Tensor polarization of deuterons passing through matter

V G Baryshevsky and A R Bartkevich

Research Institute for Nuclear Problems, Belarusian State University, 11 Bobruiskaya Str., Minsk 220050, Belarus

E-mail: bar@inp.bsu.by; v_baryshevsky@yahoo.com

Abstract.

It is shown that the magnitude of tensor polarization of the deuteron beam, which arises owing to the spin dichroism effect, depends appreciably on the angular width of the detector that registers the deuterons transmitted through the target. Even when the angular width of the detector is much smaller than the mean square angle of multiple Coulomb scattering, the beam's tensor polarization depends noticeably on rescattering. When the angular width of the detector is much larger than the mean square angle of multiple Coulomb scattering (as well as than the characteristic angle of elastic nuclear scattering), tensor polarization is determined only by the total reaction cross sections for deuteron-nucleus interaction, and elastic scattering processes make no contribution to tensor polarization.

PACS numbers: 13.88.+e

Submitted to: *J. Phys. G: Nucl. Phys.*

1. Introduction

The quasi-optical macroscopic quantum phenomenon of birefringence, first described in [1, 2], combines the effects of particle spin rotation (oscillation) and spin dichroism (i.e., the dependence of the absorption coefficient on the particle spin state) and occurs when high-energy particles with spin $S \geq 1$ traverse matter. It is analogous to the phenomenon known in optics as birefringence of light in optically anisotropic media due to the dependence of the refractive index on the polarization state of light, i.e., on the photon spin state. In contrast to light, whose wavelength largely exceeds the interatomic distance in matter, the de Broglie wavelength for a fast particle is much shorter than this distance. According to [1, 2, 3], however, in this case one can also introduce for particles a spin-dependent index of refraction. Under such conditions, the birefringence effect for deuterons takes place even in homogeneous, isotropic matter and results from the intrinsic anisotropy of particles with spin $S \geq 1$.

An important implication of the spin dichroism effect is tensor polarization arising in the initially unpolarized beam after it has passed through an unpolarized target [1, 2].

In 2005, the spin dichroism effect was first experimentally observed for 5–20 MeV deuterons passing through a carbon target [4, 5, 6]. In 2007, tensor polarization arising in a nuclotron extracted beam of initially unpolarized deuterons with a momentum of 5 GeV/c was measured [7, 8].

The magnitudes of spin dichroism and tensor polarization of the transmitted deuterons are proportional to the difference between the total interaction cross sections of the particle with the nucleus for the states with magnetic quantum numbers $M = 0$ and $M = \pm 1$ [1, 2, 3].

According to [1, 2, 3, 4, 5, 6, 7, 8], this effect can be used not only for studying the total spin–dependent interaction cross section, but also for obtaining tensor-polarized deuteron beams. Besides, this effect must be taken into account when interpreting the results of planned experiments [9] for measuring the electric dipole moment of deuterons [3, 10].

In theoretical description of this phenomenon, one should bear in mind that a deuteron colliding with a nucleus undergoes two interactions: the Coulomb and nuclear ones. The Coulomb interaction not only causes multiple Coulomb scattering, but in some cases, it can also affect significantly the magnitude of tensor polarization of the deuterons that have passed through the target. It has been shown in [11], for example, that the Coulomb–nuclear interference gives a clue to the explanation of the experimentally observed fact that tensor polarization reverses sign as the energy of deuterons changes from 5 to 20 MeV.

Multiple scattering of deuterons in matter and the scattering–angle dependence of the Coulomb–nuclear interference account for the necessity to use the kinetic equation for the spin density matrix in order to describe the spin–state dynamics of the deuteron passing through matter [12].

In the present paper, it is shown that tensor polarization of the deuterons escaping from the target in the direction of the in-coming primary beam depends on the collimator's angular width of the detector that registers the transmitted deuteron beam. It is demonstrated that when the angular width of the detector's collimator is much larger than the characteristic angles of deuteron scattering in the target, tensor polarization of deuterons is determined only by the total cross sections of inelastic processes in the interaction between the deuterons and the nuclei.

This paper is arranged as follows. Section 2 gives the kinetic equation of the spin density matrix and the spin–dependent nuclear scattering amplitude. Section 3.1 considers deuteron spin dichroism in a thin target,where only single scattering is essential. Section 3.2 discusses the effect that multiple scattering makes on the magnitude of tensor polarization of the deuteron beam moving in matter. A brief summary of the obtained results is given in Conclusion.

