of φ_{eff}, equal to $\varphi_0 \equiv \frac{2\pi N}{k}\mathrm{Re}f_1(0)z$, is the rotation angle of the polarization vector due to coherent scattering of the deuterons in unpolarized matter.

When incoherent scattering by target nuclei is taken into account, then according to (3.5), the rotation angle includes the additional contribution, which depends on $\Delta\Omega$.

To provide a more detailed treatment of this dependence, it would be recalled that the deuteron interacts with target nuclei via nuclear and Coulomb interactions (2.4). The general spin structure of the corresponding amplitudes can be represented as in (3.2). This enables one to write the spinless and spin parts of the amplitude $\hat{f}$, which appear in (2.4) as follows:

$$f_0(\theta) = a(\theta) + d(\theta), \quad f_1(\theta) = d_1(\theta), \tag{3.6}$$

where $a(\theta)$ is the amplitude of the Coulomb scattering of the deuteron by a nucleus with the charge Z (the amplitude in $f_1(\theta)$ describing the spin part of the Coulomb interaction is small in comparison with $d_1(\theta)$ [19], and so it is assumed that $\hat{f}_{coul}(\theta) = a(\theta)\hat{\mathrm{I}}$); $d(\theta)$ is the spin-independent and $d_1(\theta)$ is the spin dependent parts of the modified nuclear amplitude. The explicit form of the dependence of d on θ in the limit of small scattering angles for the case of interaction of structure particles was obtained, for example, in [20]. To estimate the major parameters, let us consider the most simple form of the dependence of d and d_1 on θ, namely:

$$d(\theta) = d(0)e^{-k^2R_d^2\theta^2/4}, \quad d_1(\theta) = d_1(0)e^{-k^2R_d^2\theta^2/4}, \tag{3.7}$$

where R_d is the deuteron radius.

3.2 Contributions to the rotation angle due to incoherent scattering

Let us consider the terms in (3.4) which are integrated over the angular width $\Delta\Omega$ of the detector. According to (3.6), the functions $f_0(\theta)$ and $f_1(\theta)$ included in the obtained solutions contain the contributions from the Coulomb and nuclear amplitudes, which, in turn, have different angular dependence. As a result, in analyzing (3.4) for the case of high energies, we shall select such limits of ϑ_{det} that are bounded by the diffraction angles θ_c and θ_n of scattering of fast deuterons, for Coulomb and nuclear scattering, respectively. For a screened Coulomb potential, $\theta_c \sim 1/kR_c$, where R_c is the shielding radius (in the case of scattering by a carbon target, $R_c = 3\cdot 10^{-9}$ cm). When deuterons are scattered by nuclei whose radius is smaller than that of the deuterons, $\theta_n \sim 1/kR_d$.

In view of the representation of the amplitude $\hat{f}(\theta)$ in (3.6), we obtain the following general expression for the rotation angle of the polarization vector (3.5):

$$\varphi_{eff} = \frac{2\pi N}{k}\mathrm{Re}\left[d_1(0) - \frac{ik}{2\pi}\int_{\Delta\Omega} a^*(\theta)d_1(\theta)\mathrm{d}\Omega - \frac{ik}{2\pi}\int_{\Delta\Omega} d^*(\theta)d_1(\theta)\mathrm{d}\Omega\right] z. \tag{3.8}$$

where the dependence of the nuclear amplitude on the scattering angle for high energies is determined by (3.7). For further analysis of the magnitude of the effect under consideration, it is convenient to represent φ_{eff} in the form

$$\varphi_{eff}(\vartheta_{det}) = \varphi_0 + \varphi_{nc}(\vartheta_{det}) + \varphi_{nn}(\vartheta_{det}), \tag{3.9}$$

where $\varphi_0 = \frac{2\pi N}{k}\mathrm{Re}d_1(0)z$ is the contribution of the coherent scattering to the rotation angle of the polarization vector; $\varphi_{nc}(\vartheta_{det}) = N\int_{\Delta\Omega} \mathrm{Im}\left[a^*(\theta)d_1(\theta)\right]\mathrm{d}\Omega z$ is the part of the rotation angle, which describes the effect of the Coulomb-nuclear interference; $\varphi_{nn}(\vartheta_{det}) = N \int_{\Delta\Omega} \mathrm{Im}\left[d^*(\theta) d_1(\theta)\right] \mathrm{d}\Omega z$ is the correction to the deuteron spin rotation from nuclear scattering.

Numerical values of these contributions obtained, for example, for the energy of 500 MeV ($k = 0.74 \cdot 10^{14}$ cm^{-1}). Then in the case of scattering by a carbon target, we have $\theta_c \sim 10^{-5}$rad, $\theta_n \sim 10^{-2}$rad.

For the imaginary and real parts of the nuclear amplitude of deuteron scattering by a carbon nucleus at zero angle, take the values calculated in the eikonal approximation in the energy region $E = 0.5 \div 1$ GeV: $f^{nucl}_{M=0}(0) = -5.97 + 74.81i$ fm, $f^{nucl}_{M=\pm 1}(0) = -5.84 + 73.9i$ fm [21]. From this $\mathrm{Im}d = 0.75 \cdot 10^{-11}$cm, $\mathrm{Re}d = -0.6 \cdot 10^{-12}$cm, $\mathrm{Im}d_1 = -0.91 \cdot 10^{-13}$cm, $\mathrm{Re}d_1 = 0.13 \cdot 10^{-13}$cm. As a result, we have $\varphi_0 \simeq 1.2 \cdot 10^{-4} z$.

Let us consider the ratio φ_{nc}/φ_0. Perform integration over the angular width of the detector, using the expression for the Coulomb amplitude in the first Born approximation. This is possible because for the deuteron energies considered here, in scattering by a screened Coulomb potential, the ratio of the real part of the amplitude in the second-order perturbation theory approximation $a^{(2)}$ to that in the Born approximation is $\left|\mathrm{Re}a^{(2)}/\mathrm{Re}a^{(1)}\right| \sim \frac{mZe^2}{\hbar^2 k}\frac{1}{kR_c} \ll 1$ for the whole range of scattering angles.

For the range of angles $\theta \ll \theta_c$, the ratio is $\left|\mathrm{Im}a^{(2)}/\mathrm{Re}a^{(1)}\right| \sim \frac{mZe^2}{\hbar^2 k} = 0.057$; this estimate also remains unchanged with increasing $k\theta R_c$ [22]. For $k\theta R_c \gg 1$ the ratio $\mathrm{Im}a^{(2)}/\mathrm{Re}a^{(1)} \simeq -2\frac{mZe^2}{\hbar^2 k}\ln(k\theta R_c)$. That is why for, e.g., $\theta \sim \theta_n$, this ratio is of the order of unity. Taking into account the magnitude of $|\mathrm{Im}d_1/\mathrm{Re}d_1| \simeq 7$, as well as a fast decrease in the integrand due to the nuclear amplitude, one can demonstrate that for $\theta \gtrsim \theta_n$, the contribution to φ_{nc}/φ_0 of the term containing $\mathrm{Im}a(\theta)$ will be an order of magnitude smaller than the magnitude of φ_{nc}/φ_0, calculated using the Coulomb amplitude in Born approximation. It should be emphasized here that this term should not be ignored in those rare cases, when $|\mathrm{Im}d_1/\mathrm{Re}d_1| \sim 1$, as well as in the cases of calculating the cross-section of Coulomb-nuclear interaction and spin dichroism (for calculations in the case of high energies $\frac{mZe^2}{\hbar^2 k} \ll 1$, it is sufficient to use the imaginary part of the Coulomb amplitude calculated in the second order perturbation theory).

Thus, in the whole range of scattering angles one can obtain for φ_{nc}/φ_0:

$$\frac{\varphi_{nc}}{\varphi_0} = -\frac{\mathrm{Im}d_1}{\mathrm{Re}d_1}\frac{mZe^2}{\hbar^2 k}\left\{\mathrm{Ei}\left(-\frac{R_d^2}{4R_c^2} - \frac{k^2 R_d^2 \vartheta_{det}^2}{4}\right) - \mathrm{Ei}\left(-\frac{R_d^2}{4R_c^2}\right)\right\}, \tag{3.10}$$

where $\mathrm{Ei}(-z) = -\int_z^\infty \frac{\mathrm{d}x}{x}e^{-x}$ is the integral exponential function. The relative contribution to φ_{nn}/φ_0 of nuclear scattering of the deuterons by the target nuclei is obtained, using the approximate expression (3.7):

$$\frac{\varphi_{nn}}{\varphi_0} = \frac{1}{kR_d^2}\left\{\mathrm{Re}d\frac{\mathrm{Im}d_1}{\mathrm{Re}d_1} - \mathrm{Im}d\right\}\left(1 - e^{-R_d^2 k^2 \vartheta_{det}^2/2}\right). \tag{3.11}$$

The analysis shows that for the range of values $\vartheta_{det} \ll 1/kR_c$ the relationships (3.10) and (3.11) are much smaller than unity, i.e., the rotation angle of the polarization vector in this case is determined by coherent scattering of particles in matter (is determined by refraction of particles in matter).

Note also that for stated ϑ_{det}, the beam depolarization and spin dichroism will only be determined by the total cross-section σ^0_{tot} and $\sigma^{\pm 1}_{tot}$.

As a result, for $\vartheta_{det} \ll 10^{-5}$, the system of solutions (3.4) is reduced to the solutions describing the evolution of the spin state of the deuteron beam only due to particle refraction in the target. These solutions were obtained in [1, 2].

With increasing ϑ_{det} (even for $\vartheta_{det} \sim 1/kR_c$), singly scattered particles start affecting the polarization characteristics of the beam. For instance, for $\vartheta_{det} \sim 10^{-3}$ rad, the contribution of the Coulomb-nuclear interference to the rotation angle φ_{nc} of the polarization vector of the deuteron beam is of the order of φ_0, moreover, with growing ϑ_{det} ($\vartheta_{det} \ll \theta_n$), according to (3.10), φ_{nc} grows logarithmically: $\varphi_{nc}/\varphi_0 \sim \ln(kR_c\vartheta_{det})$.

3.3 Rotation angle including the Coulomb contribution to the spin part of the nuclear amplitude $d_1(0)$

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It should be noted that if in (3.8), integration over dΩ is made over the entire solid angle (a 4π experimental geometry), then according to the analysis given in [11, 12], the considered interference term and the contribution to the amplitude $d_1(0)$ due to distortion of the incident waves by Coulomb interaction compensate one another, i.e., the sum of the contributions $\varphi_0 + \varphi_{nc}(\vartheta_{det} = \pi)$ is in this case determined by the spin-dependent part of a pure nuclear amplitude of scattering at zero angle $\varphi_0' \equiv \varphi_0 + \varphi_{nc}(\pi) = \frac{2\pi N}{k}\mathrm{Re}d_1'$. This will be demonstrated below.

The modified nuclear amplitude $\hat{f}_{nucl,coul}(\vec{k},\vec{k}')$ is associated with "pure" Coulomb and "pure" nuclear amplitudes by formula (2.10). Taking into account that the solutions of (3.4) and φ_{eff} include only the spinless part of the scattering amplitude $a(\theta)$, according to (2.9), the spin-dependent part of d_1 can be written in the form:

$$d_1(\vec{k},\vec{k}') \simeq d_1'(\vec{k},\vec{k}') - \frac{1}{(2\pi)^2 m}\int \frac{d_1'(\vec{k},\vec{k}'')a(\vec{k}'',\vec{k}')}{\varepsilon_k - \varepsilon_{k''} + i\eta}d^3\vec{k}'' - \frac{1}{(2\pi)^2 m}\int \frac{a(\vec{k},\vec{k}'')d_1'(\vec{k}'',\vec{k}')}{\varepsilon_k - \varepsilon_{k''} + i\eta}d^3\vec{k}''. \quad (3.12)$$

The amplitude d_1' is a "pure" nuclear spin-dependent amplitude of scattering at angle θ.

Substituting the explicit form of the amplitude $d_1(\vec{k},\vec{k}')$ from (3.12) into the expression for φ_{eff} in (3.8), and again taking into account only the first degree of the product of d_1' and a, one obtains

$$\varphi_{eff} = \frac{2\pi N}{k}\mathrm{Re}\left[d_1'(0) + \frac{ik}{2\pi}\int a(\theta)d_1'(\theta)\mathrm{d}\Omega - \frac{ik}{2\pi}\int_{\Delta\Omega} a^*(\theta)d_1'(\theta)\mathrm{d}\Omega - \frac{ik}{2\pi}\int_{\Delta\Omega} d^*(\theta)d_1'(\theta)\mathrm{d}\Omega\right]z. \quad (3.13)$$

Integration in (3.12) was carried out subject to the following representation of the stationary Green's function:

$$\frac{1}{\varepsilon_k - \varepsilon_{k'} + i\eta} = P\frac{1}{\varepsilon_k - \varepsilon_{k'}} - i\pi\delta(\varepsilon_k - \varepsilon_{k'}), \quad (3.14)$$

where P is the principal value integration.

As is seen from (3.13), when the detector registers particles in the 4π solid angle geometry ($\Delta\Omega = 4\pi$), the Coulomb-nuclear contributions are mutually canceled. The compensation, however, only occurs when we can confine ourselves to the consideration of the Coulomb scattering amplitude in the first Born approximation.

Allowing for the Coulomb contribution to $d_1(0)$, equation (3.9) can be written as:

$$\varphi_{eff} = \varphi_0' + \varphi_{nc}^{tot} + \varphi_{nn}, \quad (3.15)$$

where φ_{nc}^{tot} is the total contribution of the Coulomb-nuclear interference to the rotation angle of the polarization vector. Its magnitude is determined by the sum of the second and third terms in (3.13). Within the limits of small scattering angles, the explicit form of φ_{nc}^{tot} reads as follows:

$$\varphi_{nc}^{tot} = 2\pi N z\int_{\vartheta_{det}}^{\infty} \mathrm{Im}\left[a(\theta)d_1'(\theta)\right]\theta\mathrm{d}\theta = -2\pi N\mathrm{Im}d_1'\frac{mZe^2}{\hbar^2k^2}\mathrm{Ei}\left(-\frac{R_d^2}{4R_c^2} - \frac{k^2R_d^2\vartheta_{det}^2}{4}\right)z.$$

In this case the relative contribution of the Coulomb-nuclear interference to the rotation angle is determined by formula

$$\frac{\varphi_{nc}^{tot}}{\varphi_0'} = -\frac{\mathrm{Im}d_1'}{\mathrm{Re}d_1'}\frac{mZe^2}{\hbar^2k^2}\mathrm{Ei}\left(-\frac{R_d^2}{4R_c^2} - \frac{k^2R_d^2\vartheta_{det}^2}{4}\right) \quad (3.16)$$

For high energies, when nuclear scattering has a pronounced diffraction character, one can estimate the magnitude of the lower limit of the angle ϑ_{det}^{comp}, at which the interference contributions can be considered to compensate one another with the selected accuracy. We have that for the considered energy and the form of nuclear amplitude, the total contribution of the Coulomb-nuclear interference can be neglected even at $\vartheta_{det} \sim \theta_n$ (the diffraction angle of nuclear scattering) ($|\varphi_{nc}^{tot}|$ is an order of magnitude smaller than φ_0').

For example, if $\vartheta_{det} = 0.8 \cdot 10^{-2}$ rad, $\varphi_{nc}^{tot}/\varphi_0' = -0.2$. From this it can be assumed that $\vartheta_{det}^{comp} \sim \theta_n$.

If the collimator of the detector registers particles moving within a certain solid angle $\Delta\Omega$ corresponding to $\vartheta_{det} \ll \vartheta_{det}^{comp}$, the stated Coulomb-nuclear terms will not be compensated. For example, when $\vartheta_{det} \simeq 0.4 \cdot 10^{-3}$ rad, the part of the rotation angle φ_{nc}^{tot} is comparable in magnitude with φ_0'.

It should be emphasized that the minimum value of ϑ_{det}, when the Coulomb-nuclear contributions are compensated (ϑ_{det}^{comp}), depends on the deuteron energy ($\sim 1/\sqrt{E}$) as well as on the type of the dependence of the spin part of the nuclear amplitude on the scattering angles.

Consider now the nuclear contribution to the rotation of the deuteron spin. The estimate of relation (3.11) for $\vartheta_{det} \sim \theta_n$ gives $\varphi_{nn}/\varphi_0 \simeq -10^{-2}$. With growing ϑ_{det}, namely for $\vartheta_{det} \gg 1/kR_d$, the ratio $|\varphi_{nn}/\varphi_0|$ achieves its maximum value: $|\varphi_{nn}/\varphi_0| \simeq 0.3$, while the magnitude of the ratio $|\varphi_{nn}/\varphi_0'|$ is of the order of 10^{-2}.

Let us mention the following possibility of experimental measuring of the spin-dependent part of the zero-angle nuclear amplitude. Measuring the rotation angle of the deuteron polarization vector for two arbitrary values of the angles ϑ_1 and ϑ_2 of the detector and considering the difference $\varphi_{eff}(\vartheta_1) - \varphi_{eff}(\vartheta_2)$, one obtains that $\varphi_{eff}(\vartheta_1) - \varphi_{eff}(\vartheta_2) \approx \varphi_{nc}(\vartheta_1) - \varphi_{nc}(\vartheta_2)$. Using equation (3.10), one can thus find the value of Imd_1 and, hence the value of φ_{nc} for any ϑ_{det}.

Knowing them and taking into account that $|\varphi_{nn}| \ll |\varphi_{nc}|$ for any ϑ_{det}, it is possible, using (3.9), to estimate the magnitude of the rotation angle φ_0 due coherent scattering and, hence, the magnitude of Red_1. This part of $d_1(0)$ can also be obtained directly measuring the rotation angle of the polarization vector for $\vartheta_{det} \ll 1/kR_c$. Basing on the assumption that the Coulomb-nuclear contributions in φ_{eff} are compensated and choosing the angle of the detector so that $\vartheta_{det} \gg \vartheta_{det}^{comp}$, one can obtain the magnitude of Red_1'.

The domain of applicability of the solutions of (3.4) corresponds to such target thicknesses z for which multiple scattering in matter can be neglected, i.e., z is much smaller than the mean free path of the deuterons due to Coulomb scattering: $z \leq 1/N\sigma_{coul}$. For energies 0.5÷1 GeV, thickness z is of the order of 10^{-5} cm.

In the next section we shall demonstrate the validity of the assertion that in the cases when z is much larger than the mean free path $1/N\sigma_{coul}$ (i.e., in the targets where multiple Coulomb scattering takes place), the contribution of the Coulomb-nuclear interference to the rotation angle of the polarization vector also depends on the angle of the detector collimator.

4 Polarization characteristics of deuterons under the conditions of multiple Coulomb scattering in matter

With increasing target thickness, at least the condition of the smallness of the mean free path due to Coulomb interaction $N\sigma_{coul}z \ll 1$ is violated. In this case the solution of the kinetic equation, which describes the Coulomb scattering of a deuteron by the nuclei of matter should be found. It will describe the beam distribution due to single and multiple Coulomb collisions of particles in the target.

As has been stated above, if in this case $z \ll 1/N\sigma_{nucl}$, which is realized in the majority of practical cases, one can find the nuclear and Coulomb nuclear contributions in the scope of the perturbation theory.

4.1 Solution of the kinetic equation

Write equation (1.18) as follows:

$$\frac{d\hat\rho(\vec k)}{dz}=-N\sigma^0_{tot}\hat\rho(\vec k)-\frac{N}{2}\left(\sigma^{\pm1}_{tot}-\sigma^0_{tot}\right)\left\{(\vec S\vec n)^2,\rho(\vec k)\right\}+\frac{2\pi i}{k}N\mathrm{Re}f_1(0)\left[(\vec S\vec n)^2,\rho(\vec k)\right]+ \\ +N\int d\Omega_{\vec k'}\hat f(\vec k,\vec k')\rho(\vec k')\hat f^+(\vec k',\vec k) \tag{4.1}$$

where $\sigma^{0,\pm1}_{tot}=Sp\int \mathrm{d}\Omega_{\vec k'}\hat f(\vec k,\vec k')\hat\rho_0^{0,\pm1}\hat f^+(\vec k,\vec k')+\sigma_r^{0,\pm1}$, $\hat\rho_0^{0,\pm1}$ is the deuteron density matrix describing the initial state of the beam for $M=0$ and $M=\pm1$, correspondingly; $\vec n$ is the unit vector in the direction of $\vec k$; σ_r denotes the part of the total cross-section, which is responsible for all possible inelastic interactions between the deuteron and the nuclei of matter, including nuclear reactions.