2. Kinetic equation for the spin density matrix

In order to describe the behavior of a beam of spin particles in matter, let us introduce the spin density matrix $\hat{\rho}$ of the system "incident particle + target". The spin density matrix $\hat{\rho}_{\mathrm{d}}$ of particles (deuterons, protons) moving in matter is defined by the expression $\hat{\rho}_{\mathrm{d}} = \mathrm{Tr}_{\mathrm{T}}\,\hat{\rho}$, where Tr_{T} means summation of the diagonal elements of the matrix over all states of the target. The equation for time evolution of the spin density matrix $\hat{\rho}_{\mathrm{d}}$ has the form [13]:

$$\frac{\mathrm{d}\hat{\rho}_{\mathrm{d}}}{\mathrm{d}t} = -\frac{i}{\hbar}\left[\hat{H}_0, \hat{\rho}_{\mathrm{d}}\right] + \left(\frac{\partial\hat{\rho}_{\mathrm{d}}}{\partial t}\right)_{\mathrm{sct}},$$

where the Hamiltonian $\hat{H}_0$ describes the interaction between the particles and macroscopic external fields, and the collision term $(\partial\hat{\rho}_{\mathrm{d}}/\partial t)_{\mathrm{sct}}$ describes the density matrix evolution due to scattering by the target nuclei (atoms).

The explicit form of $(\partial\hat{\rho}_{\mathrm{d}}/\partial t)_{\mathrm{sct}}$ can be obtained when considering scattering of the incident particle by the bound target particle, provided that the following inequality holds: $W_0/\varepsilon_k \ll 1$. Here W_0 is the binding energy of the scatterer in the target, and ε_k is the kinetic energy of the incident particle. In this case, impulse approximation applies to calculating the scattering process [14]. We shall further consider the case when the external electric and magnetic fields are absent. This enables one to write the below integro–differential representation of the kinetic equation for the density matrix that describes the behavior of a polarized particle beam as it passes through matter whose nuclei (atoms) in the general case are polarized [12]:

$$\begin{aligned}\frac{\mathrm{d}\hat{\rho}_{\mathrm{d}}(\vec{k},t)}{\mathrm{d}t} &= \frac{2\pi\mathrm{i}}{M_{\mathrm{r}}} N\,\mathrm{Tr}_{\mathrm{T}}\left(\hat{F}(\vec{k},\vec{k})\hat{\rho}(\vec{k},t) - \hat{\rho}(\vec{k},t)\hat{F}^{+}(\vec{k},\vec{k})\right)\\ &\quad + \frac{N}{M_{\mathrm{r}}^2}\,\mathrm{Tr}_{\mathrm{T}}\int \mathrm{d}^3\vec{k}'\,\delta\left(\varepsilon_k - \varepsilon_{k'} - \frac{\vec{q}^{\,2}}{2M}\right)\hat{F}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',t)\hat{F}^{+}(\vec{k}',\vec{k}),\end{aligned} \tag{2.1}$$

where $\hat{F}(\vec{k},\vec{k}')$ is the amplitude of particle scattering by the target nuclei, N is the number of target particles in the unit volume, M is the mass of the scatterer, M_{r} is the reduced mass of the incident particle and the nucleus, ε_k is the kinetic energy of the incident particle, $\vec{k}$ and $\vec{k}'$ are the wave vectors of the beam particle before and after scattering, respectively. The density matrix $\hat{\rho}(\vec{k})$ is the direct product of the spin density matrix $\hat{\rho}_{\mathrm{d}}(\vec{k})$ of the incident particle and the spin density matrix $\hat{\rho}_{\mathrm{T}}(\vec{k})$ of the target nuclei: $\hat{\rho}(\vec{k}) = \hat{\rho}_{\mathrm{d}}(\vec{k};\hat{\vec{S}}_{\mathrm{d}}) \otimes \hat{\rho}_{\mathrm{T}}(\hat{\vec{S}}_{\mathrm{T}})$, where $\hat{\vec{S}}_{\mathrm{d}}$ and $\hat{\vec{S}}_{\mathrm{T}}$ are the spin operators of the beam particle and the scatterer, respectively.

Kinetic equation (2.1) applies to such values of the transmitted momentum $|\vec{q}| = |\vec{k}' - \vec{k}|$ that are governed by the inequality $q \gg r^{-1}$, where r is the correlation radius of the medium [12].

Equation (2.1) simplifies when a particle (proton, deuteron, antiproton) passes through a target whose nuclei have a mass much greater than the mass of the incoming particle. In this case, we can neglect the effect of the incident particle's energy loss through scattering. So we can neglect the recoil energy $\vec{q}^{\,2}/2M$ in the δ-function. As a result, one obtains a simple kinetic equation describing the time and spin evolution of the incident particle as it passes through the target [12, 13]:

$$\frac{\mathrm{d}\hat{\rho}_{\mathrm{d}}(\vec{k},t)}{\mathrm{d}t} = \frac{2\pi \mathrm{i}}{m} N \,\mathrm{Tr}_{\mathrm{T}} \left[\hat{F}(\vec{k},\vec{k})\hat{\rho}(\vec{k},t) - \hat{\rho}(\vec{k},t)\hat{F}^{+}(\vec{k},\vec{k})\right] + N\frac{k}{m}\,\mathrm{Tr}_{\mathrm{T}} \int \mathrm{d}\Omega_{\vec{k}'} \hat{F}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',t)\hat{F}^{+}(\vec{k}',\vec{k}), \qquad (2.2)$$

where $|\vec{k}| = |\vec{k}'|$ and m is the mass of the incident particle.