Substituting (2.4) into the expressions for the total cross-sections, the elastic part of $\sigma^{0,\pm1}_{tot}$ can be represented as a sum of the terms describing the cross-section of the Coulomb scattering, the pure nuclear cross-section and the cross-section of the interference between the Coulomb and nuclear interactions: $\sigma^{0,\pm1}_{tot}=\sigma^{0,\pm1}_{coul}+\sigma^{0,\pm1}_{nucl,coul}+\sigma^{0,\pm1}_{nucl}+\sigma_r^{0,\pm1}$.

Use the following notations: $\sigma^{0,\pm1}_{NC}=\sigma^{0,\pm1}_{nucl,coul}+\sigma^{0,\pm1}_{nucl}+\sigma_r^{0,\pm1}$. Thus, represent the cross-sections $\sigma^{0,\pm1}_{tot}$ in the form:

$$\sigma^{0,\pm1}_{tot}=\sigma^{0,\pm1}_{coul}+\sigma^{0,\pm1}_{NC}, \tag{4.2}$$

where $\sigma^{0,\pm1}_{coul}=Sp\int d\Omega_{\vec k'}\hat f_{coul}(\vec k,\vec k')\hat\rho_0^{0,\pm1}\hat f^+_{coul}(\vec k',\vec k)$, $\sigma^{0,\pm1}_{NC}=Sp\int d\Omega_{\vec k'}\Big(\hat f_{coul}(\vec k,\vec k')\hat\rho_0^{0,\pm1}\hat f^+_{nucl,coul}(\vec k',\vec k)+$
$+\hat f_{nucl,coul}(\vec k,\vec k')\hat\rho_0^{0,\pm1}\hat f^+_{coul}(\vec k',\vec k)+\hat f_{nucl,coul}(\vec k,\vec k')\hat\rho_0^{0,\pm1}\hat f^+_{nucl,coul}(\vec k',\vec k)\Big)+\sigma_r^{0,\pm1}$.

In view of the notations introduced above and the representation of the scattering amplitude (2.4), rewrite kinetic equation (4.1) as:

$$\frac{d\hat\rho(\vec k)}{dz}=-N\sigma^0_{coul}\hat\rho(\vec k)-\frac{N}{2}\left(\sigma^{\pm1}_{coul}-\sigma^0_{coul}\right)\left\{(\vec S\vec n)^2,\rho(\vec k)\right\}+\frac{2\pi i}{k}N\mathrm{Re}f_1^{coul}(0)\left[(\vec S\vec n)^2,\hat\rho(\vec k)\right]+ \\ +N\int d\Omega_{\vec k'}\hat f_{coul}(\vec k,\vec k')\hat\rho(\vec k')\hat f^+_{coul}(\vec k',\vec k)- \\ -N\sigma^0_{NC}\hat\rho(\vec k)-\frac{N}{2}\left(\sigma^{\pm1}_{NC}-\sigma^0_{NC}\right)\left\{(\vec S\vec n)^2,\hat\rho(\vec k)\right\}+\frac{2\pi i}{k}N\mathrm{Re}f_1^{nucl}(0)\left[(\vec S\vec n)^2,\hat\rho(\vec k)\right]+ \\ +N\int d\Omega_{\vec k'}\left(\hat f_{coul}(\vec k,\vec k')\hat\rho(\vec k')\hat f^+_{nucl,coul}(\vec k',\vec k)+\hat f_{nucl,coul}(\vec k,\vec k')\hat\rho(\vec k')\hat f^+_{coul}(\vec k',\vec k)\right)+ \\ +N\int d\Omega_{\vec k'}\hat f_{nucl,coul}(\vec k,\vec k')\hat\rho(\vec k')\hat f^+_{nucl,coul}(\vec k',\vec k), \tag{4.3}$$

As stated above, the solution of equation (4.3) can be presented as follows:

$$\hat\rho(\vec k,z)=\hat\rho^{(0)}(\vec k,z)+\hat\rho^{(1)}(\vec k,z)+..., \tag{4.4}$$

where $\hat\rho^{(0)}(\vec k,z)$ is the zero approximation, which is the solution of kinetic equation (4.3) and describes only the Coulomb interaction between the deuteron beam and the nuclei of matter; $\hat\rho^{(1)}(\vec k,z)$ is the first order perturbation theory correction allowing for the contribution of nuclear scattering to the evolution of the polarization characteristics of the beam.

The equation for $\hat\rho^{(0)}(z)$ has a form:

$$\frac{d\hat\rho^{(0)}(\vec k)}{dz}=-N\sigma^0_{coul}\hat\rho^{(0)}(\vec k)-\frac{N}{2}\left(\sigma^{\pm1}_{coul}-\sigma^0_{coul}\right)\left\{(\vec S\vec n)^2,\hat\rho^{(0)}(\vec k)\right\}+\frac{2\pi i}{k}N\mathrm{Re}f_1^{coul}(0)\left[(\vec S\vec n)^2,\hat\rho^{(0)}(\vec k)\right]+ \\ +N\int d\Omega_{\vec k'}\hat f_{coul}(\vec k,\vec k')\hat\rho^{(0)}(\vec k')\hat f^+_{coul}(\vec k,\vec k'), \tag{4.5}$$

where the spin structure $\hat{f}_{coul}(\vec{k},\vec{k}')$ in the general case has a form (2.1).

It can be shown [10] that the terms in the Coulomb amplitude (2.1), which are proportional to B, C_1, and to the part of C_2 depending on θ, lead to depolarization of the registered beam. The magnitude of such depolarization is determined by $b_g^2\overline{\theta^2}z$ ($b_g = \frac{g-2}{2}\frac{\gamma^2-1}{\gamma} + \frac{\gamma-1}{\gamma}$, g is the gyromagnetic ratio, $\overline{\theta^2} = N\int \theta^2 \frac{d\sigma_{coul}}{d\Omega} d\Omega$ is the average squared angle of Coulomb scattering). With due account of the small values of the scattering angles due to Coulomb scattering at high energies, this quantity will be insignificant as compared with the contribution from the scalar part of the amplitude (2.1).

For this reason, in the case of Coulomb interaction we can confine ourselves to considering the scattering amplitude of the form (3.2).

Moreover, the term in (3.2), which is equal to $f_1^{coul}(0)(\vec{S}\vec{n})^2$ and describes a coherent rotation of the polarization vector of the deuteron beam due to the elastic Coulomb scattering at zero angle, is also small and thus can be neglected [19].

As a result, equation (4.5) is rewritten as follows:

$$\frac{d\hat{\rho}^{(0)}(\vec{k},z)}{dz} = -N\sigma^0_{coul}\hat{\rho}^{(0)}(\vec{k},z) + N\int d\Omega_{\vec{k}'}\left|a(\vec{k},\vec{k}')\right|^2 \hat{\rho}^{(0)}(\vec{k}',z), \tag{4.6}$$

where $a(\vec{k},\vec{k}')$ denotes the spinless part of the amplitude $\hat{f}_{coul}(\vec{k},\vec{k}')$.

Expression (4.6) is an integro-differential equation. Its solution in the limit of small scattering angles can be obtained by expanding the function $\hat{\rho}^{(0)}(\vec{k}')$ over a relatively small parameter $\vec{q}$ (transferred momentum) [23]:

$$\hat{\rho}^{(0)}(\vec{k}',z) \approx \hat{\rho}^{(0)}(\vec{k},z) + \frac{\partial\hat{\rho}^{(0)}}{\partial k_x}q_x + \frac{\partial\hat{\rho}^{(0)}}{\partial k_y}q_y + \frac{1}{2}\frac{\partial^2\hat{\rho}^{(0)}}{\partial k_x^2}q_x^2 + \frac{\partial^2\hat{\rho}^{(0)}}{\partial k_x \partial k_y}q_x q_y + \frac{1}{2}\frac{\partial^2\hat{\rho}^{(0)}}{\partial k_y^2}q_y^2 + ..., \tag{4.7}$$

where $q_x = q\cos\varphi$, $q_y = q\sin\varphi$, $q \simeq k\theta$, where θ is the scattering angle (the angle between vector $\vec{k}$ and the z-axis).

Substituting the expansion (4.7) into (4.6) and integrating over the azimuth angle φ, one can obtain

$$\frac{d\hat{\rho}^{(0)}(\vec{k},z)}{dz} = \frac{\overline{\theta^2}}{4}\Delta\hat{\rho}^{(0)}(\vec{k},z) + \frac{\overline{\theta^4}}{64}\Delta\Delta\hat{\rho}^{(0)}(\vec{k},z) + ..., \tag{4.8}$$

operator Δ acts on the transverse components of vector $\vec{k}$: $\Delta = \frac{\partial^2}{\partial n_x^2} + \frac{\partial^2}{\partial n_y^2}$, where $\vec{n} = \vec{k}/k$; $\overline{\theta^2} = N\int\theta^2\frac{d\sigma_{coul}}{d\Omega}d\Omega$, $\overline{\theta^4} = N\int\theta^4\frac{d\sigma_{coul}}{d\Omega}d\Omega$, and etc.

If only the first term on the right-hand side of equation (4.8) is taken into account, the integro-differential equation (4.6) is reduced to a parabolic differential equation. The limiting angle for such approximation is obtained from the condition [23]:

$$\theta^2_{\max} < \frac{16\Delta\hat{\rho}^{(0)}(\vec{k},z)}{\Delta\Delta\hat{\rho}^{(0)}(\vec{k},z)}. \tag{4.9}$$

The solution of equation

$$\frac{d\hat{\rho}^{(0)}(\vec{k},z)}{dz} = \frac{\overline{\theta^2}}{4}\Delta\hat{\rho}^{(0)}(\vec{k},z) \tag{4.10}$$

for the initial condition $\hat{\rho}^{(0)}(\vec{k},z=0) = \hat{\rho}_0\delta(n_x)\delta(n_y)$ and an infinite medium is

$$\hat{\rho}^{(0)}(\vec{k},z) = \hat{\rho}_{0s}g_c(\vec{k},z), \tag{4.11}$$

where

$$\hat{\rho}_{0s} = \frac{1}{3} I_0 \hat{I} + \frac{1}{2} \vec{P}_0 \hat{S} + \frac{1}{9} Q_{0ik} \hat{Q}_{ik}$$

(see equation (3.3)), $g_c(\vec{k}, z) = \frac{1}{\pi \overline{\theta^2} z} e^{-\frac{(\vec{n}-\vec{n}_0)^2}{\overline{\theta^2} z}}$. The unit vector $\vec{n}$ is counted from an arbitrary direction $\vec{n}_0$ (then direct the z-axis along $\vec{n}_0$), $(\vec{n} - \vec{n}_0)^2 = \theta^2$, where θ is the angle between $\vec{k}$ and the z-axis directed along $\vec{n}_0$; $\overline{\theta^2}$ is the average squared angle of scattering per unit path length. The average squared angle of scattering at depth z $(\overline{\theta^2} z)$ will further be denoted by $\overline{\theta_z^2}$, and its square root, by θ_z.

The applicability condition (4.9) of this solution is equivalent to the inequality $\theta \ll \theta_z$. It should also be stated that for large θ, the solution of equation (4.6) will mainly behave as $\sim {}^1\!/_{\theta^4}$ [24]. Thus, in the approximate equation (4.10), scattering at large angles in a single scattering event is ignored. Indeed, for $\theta \gg \theta_z$, the solution (4.11) decreases exponentially, while the next term, corresponding to the solution of the initial equation (4.6), decreases according to a power law θ^{-4}. The angular distribution (4.11), therefore, does not describe the characteristics of particles scattered at angles $\theta \gg \theta_z$.

The expression for the density matrix in the zero-order approximation (4.11) enables obtaining the correction in the first order perturbation theory $\hat{\rho}^{(1)}(\vec{k}, z)$:

$$\begin{aligned}
\hat{\rho}^{(1)}(\vec{k}, z) = N \int_0^z \int G(\vec{k} - \vec{k}''; z - z') \Big(-\sigma_{NC}^0 \hat{\rho}^{(0)}(\vec{k}'', z') - \frac{1}{2} \left(\sigma_{NC}^{\pm 1} - \sigma_{NC}^0\right) \left\{ \hat{f}_s(\vec{k}'', \vec{k}''), \hat{\rho}^{(0)}(\vec{k}'', z') \right\} + \\
+ \frac{2\pi i}{k} N \mathrm{Re} f_1^{nucl}(0) \left[\hat{f}_s(\vec{k}'', \vec{k}''), \hat{\rho}^{(0)}(\vec{k}'', z') \right] \Big) d\Omega_{\vec{k}''} dz' + \\
+ N \int_0^z \int G(\vec{k} - \vec{k}''; z - z') \left(\hat{f}_{coul}(\vec{k}'', \vec{k}') \hat{\rho}^{(0)}(\vec{k}', z') \hat{f}^+_{nucl,coul}(\vec{k}', \vec{k}'') + \right. \\
\left. + \hat{f}_{nucl,coul}(\vec{k}'', \vec{k}') \hat{\rho}^{(0)}(\vec{k}', z') \hat{f}^+_{coul}(\vec{k}', \vec{k}'') \right) d\Omega_{\vec{k}'} d\Omega_{\vec{k}''} dz' \\
+ N \int_0^z \int G(\vec{k} - \vec{k}''; z - z') \hat{f}_{nucl,coul}(\vec{k}'', \vec{k}') \hat{\rho}^{(0)}(\vec{k}', z') \hat{f}^+_{nucl,coul}(\vec{k}', \vec{k}'') d\Omega_{\vec{k}'} d\Omega_{\vec{k}''} dz',
\end{aligned} \tag{4.12}$$

where $G(\vec{k}-\vec{k}''; z-z')$ is the Green function of equation (4.10): $G(\vec{k}-\vec{k}''; z-z') = \frac{1}{\pi \overline{\theta^2} |z - z'|} e^{-\frac{(\vec{n}-\vec{n}'')^2}{\overline{\theta^2} |z-z'|}}$. The amplitudes of Coulomb and nuclear scattering in the general case are expressed by formula (2.1).

Thus, the solution of kinetic equation (4.1) allowing for nuclear interaction has the following general form:

$$\hat{\rho}(\vec{k}, z) \simeq \hat{\rho}^{(0)}(\vec{k}, z) + \hat{\rho}^{(1)}(\vec{k}, z) \ , \tag{4.13}$$

where the main contribution $\hat{\rho}^{(0)}(\vec{k}, z)$ is defined by expression (4.11), the correction in the first order perturbation theory $\hat{\rho}^{(1)}(\vec{k}, z)$ equals (4.12).

To represent the solution in the explicit form, we shall confine ourselves to considering small scattering angles. In this case one can obtain the following approximate expression for the amplitude (2.1):

$$\hat{f}(\vec{k}, \vec{k}') = A(\chi) \left[1 + b\chi (\vec{S}\vec{\nu}) + c_1 \chi^2 \hat{Q}_{ik} \mu_i \mu_k + c_2 (1 - \chi^2/4) \hat{Q}_{ik} \mu_{1i} \mu_{1k} \right], \tag{4.14}$$

where the angular dependence of functions $b(\chi)$, $c_1(\chi)$, and $c_2(\chi)$ was supposed to be $b(\chi) \sim \sin\chi$, $c_1(\chi) \sim \sin^2\chi$, $c_2(\chi) \sim \cos^2\chi/2$.

Integration of the right- and left-hand sides of the solution of (4.13) over the final momentum $\vec{k}$ in the limits corresponding to the variation of the axial angle from 0 to 2π and the polar angle, from 0 to a certain ϑ_{det} gives the expression for the density matrix of the beam that has passed through the area occupied by the detector with angular width $\Delta\Omega$.

As has been stated above, for high energy deuterons, elastic scattering occurs mainly at small angles. In this case in the solution of (4.13) the general structure of the scattering amplitude in the form (4.14) should be used for the nuclear amplitude. It can be shown, however, that under the assumption of axial symmetry of the collimator of the detector, the correction terms in the polarization characteristics can be neglected with high accuracy. These polarization characteristics correspond to the terms in (4.14), which are proportional to χ and χ^2. This assertion is also valid for such target thicknesses and deuteron energies, for which the following inequalities are fulfilled: $\overline{\theta^2} z \ll 1$, $1/kR_d \ll 1$, $1/kR_c \ll 1$.

As a consequence, in the expression for the density matrix (4.13), the amplitude $\hat{f}_{nucl,coul}(\vec{k},\vec{k}')$ of scattering due to nuclear interaction is taken equal to:

$$\hat{f}_{nucl,coul}(\vec{k},\vec{k}') = d(\theta) + d_1(\theta)(\vec{S}\vec{n}_0)^2, \qquad (4.15)$$

$\vec{n}_0$ is the unit vector directed along the z-axis, $d(\theta)$ is the spin-independent and $d_1(\theta)$ is the spin-dependent parts of the nuclear amplitude, θ is the angle between $\vec{k}$ and $\vec{k}'$.

For the Coulomb scattering amplitude $\hat{f}_{coul}(\vec{k},\vec{k}')$, its spinless part $a(\theta)$ can be used with high accuracy.

Integral characteristics of the deuteron beam are obtained by substituting the explicit form of the density matrix (3.3) and the scattering amplitude (4.15) into the solution (4.13). As a result, we have:

$$\mathcal{I}(z) = (1 - e^{-\frac{\vartheta_{det}^2}{\theta_z^2}}) I_0 + \xi_1 I_0 + (\xi_2 + \xi_3)\left[\frac{2}{3} I_0 + \frac{1}{3}(\mathbf{Q}_0\vec{n}_0)\vec{n}_0\right],$$

$$\vec{P}(z) = (1 - e^{-\frac{\vartheta_{det}^2}{\theta_z^2}})\vec{P}_0 + \xi_1\vec{P}_0 + \frac{1}{2}\xi_2\left[\vec{P}_0 + \vec{n}_0(\vec{P}_0\vec{n}_0)\right] + \xi_3\vec{n}_0(\vec{P}_0\vec{n}_0)z - \frac{2}{3}\varphi_{eff}[\vec{n}_0 \times (\mathbf{Q}_0\vec{n}_0)],$$

$$\begin{aligned}
\mathcal{Q}(z) = {} & (1 - e^{-\frac{\vartheta_{det}^2}{\theta_z^2}})\mathbf{Q}_0 + \xi_1\mathbf{Q}_0 + \\
& + \xi_2\left[\mathbf{Q}_0 + \frac{1}{3}(3\vec{n}_0 \otimes \vec{n}_0 - \mathbf{I})I_0 - \frac{1}{2}\left((\mathbf{Q}_0\vec{n}_0)\otimes\vec{n}_0 + \vec{n}_0 \otimes (\mathbf{Q}_0\vec{n}_0)\right) + \frac{1}{3}\mathbf{I}(\mathbf{Q}_0\vec{n}_0)\vec{n}_0\right] + \\
& + \xi_3\left[\frac{1}{3}(3\vec{n}_0 \otimes \vec{n}_0 - \mathbf{I})I_0 - \frac{1}{2}\left((\mathbf{Q}_0\vec{n}_0)\otimes\vec{n}_0 + \vec{n}_0 \otimes (\mathbf{Q}_0\vec{n}_0)\right) + \frac{1}{2}\vec{n}_0^{\times}\mathbf{Q}_0\vec{n}_0^{\times} + \right. \\
& \left. + \vec{n}_0 \otimes \vec{n}_0\ (\mathbf{Q}_0\vec{n}_0)\vec{n}_0 - \frac{1}{6}\mathbf{I}(\mathbf{Q}_0\vec{n}_0)\vec{n}_0 + \frac{1}{2}\mathbf{Q}_0\right] - \frac{3}{2}\varphi_{eff}\left([\vec{n}_0 \times \vec{P}_0]\otimes\vec{n}_0 + \vec{n}_0 \otimes [\vec{n}_0 \times \vec{P}_0]\right).
\end{aligned} \qquad (4.16)$$

The parameters ξ_1, ξ_2, and ξ_3 are defined in terms of the relations:

$$\xi_1(z) = -N\sigma^0_{NC}\int\limits_0^z\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')g_c(\vec{k}'',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}}dz'+$$

$$+N\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(a(\vec{k}'',\vec{k}')d^+(\vec{k}',\vec{k}'')+d(\vec{k}'',\vec{k}')a^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'+$$

$$+N\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')d(\vec{k}'',\vec{k}')d^+(\vec{k}',\vec{k}'')g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz',$$

$$\xi_2(z) = -N(\sigma^{\pm1}_{NC}-\sigma^0_{NC})\int\limits_0^z\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')g_c(\vec{k}'',z')d\Omega''_{\vec{k}}d\Omega_{\vec{k}}dz'+$$

$$+N\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1(\vec{k}'',\vec{k}')a^+(\vec{k}',\vec{k}'')+a(\vec{k}'',\vec{k}')d_1^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'+$$

$$+N\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d(\vec{k}'',\vec{k}')d_1^+(\vec{k}',\vec{k}'')+d_1(\vec{k}'',\vec{k}')d^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega''_{\vec{k}}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz',$$

$$\xi_3(z) = N\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')d_1(\vec{k}'',\vec{k}')d_1^+(\vec{k}',\vec{k}'')g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'. \tag{4.17}$$

The general structure of the solution (4.16) is written in the same way as the solution (3.4) for a thin target. For this reason, all the conclusions derived in analyzing each of the terms appearing in the expression for the polarization vector in (3.4) are automatically (naturally) extended to the case of a thick target.