The first term on the right-hand side of (2.2) can be represented as follows:

$$\hat{F}(0)\hat{\rho}(\vec{k},t) - \hat{\rho}(\vec{k},t)\hat{F}^{+}(0) = \left[\frac{1}{2}\left(\hat{F}(0) + \hat{F}^{+}(0)\right), \hat{\rho}(\vec{k},t)\right] + \left\{\frac{1}{2}\left(\hat{F}(0) - \hat{F}^{+}(0)\right), \hat{\rho}(\vec{k},t)\right\}, \qquad (2.3)$$

where $[,]$ is the commutator, $\{\,,\}$ is the anticommutator.

The part proportional to the commutator leads to rotation of the polarization vector due to elastic coherent scattering [13] (as a result of the refraction effect [1, 2, 3]); the anticommutator describes the reduction in the intensity of the transmitted beam.

The last term in (2.2) determines the effect of incoherent scattering on the change of $\hat{\rho}_d$ (in the general case, the effect of single and multiple scattering).

For the sake of concreteness, let us consider the process of deuteron passage through the target with spinless nuclei. In this case, the density matrix $\hat{\rho}(\vec{k})$, as well as the amplitude $\hat{F}(\vec{k},\vec{k}')$, contains only spin variables of the scattered beam. For the amplitude $\hat{F}(\vec{k},\vec{k}')$, we shall introduce the notation $\hat{f}(\vec{k},\vec{k}') = \hat{F}(\vec{k},\vec{k}')$, where $\hat{f}(\vec{k},\vec{k}') \equiv \hat{f}(\vec{k},\vec{k}';\hat{\vec{S}}_{\mathrm{d}})$.

As a result, (ref1.2) can be written as follows:

$$\frac{\mathrm{d}\hat{\rho}_{\mathrm{d}}(\vec{k},z)}{\mathrm{d}z} = \frac{\pi \mathrm{i}}{k} N \left[(\hat{f}(\vec{k},\vec{k}) + \hat{f}^{+}(\vec{k},\vec{k})), \hat{\rho}_{\mathrm{d}}(\vec{k},z)\right] + \frac{\pi \mathrm{i}}{k} N \left\{(\hat{f}(\vec{k},\vec{k}) - \hat{f}^{+}(\vec{k},\vec{k})), \hat{\rho}_{\mathrm{d}}(\vec{k},z)\right\} + N \int \mathrm{d}\Omega_{\vec{k}'} \hat{f}(\vec{k},\vec{k}')\hat{\rho}_{\mathrm{d}}(\vec{k}',z)\hat{f}^{+}(\vec{k}',\vec{k}), \qquad (2.4)$$

where $z = vt$ (v is the particle velocity) is the distance travelled by the incident particle in matter. Hereinafter, the subscript d of the density matrix will be dropped.

In the case of deuterons (particles with spin 1), the polarization state is characterized by the polarization vector $\vec{P}(\vec{k}) = \mathrm{Tr}\,\hat{\rho}(\vec{k})\hat{\vec{S}}$ and the quadrupolarization tensor (tensor of rank 2) $\mathbf{Q}$, whose components are defined as $Q_{ik}(\vec{k}) = \mathrm{Tr}\,\hat{\rho}(\vec{k})\hat{Q}_{ik}$, where the operators $\hat{Q}_{ik}$ can be represented in the form: $\hat{Q}_{ik} = \frac{3}{2}\left(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i - \frac{4}{3}\delta_{ik}\hat{\mathrm{I}}\right)$.

The spin density matrix $\hat{\rho}(\vec{k})$ can be written in the following general form:

$$\hat{\rho}(\vec{k}) = \frac{1}{3}I(\vec{k})\hat{\mathrm{I}} + \frac{1}{2}\vec{P}(\vec{k})\hat{\vec{S}} + \frac{1}{9}Q_{ik}(\vec{k})\hat{Q}_{ik}, \tag{2.5}$$

where $I(\vec{k}) = \mathrm{Tr}\,\hat{\rho}(\vec{k})$, $\hat{\mathrm{I}}$ is the 3×3 identity matrix in the spin space. Note that alongside with the quantities $\vec{P}$ and $\mathbf{Q}$, normalized spin characteristics of the beam are also used to describe the beam polarization [15, 16].