The rotation angle of the polarization vector with respect to vector $\vec{n}_0$ appears equal to

$$\varphi_{eff} = \frac{2\pi N}{k}\int\limits_0^z\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1(\vec{k},\vec{k})+d_1^+(\vec{k},\vec{k})\right)g_c(\vec{k}'',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}}dz'-$$

$$-\frac{iN}{2}\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1(\vec{k}'',\vec{k}')a^+(\vec{k}',\vec{k}'')-a(\vec{k}'',\vec{k}')d_1^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'-$$

$$-\frac{iN}{2}\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1(\vec{k}'',\vec{k}')d^+(\vec{k}',\vec{k}'')+d(\vec{k}'',\vec{k}')d_1^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'. \tag{4.18}$$

4.2 Analysis of the problem of compensation of the Coulomb-nuclear contributions in the expression for the rotation angle of the polarization vector

Let us consider in detail the contributions that are included in the expression for the rotation angle of the polarization vector of deuterons. The expression between the first brackets that contain the nuclear amplitude d_1 is the part of the rotation angle, which is due to the coherent scattering of deuterons in matter; the expression between the second brackets, which consists of the product of d_1 and the Coulomb amplitude a, describes the effect of the Coulomb-nuclear interference; the last part in φ_{eff}, which contains the product of the spin-dependent and spinless parts of the nuclear amplitude, is the correction to the deuteron spin rotation from nuclear scattering.

It would be recalled, however, that nuclear amplitudes d and d_1 are actually the scattering amplitudes modified by the Coulomb interaction. The relation of $\hat{f}_{nucl,coul}(\vec{k},\vec{k}')$ to "pure" Coulomb and "pure" nuclear amplitudes is expressed by a general (2.9) or an approximate (3.12) formula.

Substitution of $d_1(\vec{k},\vec{k}')$ from (3.12) into the expression for φ_{eff} (4.18) in the accepted approximation gives

$$\varphi_{eff} = \frac{2\pi N}{k}\mathrm{Re}d_1'(0)\int\limits_0^z\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')g_c(\vec{k}'',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}}dz'+$$
$$+\frac{iN}{4}\int\limits_0^z\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1'(\vec{k},\vec{k}')a(\vec{k}',\vec{k})+a(\vec{k},\vec{k}')d_1'(\vec{k}',\vec{k})-\right.$$
$$\left.-d_1'^*(\vec{k},\vec{k}')a^*(\vec{k}',\vec{k})-a^*(\vec{k},\vec{k}')d_1'^*(\vec{k}',\vec{k})\right)g_c(\vec{k}'',z')d\Omega'_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'-$$
$$-\frac{iN}{2}\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1'(\vec{k}'',\vec{k}')a^+(\vec{k}',\vec{k}'')-a(\vec{k}'',\vec{k}')d_1^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega'_{\vec{k}'}d\Omega_{\vec{k}}dz'-$$
$$-\frac{iN}{2}\int\limits_0^z\int\int\int\limits_{\Delta\Omega} G(\vec{k}-\vec{k}'';z-z')\left(d_1'(\vec{k}'',\vec{k}')d'^+(\vec{k}',\vec{k}'')+d'(\vec{k}'',\vec{k}')d_1'^+(\vec{k}',\vec{k}'')\right)g_c(\vec{k}',z')d\Omega_{\vec{k}''}d\Omega_{\vec{k}'}d\Omega_{\vec{k}}dz'. \tag{4.19}$$

Expand the Green function $G(\vec{k}-\vec{k}';z-z')$ and the function $g_c(\vec{k},z')$ into a series of orthogonal Legendre polynomials:

$$\begin{aligned} G(\vec{k}-\vec{k}';z-z') &= \sum_n\left(n+\frac{1}{2}\right)G_n(z-z')P_n(\cos\vartheta),\\ g_c(\vec{k}',z) &= \sum_l\left(l+\frac{1}{2}\right)g_{c\,l}(z)P_l(\cos\vartheta'), \end{aligned} \tag{4.20}$$

where ϑ is the angle between vectors $\vec{k}$ and $\vec{k}'$; ϑ' is the angle between vector $\vec{k}'$ and the z-axis.

In substitution of relations (4.20) into (4.19), one should take into account the condition of the orthogonality of the polynomials:

$$\int\limits_0^\pi P_j^m(\cos\vartheta)P_r^m(\cos\vartheta)\sin\vartheta d\vartheta = \frac{1}{j+1/2}\frac{(j+m)!}{(j-m)!}\delta_{jr} \tag{4.21}$$

and summation theorem:

$$P_n(\cos\chi) = P_n(\cos\vartheta)P_n(\cos\vartheta_1)+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\vartheta)P_n^m(\cos\vartheta_1)\cos m(\varphi-\varphi_1), \tag{4.22}$$

where χ is the angle between the two vectors that make with the z-axis the angles ϑ and ϑ_1, respectively. Considering small deviation angles of particles from the initial direction (the z-axis), let us take account of the fact that large n and l play the main part in the expansions (4.20). So in view of of the below relation, one can pass from the expansion into a series of Legendre functions to that of Bessel functions

$$\lim_{n\to\infty}P_n(\cos\frac{\vartheta}{k}) = J_0(\vartheta), \tag{4.23}$$

where $J_0(\vartheta)$ is the zero-order Bessel function. In this approximation the expansion coefficients $G_n(z-z')$ and $g_{c\,l}(z)$ have a form:

$$\begin{aligned} G_n(z-z') &= \frac{1}{2\pi}e^{-\frac{n^2}{4}\overline{\theta^2}(z-z')},\\ g_{c\,l}(z) &= \frac{1}{2\pi}e^{-\frac{l^2}{4}\overline{\theta^2}z}. \end{aligned} \tag{4.24}$$

As a result, expression (4.19) can be rewritten as follows:

$$\varphi_{eff} = \frac{2\pi N}{k}\mathrm{Re}d_1'(0)\int\limits_{\Delta\Omega} g_c(\vec{k},z)d\Omega_{\vec{k}}z+$$
$$+\frac{iN}{4}z\int\int\limits_{\Delta\Omega}\Big(d_1'(\vec{k},\vec{k}')a(\vec{k}',\vec{k})+a(\vec{k},\vec{k}')d_1'(\vec{k}',\vec{k})-d_1'^*(\vec{k},\vec{k}')a^*(\vec{k}',\vec{k})-a^*(\vec{k},\vec{k}')d_1'^*(\vec{k}',\vec{k})\Big)g_c(\vec{k},z)d\Omega_{\vec{k}'}d\Omega_{\vec{k}}-$$
$$-\frac{iN}{2}z\int\int\limits_{\Delta\Omega}\Big(d_1'(\vec{k},\vec{k}')a^*(\vec{k},\vec{k}')-a(\vec{k},\vec{k}')d_1'^*(\vec{k},\vec{k}')\Big)g_c(\vec{k}',z)d\Omega_{\vec{k}'}d\Omega_{\vec{k}}-$$
$$-\frac{iN}{2}z\int\int\limits_{\Delta\Omega}\Big(d_1'(\vec{k},\vec{k}')d'^+(\vec{k}',\vec{k})+d'(\vec{k},\vec{k}')d_1'^+(\vec{k}',\vec{k})\Big)g_c(\vec{k}',z)d\Omega_{\vec{k}'}d\Omega_{\vec{k}}. \tag{4.25}$$

Let now integration over the directions of the final momentum $\vec{k}$ be made in the domain of the entire range of variation in the scattering angle. In this case in (4.25), it is possible to replace the integration variables $\vec{k}$ by $\vec{k}'$ and vice versa.

For such deuteron energies at which only Coulomb amplitude in the first Born approximation can be taken into account, the Coulomb-nuclear contributions in the expression for the rotation angle compensate one another.

As a result, registration of the scattered and transmitted particles in a 4π experimental geometry gives the following magnitude of φ_{eff}:

$$\varphi_{eff} = \frac{2\pi N}{k}z\mathrm{Re}\left[d_1'(0)-\frac{ik}{2\pi}\iint d_1'(\vec{k},\vec{k}')d'^*(\vec{k},\vec{k}')g_c(\vec{k}',z)d\Omega_{\vec{k}'}d\Omega_{\vec{k}}\right]. \tag{4.26}$$

In the case under consideration, the rotation angle is determined by the sum of two terms: the coherent contribution, depending only on a pure nuclear amplitude of scattering at zero angle and the incoherent contribution due to single nuclear scattering.

In a real experiment, however, this case of a 4π-geometry is generally not realized. That is why, integration over $\vec{k}$ in (4.17) and (4.18) or (4.19) should be made within finite limits.

4.3 Calculation of the parameters of integral characteristics of the beam.

Integration over the angular variables in the parameters ξ_1, ξ_2, ξ_3, and φ_{eff} is performed using the expansion of the Green function $G(\vec{k}-\vec{k}';z-z')$ and the function $g_c(\vec{k},z)$ into a series of orthogonal Legendre polynomials according to formulas (4.20)–(4.24).

In the explicit form, the parameters of the system (4.16) are as follows:

$$\xi_1(z) = -N\sigma^0_{NC}(1-e^{-\frac{\vartheta^2_{det}}{\overline{\theta^2_z}}})z+2\pi Nz\int\limits_0^\infty P(\chi;\vartheta_{\mathrm{det}},\overline{\theta^2_z})\Big(2\mathrm{Re}[a(\chi)d^*(\chi)]+|d(\chi)|^2\Big)\chi d\chi,$$
$$\xi_2(z) = -N(\sigma^{\pm1}_{NC}-\sigma^0_{NC})(1-e^{-\frac{\vartheta^2_{det}}{\overline{\theta^2_z}}})z+2\pi Nz\int\limits_0^\infty P(\chi;\vartheta_{\mathrm{det}},\overline{\theta^2_z})\Big(\mathrm{Re}[a(\chi)d_1^*(\chi)]+$$
$$+\mathrm{Re}[d(\chi)d_1^*(\chi)]\Big)\chi d\chi, \tag{4.27}$$
$$\xi_3(z) = 2\pi Nz\int\limits_0^\infty P(\chi;\vartheta_{\mathrm{det}},\overline{\theta^2_z})|d_1(\chi)|^2\chi d\chi,$$

$$\varphi_{eff} = \frac{2\pi N}{k}\mathrm{Re}\left[d_1(0)\left(1-e^{-\frac{\vartheta^2_{det}}{\overline{\theta^2_z}}}\right)-ik\int\limits_0^\infty P(\chi;\vartheta_{\mathrm{det}},\overline{\theta^2_z})\Big(a^*(\chi)d_1(\chi)+d^*(\chi)d_1(\chi)\Big)\chi d\chi\right]z, \tag{4.28}$$

where the integral $P(\chi;\vartheta_{det},\overline{\theta_z^2}) \equiv \int\limits_0^{\vartheta_{det}}\int\limits_0^{\infty} \vartheta d\vartheta n\, dn\, e^{-\frac{n^2}{4}\overline{\theta_z^2}} J_0(n\vartheta)J_0(n\chi)$ is denoted in terms of the function $P(\chi;\vartheta_{\rm det},\overline{\theta_z^2})$ The last expression is integrated over n. As a result, the function P can be represented as the integral over ϑ of the expression containing a modified zero-order Bessel function $(I_0(z) = J_0(iz))$: $P(\chi;\vartheta_{det},\overline{\theta_z^2}) = \frac{2}{\overline{\theta_z^2}}\int\limits_0^{\vartheta_{det}} e^{-\frac{\vartheta^2+\chi^2}{\overline{\theta_z^2}}} I_0\left(\frac{2\vartheta\chi}{\overline{\theta_z^2}}\right)\vartheta d\vartheta$. The explicit form of the introduced function, which can be written in a form of the following infinite series is $P(\chi;\vartheta_{\rm det},\overline{\theta_z^2}) = \sum\limits_{m=0}^{\infty}\frac{\Gamma(m+1)}{m!\Gamma(m+2)}(-1)^m\left(\frac{\vartheta_{det}^2}{\overline{\theta_z^2}}\right)^{m+1} {\rm F}\left(-m;-1-m;1;\frac{\chi^2}{\vartheta_{det}^2}\right)$, where F is the gaussian hypergeometric function.

4.4 Analysis of the contributions to the rotation angle φ_{eff} for the case of scattering by a thick target

Rewrite expression (4.28) for two limiting values of the angle ϑ_{det} of the collimator of the detector: $\vartheta_{det} \ll \theta_z$ and $\vartheta_{det} \gg \theta_z$.

Before considering the angles $\vartheta_{\rm det} \gg \theta_z$, let us indicate the following points. As has been stated above, the solution (4.11), which describes the angular distribution of the deuterons due to Coulomb interaction holds true for the scattering angles $\theta \ll \theta_z$. Since we analyze the integral characteristics of the beam, the solution (4.11) can also be used for $\vartheta_{\rm det} \gg \theta_z$. The stated approximation means that the contribution of singly scattered particles due to Coulomb interaction is neglected. Moreover, large values of θ make zero contribution to the integral $\int\limits_0^{\vartheta_{det}} \rho^{(0)}(\vec{k},z)d\Omega$ and a small contribution to $\int\limits_0^{\vartheta_{det}} \rho^{(1)}(\vec{k},z)d\Omega$ (the major contribution to the integral comes from small θ).

In the first case, in integration of the second term in (4.8), it can be assumed that $J_0(n\vartheta) \approx 1$. Let $\varphi^{\rm I}_{eff}$ denote the rotation angle of the beam that has passed through the area occupied by the detector with angular width $\vartheta_{\rm det} \ll \theta_z$. The explicit expression for $\varphi^{\rm I}_{eff}$ has a form:

$$\varphi^{\rm I}_{eff} = \frac{2\pi N}{k}\frac{\vartheta_{det}^2}{\overline{\theta_z^2}}{\rm Re}\left[d_1(0) - ik\int\limits_0^{\infty}\chi d\chi e^{-\frac{\chi^2}{\overline{\theta_z^2}}}\left(a^*(\chi)d_1(\chi) + d^*(\chi)d_1(\chi)\right)\right] z. \tag{4.29}$$

For another limiting case $\vartheta_{det} \gg \theta_z$, relation (4.28) is written as:

$$\varphi^{\rm II}_{eff} \simeq \frac{2\pi N}{k}{\rm Re}\left[d_1(0) - ik\int\limits_0^{\infty}\chi d\chi\left(a^*(\chi)d_1(\chi) + d^*(\chi)d_1(\chi)\right)\right] z. \tag{4.30}$$

One can easily see that in the case when the whole beam gets into the detector, the expression for the rotation angle $\varphi^{\rm II}_{eff}$ is similar to that for φ_{eff}, which was obtained in describing the passage of the deuteron beam through a very thin target in the case of a wide experimental geometry (3.8).

As it has been done in the previous section, let us represent φ_{eff} as a sum of three terms:

$$\varphi_{eff}(\vartheta_{det}) = \varphi_0(\vartheta_{det}) + \varphi_{nc}(\vartheta_{det}) + \varphi_{nn}(\vartheta_{det}), \tag{4.31}$$

where $\varphi_0(\vartheta_{det}) = \frac{2\pi N}{k}{\rm Re}\, d_1(0)\left(1 - e^{-\vartheta_{det}^2/\overline{\theta_z^2}}\right) z$ is the contribution of the coherent scattering to the rotation of the polarization vector; $\varphi_{nc}(\vartheta_{det}) = 2\pi N {\rm Im}\left[\int\limits_0^{\infty} P(\chi;\vartheta_{det},\overline{\theta_z^2})a^*(\chi)d_1(\chi)\chi d\chi\right] z$ is the part of the rotation angle, which describes the effect of the Coulomb-nuclear interference; $\varphi_{nn}(\vartheta_{det}) =$

$2\pi N \mathrm{Im}\left[\int_0^\infty P(\chi;\vartheta_{det},\overline{\theta_z^2}) d^*(\chi) d_1(\chi)\chi d\chi\right] z$ is the correction to the rotation of the deuteron spin from a pure nuclear scattering. Obtain the numerical values of these contributions for the energy of 500 MeV, $k = 0.74 \cdot 10^{14}\mathrm{cm}^{-1}$. Then we have $kR_d = 31.82$ and the value of the average squared angle of deuteron scattering by a carbon target per unit length $\overline{\theta^2} = 16\pi N Z^2 \left(\frac{e^2}{pv}\right)^2 \ln(137\, Z^{-1/3})$ equals $0.23 \cdot 10^{-4}\mathrm{cm}^{-1}$. The maximum value of the rotation angle φ_0 is obtained at $\vartheta_{det} \gg \theta_z$: $\varphi_0 \simeq 0.12 \cdot 10^{-3} z$.

Consider the relation φ_{nc}/φ_0. Calculate the explicit form of the Coulomb-nuclear correction, using the expression for the Coulomb amplitude corresponding to scattering by a screened Coulomb potential in the first Born approximation:

$$a(\theta) = -2\frac{mZe^2}{\hbar^2} R_c^2 \frac{1}{1 + k^2 R_c^2 \theta^2}. \tag{4.32}$$

In the case of scattering by carbon, $kR_{coul} = 2 \cdot 10^5$. Using (3.7) and (4.32), one can obtain for the whole range of scattering angles:

$$\frac{\varphi_{nc}}{\varphi_0} = -2\frac{\mathrm{Im} d_1}{\mathrm{Re} d_1}\frac{mZe^2}{\hbar^2 k}\frac{k^2 R_{coul}^2}{1 - e^{-\vartheta_{det}^2/\overline{\theta_z^2}}}\int_0^\infty P(\chi;\vartheta_{det},\overline{\theta_z^2})\frac{1}{1 + k^2 R_c^2 \chi^2} e^{-\frac{\chi^2 k^2 R_d^2}{4}}\chi d\chi \tag{4.33}$$

Plot this dependence for three values of $\overline{\theta_z^2}$ (since the value of energy is fixed, this corresponds to three values of the target thickness z):

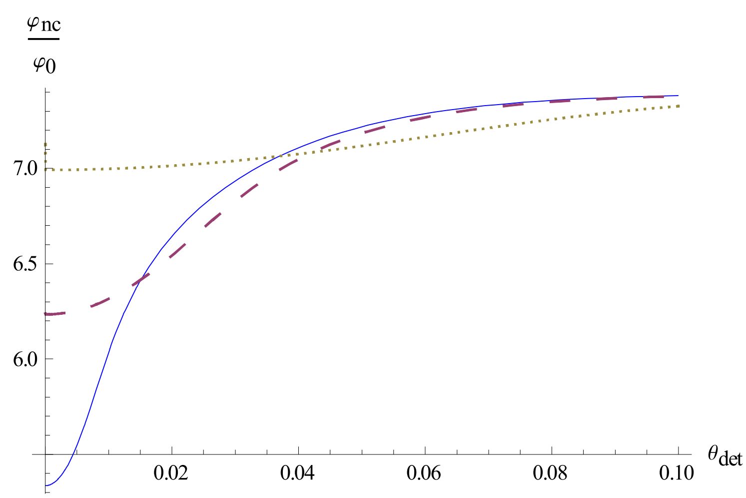

Fig. 3: Relative contribution of the Coulomb-nuclear interference as a function of the angle of the detector for $\overline{\theta_{z1}^2} = 0.23 \cdot 10^{-4}$ cm^{-1} (solid curve), $\overline{\theta_{z2}^2} = 0.23 \cdot 10^{-3}$ cm^{-1} (dashed curve) and $\overline{\theta_{z3}^2} = 0.23 \cdot 10^{-2}$ cm^{-1} (dotted curve).

A rather simple analytical expression for φ_{nc}/φ_0 can be obtained for two limiting values of ϑ_{det}.

Let $\vartheta_{\mathrm{det}} \ll \theta_z$. From (4.29) we have that for small values of the angles of the detector, the relation φ_{nc}/φ_0 is the function independent of ϑ_{det}:

$$\left(\frac{\varphi_{nc}}{\varphi_0}\right)^{\mathrm{I}} = \frac{\mathrm{Im} d_1}{\mathrm{Re} d_1}\frac{mZe^2}{\hbar^2 k}\mathrm{Ei}\left[-\frac{1}{k^2 R_c^2}\left(\frac{1}{\overline{\theta_z^2}} + \frac{k^2 R_d^2}{4}\right)\right]\exp\left[-\frac{1}{k^2 R_c^2}\left(\frac{1}{\overline{\theta_z^2}} + \frac{k^2 R_d^2}{4}\right)\right]. \tag{4.34}$$

When $\vartheta_{\mathrm{det}} \to \infty$ the expression φ_{nc}/φ_0 tends to its limiting value (see Fig. 1):

$$\left(\frac{\varphi_{nc}}{\varphi_0}\right)^{\mathrm{II}} = \frac{\mathrm{Im}d_1}{\mathrm{Re}d_1}\frac{mZe^2}{\hbar^2 k}\mathrm{Ei}\left[-\frac{R_d^2}{4R_c^2}\right]. \tag{4.35}$$

According to (4.34) for such energies and target thicknesses when $\theta_z \gg \theta_n$, the relation $(\varphi_{nc}/\varphi_0)^{\mathrm{I}}$ tends to $(\varphi_{nc}/\varphi_0)^{\mathrm{II}}$, i.e., weakly depends on ϑ_{det}. If the parameter $\theta_z \ll \theta_n$, then for $\vartheta_{det} \to 0$ the relation $(\varphi_{nc}/\varphi_0)^{\mathrm{I}}$ decreases with decreasing target thickness. The corresponding contribution can be neglected in the case when $\overline{\theta_z^2} \ll \theta_n^2 \exp\left(\mathrm{C} - \frac{\hbar^2 k}{mZe^2}\frac{\mathrm{Re}d_1}{\mathrm{Im}d_1}\right)$, where C is the Euler–Mascheroni constant, C =0.5772. For the energy of 500 MeV, the value of z that satisfies this condition is $z \ll 10^{-6}$ cm. When $\overline{\theta_z^2} \gtrsim \theta_n^2 \exp\left(\mathrm{C} - \frac{\hbar^2 k}{mZe^2}\frac{\mathrm{Re}d_1}{\mathrm{Im}d_1}\right)$, the value of φ_{nc} at $\vartheta_{det} \ll \theta_z$ is of the order of or greater than the main contribution φ_0.

With increasing ϑ_{det}, the relation φ_{nc}/φ_0 grows, approaching $(\varphi_{nc}/\varphi_0)^{\mathrm{I}} = \frac{\mathrm{Im}d_1}{\mathrm{Re}d_1}\frac{mZe^2}{\hbar^2 k}\left[\mathrm{C} + \ln\left(\frac{R_d^2}{4R_c^2}\right)\right]$, and this value of the contribution does not depend on the target thickness. From this follows that when $\vartheta_{\mathrm{det}} \gg \theta_z$, the contribution of the Coulomb-nuclear scattering cannot be neglected for any z: φ_{nc} is one order of magnitude larger than the main contribution φ_0.

4.5 Rotation angle including the Coulomb contribution to the spin part of nuclear amplitude $d_1(0)$

Let us consider in more detail the expression for the rotation angle including the Coulomb contribution to the amplitude $d_1(0)$. For the analysis of the total contribution of the Coulomb-nuclear interference, it is convenient to write equation (4.31) in a form:

$$\varphi_{eff} = \varphi_0' + \varphi_{nc}^{tot} + \varphi_{nn}, \tag{4.36}$$

where φ_{nc}^{tot} is the total contribution of the Coulomb-nuclear interference into the rotation angle of the polarization vector. Within the limit of small scattering angles, its explicit form reads: : $\varphi_{nc}^{tot} = -\frac{2\pi N}{k}\frac{mZe^2}{\hbar^2 k^2}\mathrm{Im}d_1\left\{2k^2 R_{coul}^2\int\limits_0^\infty f(\chi;\vartheta_{\mathrm{det}},\overline{\theta_z^2})\frac{1}{1+k^2R_c^2\chi^2}e^{-\frac{\chi^2k^2R_d^2}{4}}\chi d\chi + \mathrm{Ei}\left(-\frac{R_d^2}{4R_c^2}\right)\left(1-e^{-\vartheta_{det}^2/\overline{\theta_z^2}}\right)\right\}z.$

Basing on the above considerations, one may immediately see that when the condition $\theta_z \gg \theta_n$ is fulfilled, the total Coulomb-nuclear contribution vanishes.

Let us analyze the behavior of the Coulomb-nuclear interference when $\theta_z \ll \theta_n$. With this aim in view, consider the relative contribution of φ_{nc}^{tot}:

$$\begin{aligned}\frac{\varphi_{nc}^{tot}}{\varphi_0'} = -\frac{\mathrm{Im}d_1'}{\mathrm{Re}d_1'}\frac{mZe^2}{\hbar^2 k}\frac{1}{1-e^{-\vartheta_{\mathrm{det}}^2/\overline{\theta_z^2}}}&\left\{2k^2R_{coul}^2\int\limits_0^\infty f(\chi;\vartheta_{\mathrm{det}},\overline{\theta_z^2})\frac{1}{1+k^2R_c^2\chi^2}e^{-\frac{\chi^2k^2R_d^2}{4}}\chi d\chi +\right.\\ &\left. + \mathrm{Ei}\left(-\frac{R_d^2}{4R_c^2}\right)\left(1-e^{-\vartheta_{det}^2/\overline{\theta_z^2}}\right)\right\},\end{aligned} \tag{4.37}$$

where $\mathrm{Re}d_1'$ is the spin-dependent part of a pure nuclear amplitude of scattering. According to (3.12), the relation between the amplitudes $\mathrm{Re}d_1'$ and $\mathrm{Re}d_1$ is determined by formula

$$\mathrm{Re}d_1' = \mathrm{Re}d_1 + \mathrm{Im}d_1\frac{mZe^2}{\hbar^2 k}\mathrm{Ei}\left[-\frac{R_d^2}{4R_c^2}\right]. \tag{4.38}$$

The calculated value is $\mathrm{Re}d_1/\mathrm{Re}d_1' = 0.12$. Using this estimate, let us plot the dependence $\varphi_{nc}^{tot}/\varphi_0'$ for three values of the average squared angle $\overline{\theta_z^2}$ of multiple scattering at depth z:

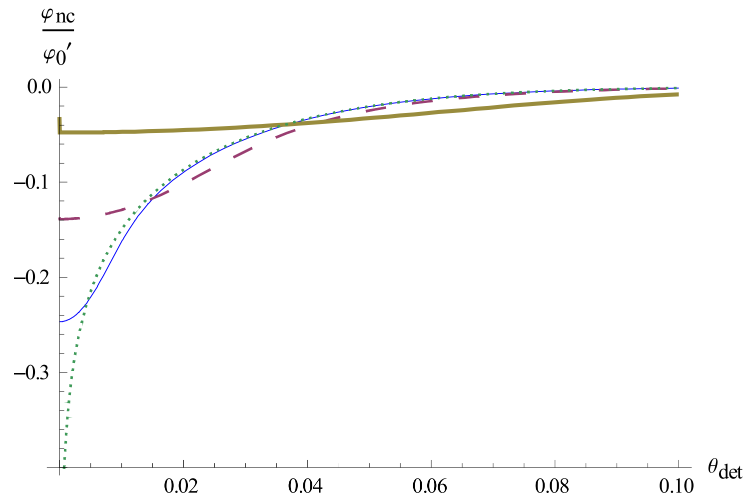

Fig. 4: Relative contribution of the Coulomb-nuclear interference $\varphi_{nc}^{tot}/\varphi_0'$ as a function of the angle of the detector for $\overline{\theta_z^2}_1 = 0.23 \cdot 10^{-4}$ cm^{-1} (solid curve), $\overline{\theta_z^2}_2 = 0.23 \cdot 10^{-3}$ cm^{-1} (dashed curve) and $\overline{\theta_z^2}_3 = 0.23 \cdot 10^{-2}$ cm^{-1} (thick solid curve).

It is seen in Fig. 4 that the maximum absolute value of $|\varphi_{nc}^{tot}|$ is achieved at the angles of the detector $\vartheta_{\rm det} \ll \theta_z$:

$$\left(\frac{\varphi_{nc}^{tot}}{\varphi_0'}\right)_{\rm max} = \frac{{\rm Im} d_1}{{\rm Re} d_1'}\frac{mZe^2}{\hbar^2 k}\left\{{\rm Ei}\left[-\frac{1}{k^2R_c^2}\left(\frac{1}{\overline{\theta_z^2}}+\frac{k^2R_d^2}{4}\right)\right]\exp\left[-\frac{1}{k^2R_c^2}\left(\frac{1}{\overline{\theta_z^2}}+\frac{k^2R_d^2}{4}\right)\right]- {\rm Ei}\left[-\frac{R_d^2}{4R_c^2}\right]\right\}. \tag{4.39}$$

For considered energy, at target thickness $z \leq 0.01$ cm, the magnitude of $|\varphi_{nc}^{tot}|$ is of the order of φ_0'. With further decrease in z, the contribution of $|\varphi_{nc}^{tot}|$ grows and reaches its maximum value $\frac{{\rm Im} d_1}{{\rm Re} d_1'}\frac{mZe^2}{\hbar^2 k}{\rm Ei}\left[-\frac{R_d^2}{4R_c^2}\right]$ at $z \to 0$. It is easy to see that it is exactly equal to the maximum absolute value of the relative value of the total Coulomb-nuclear contribution $\left|\varphi_{nc}^{tot}/\varphi_0'\right|$ for the case of a thin target (3.16). The maximum value of this quantity is also achieved for small ϑ_{det}, namely, within the limit $\vartheta_{det} \to 0$. Moreover, for a thin target, the dependence of the relation $\varphi_{nc}^{tot}/\varphi_0'$ on the detector angle ϑ_{det} is, in fact, the limit to which the Coulomb-nuclear contribution tends when deuterons are scattered by a thick target, if the parameter $\overline{\theta_z^2} \to 0$ (dotted line in Fig. 4).

Indeed, at $\overline{\theta_z^2} \to 0$, the introduced function $f(\chi;\vartheta_{\rm det},\overline{\theta_z^2})$ becomes equal to $f(\chi;\vartheta_{det},0) = \int_0^{\vartheta_{det}} \delta(\vartheta-\chi)d\vartheta$. As a result, for the rotation angle φ_{eff} defined by formula (4.28), we obtain the expression for φ_{eff} in the case of deuteron scattering by a thin target (3.8). It is clear because within the limit $\overline{\theta_z^2} \to 0$, the contribution of multiple scattering can be neglected for given energies and (or) target thicknesses. This statement is true even for $\theta_z \ll \theta_c$, i.e., for the case when the value of the average squared angle of multiple scattering is much smaller than the diffraction angle for Coulomb scattering. In this case, similarly to the case of scattering by a thin target, one can introduce the detector angle ϑ_{det}^{comp} at which one may consider that the interference contributions compensate each other. Thus, for $\vartheta_{det} \ll \theta_n$, the contribution of $|\varphi_{nc}^{tot}|$ is comparable in magnitude to φ_0', i.e., the Coulomb-nuclear terms are not compensated. When $\vartheta_{det} > \theta_n$, the total contribution of the Coulomb-nuclear interference can be neglected.

On the other hand, when $\theta_z \gg \theta_c$, the dependence of $\varphi_{nc}^{tot}/\varphi_0'$ on ϑ_{det} becomes more smooth (Fig.

4). In this case the the maximum absolute value of $|\varphi_{nc}^{tot}/\varphi_0'|$ diminishes with growing parameter $\overline{\theta_z^2}$:

$$\left(\frac{\varphi_{nc}^{tot}}{\varphi_0'}\right)_{\max} = \frac{\mathrm{Im}d_1'}{\mathrm{Re}d_1'}\frac{mZe^2}{\hbar^2 k}\ln\left[\frac{4}{k^2R_d^2}\left(\frac{1}{\overline{\theta_z^2}}+\frac{k^2R_d^2}{4}\right)\right]. \tag{4.40}$$

From this follows that at the energy of 500 MeV and the target thickness $z \gg 1$ cm, up to the accuracy of 10%, one may consider that the Coulomb-nuclear interference due to incoherent scattering compensates the Coulomb-nuclear contribution to the amplitude $d_1(0)$ for all values of ϑ_{det}. The expression for the rotation angle φ_{eff} will also take quite a simple form (below, it is shown that the nuclear part of φ_{nn} can as well be neglected in comparison with φ_0'):

$$\varphi_{eff} \simeq \varphi_0' = \frac{2\pi N}{k}\mathrm{Re}d_1'(1-e^{-\frac{\vartheta_{det}^2}{\theta^2 z}})z. \tag{4.41}$$

Using (4.38), we obtain that the maximum value of this quantity is $\varphi_0' \simeq 10^{-2}z$, i.e., one order of magnitude larger than φ_0.

Thus, for the energies in the region $0.1 \div 1$ GeV, one can indicate two main domains of variability of the value of the multiple scattering parameter $\overline{\theta_z^2}$:

1. $\theta_z \ll \theta_c$. Here the dependence of the function $\varphi_{nc}^{tot}/\varphi_0'$ on the angle ϑ_{det} of the detector is close to the similar dependence in the case of a thin target.

2. $\theta_z \gtrsim \theta_c$. In this range of energies and target thicknesses the dependence of the magnitude of the relative Coulomb-nuclear contribution on ϑ_{det} is quite smooth. The total contribution of the Coulomb-nuclear interference can be neglected with quite a good accuracy.

4.6 Nuclear contribution to the rotation angle

The relative contribution of nuclear scattering of deuterons by target nuclei φ_{nn}/φ_0 is obtained, using the approximate expression (3.7):

$$\frac{\varphi_{nn}}{\varphi_0} = \frac{1}{kR_d^2}\left(\mathrm{Re}d\frac{\mathrm{Im}d_1}{\mathrm{Re}d_1} - \mathrm{Im}d\right)\left[1-\exp\left(-\frac{k^2R_d^2\vartheta_{\mathrm{det}}^2}{2+k^2R_d^2\,\overline{\theta_z^2}}\right)\right]\frac{1}{1-e^{-\vartheta_{\mathrm{det}}^2/\overline{\theta_z^2}}}. \tag{4.42}$$

Similarly to the previous case, let us represent the dependence of φ_{nn}/φ_0 on the angle of the detector for three values of average squared angle of multiple scattering at depth z:

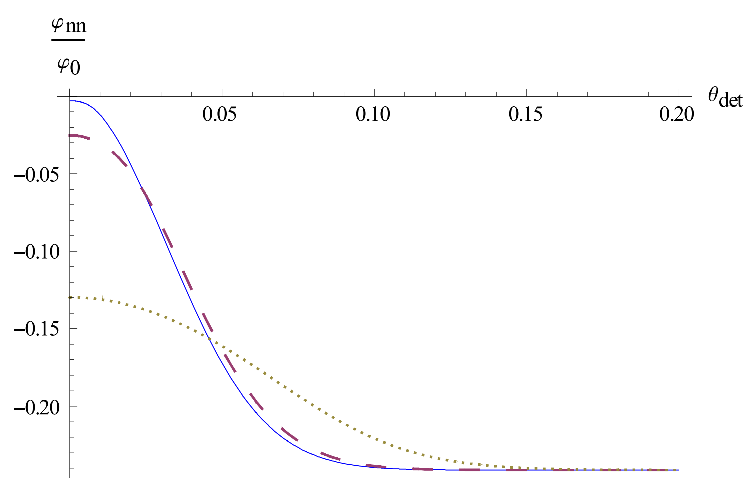

Fig. 5: Relative nuclear contribution s as a function of the angle of the detector for $\overline{\theta_{z1}^2} = 0.23 \cdot 10^{-4}$ cm^{-1} (solid curve), $\overline{\theta_{z2}^2} = 0.23 \cdot 10^{-3}$ cm^{-1} (dashed curve) and $\overline{\theta_{z3}^2} = 0.23 \cdot 10^{-2}$ cm^{-1} (dotted curve).

In the case of a thick target $\theta_z \gg \theta_n$, the relation φ_{nn}/φ_0 at large thickness becomes independent of $\vartheta_{\rm det}$ and equals its maximum value.

$$\left(\frac{\varphi_{nn}}{\varphi_0}\right)^{\rm I} = \frac{1}{kR_d^2}\left(\mathrm{Re}d\frac{\mathrm{Im}d_1}{\mathrm{Re}d_1} - \mathrm{Im}d\right). \tag{4.43}$$

The numerical value of $|\varphi_{nn}/\varphi_0|_{\rm max} = 0.24$.

For $\theta_z \ll \theta_n$, the contribution of φ_{nn}/φ_0 depends on $\vartheta_{\rm det}$ (see Fig.5); for $\vartheta_{\rm det} \ll \theta_z$ this relation is $\left(\frac{\varphi_{nn}}{\varphi_0}\right)^{\rm I} = \frac{1}{kR_d^2}\left(\mathrm{Re}d\frac{\mathrm{Im}d_1}{\mathrm{Re}d_1} - \mathrm{Im}d\right)\frac{k^2R_d^2}{2}\overline{\theta_z^2}$. It increases gradually with growing angular width of the detector and achieves the maximum absolute value (4.43) at $\vartheta_{\rm det} \gg \theta_n$.

From this follows that the nuclear contribution φ_{nn} can be neglected for any $\vartheta_{\rm det}$. In this case the magnitude of the relation $|\varphi_{nn}/\varphi_0'|$ is of the order of 10^{-2}.

Conclusion

Experimental observation of the phenomenon of birefringence of particles naturally always implies such experimental arrangement, in which the beam of particles is directed to the target, passes through it and then the detector registers the polarization characteristics of particles moving in the direction of incidence of the initial beam and detected within a certain small angular interval relative to this direction. A comprehensive description of the interaction between particles (deuterons) and the nuclei of matter is given by a kinetic equation for the density matrix. Using this equation one can also see that the scattering process may be both coherent and incoherent. The birefringence effect itself, and, in particular, one of its parameters, the rotation angle of the polarization vector is the result of a coherent interaction between the particle and the target. Alongside with coherent interaction, incoherent interaction also leads to the rotation of the polarization vector relative to the chosen direction. That is why the total effective rotation angle is the characteristic, which is measured in the experiment. As it has been shown, real experimental conditions also influence the value of φ_{eff}: being defined as an integral characteristic, it depends on the value of the angle of the detector collimator. In the general case, the magnitude of the Coulomb-nuclear and nuclear-nuclear contributions increases with growing ϑ_{det}. In particular, at $\vartheta_{det} > \theta_n$, the correction from the Coulomb-nuclear interference exceeds by one order of magnitude the main contribution of φ_0. Here arises the question about the magnitude of the total contribution of the Coulomb-nuclear interference to the rotation angle φ_{eff}. It has been shown that these terms are compensated completely only when the beam particles are registered in a 4π geometry of the experiment and the deuteron energy is such that one can take into account only the first Born approximation to the Coulomb amplitude. Therefore, in a real experimental arrangement, the compensation of the Coulomb-nuclear terms will not be observed. The quantitative analysis demonstrated that for a thin target, the magnitude of the total Coulomb-nuclear interference for $\vartheta_{det} \gg \theta_n$ equals zero with high accuracy. At the same time, as a result of multiple scattering of particles in the target, the contribution of the Coulomb-nuclear terms becomes dependent on one more parameter: the average squared angle of multiple scattering. It has been shown that if its magnitude at depth z is of the order of or larger than the magnitude of the squared diffraction angle of nuclear scattering, the Coulomb-nuclear interference can be neglected for any angles of the detector. If this condition is not fulfilled, the contribution of the total Coulomb-nuclear interference to the rotation angle should be taken into account.

The contribution of the nuclear-nuclear interaction for the two cases of target thicknesses considered above is small as compared to the main contribution.

Thus, the elastic coherent scattering of deuterons in the target leads to additional corrections to the birefringence effect, the magnitude of these contributions appreciably depends on both the average squared angle of multiple scattering and on the specific geometry of the experiment, namely on the angular width of the collimator.

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