For particles with spin 1 that are scattered by an unpolarized nucleus, the amplitude $\hat{f}(\vec{k},\vec{k}')$ can be expressed in terms of the deuteron spin operator $\hat{\vec{S}}$, the quadrupolarization tensor $\hat{Q}_{ik}$, and the combination of vectors $\vec{k}$ and $\vec{k}'$:

$$\hat{f}(\vec{k},\vec{k}') = A\hat{I} + B(\hat{\vec{S}}\vec{\nu}) + C_1\hat{Q}_{ik}\mu_i\mu_k + C_2\hat{Q}_{ik}\mu_{1i}\mu_{1k}, \tag{2.6}$$

where A, B, C_1, and C_2 are the parameters depending on the scattering angle θ; $\vec{\nu} = [\vec{k}\times\vec{k}']/|[\vec{k}\times\vec{k}']|$; $\vec{\mu} = (\vec{k}-\vec{k}')/|\vec{k}-\vec{k}'|$, and $\vec{\mu}_1 = (\vec{k}+\vec{k}')/|\vec{k}+\vec{k}'|$.

In view of (2.6), for the zero-angle scattering amplitude $\hat{f}(\vec{k},\vec{k})$, we have

$$\hat{f}(\vec{k},\vec{k}) = f_0(0) + f_1(0)(\vec{S}\vec{n})^2, \tag{2.7}$$

where $\vec{n} = \vec{k}/k$ is the unit vector in the direction of $\vec{k}$, and the following notation is introduced: $f_0 = A - 2C_1 - 2C_2$, $f_1 = 3C_2$.

In the general case, f_0 and f_1 are complex functions, and according to the optical theorem, the imaginary parts of f_0 and f_1 can be expressed in terms of the corresponding total cross sections, namely,

$$\mathrm{Im} f_0(0) = \frac{k}{4\pi}\,\sigma^0_{\mathrm{tot}}, \qquad \mathrm{Im} f_1(0) = \frac{k}{4\pi}\left[\sigma^{\pm 1}_{\mathrm{tot}} - \sigma^0_{\mathrm{tot}}\right], \tag{2.8}$$

where σ^0_{tot}, $\sigma^{\pm 1}_{\mathrm{tot}}$ are the total scattering cross sections for the initial spin state of the deuteron with magnetic quantum numbers $M = 0$ and $M = \pm 1$, respectively (the quantization axis z is directed along $\vec{n}$).

Deuterons interact with the target nuclei via the Coulomb and nuclear interactions. The amplitude $\hat{f}(\vec{k},\vec{k}')$ of deuteron scattering by the target nuclei in this case can be represented in the form [12, 17]:

$$\hat{f}(\vec{k},\vec{k}') = \hat{f}_{\mathrm{coul}}(\vec{k},\vec{k}') + \hat{f}_{\mathrm{nucl,coul}}(\vec{k},\vec{k}'), \tag{2.9}$$

where $\hat{f}_{\mathrm{coul}}$ is the amplitude of Coulomb scattering of the deuteron by the nucleus in the absence of nuclear interaction, $\hat{f}_{\mathrm{nucl,coul}}$ is the amplitude of scattering of Coulomb–distorted waves by the nuclear potential.

3. The phenomenon of spin dichroism

The phenomenon of deuteron spin dichroism (i.e., the dependence of the absorption coefficient on the spin state of the particle moving in matter) leads to the appearance of tensor polarization in the initially unpolarized beam transmitted through the unpolarized target.

The unpolarized deuteron beam can be considered as an incoherent mixture of three polarized beams having the same intensity and spin states with magnetic quantum numbers $M = -1$, 0, and +1, respectively (the quantization axis is directed parallel to the deuteron momentum). The total scattering cross section σ_{tot} of deuterons by a nucleus depends on the polarization state of the incident particle; namely, in the interaction with an unpolarized nucleus, $\sigma_{\text{tot}}^{+1} = \sigma_{\text{tot}}^{-1} \neq \sigma_{\text{tot}}^{0}$. As a result, the partial intensities $I^{(\pm1)}$ and $I^{(0)}$ of the beam transmitted through the target appear to be unequal, and so the spin dichroism phenomenon can be characterized by the parameter

$$D = \frac{I^{(\pm1)} - I^{(0)}}{I^{(\pm1)} + I^{(0)}}, \tag{3.1}$$

where $I^{(0)}$, $I^{(\pm1)}$ are the intensities of the deuteron beam transmitted through the unpolarized target for the case when the initial state of the deuteron is defined by quantum numbers $M = 0$ and $M = \pm1$, respectively.

Spin dichroism leads to the appearance of tensor polarization in the transmitted beam. According to the definition (see e.g., [15, 16]), the diagonal components of the normalized tensor polarization can be written in the form:

$$p_{xx} = p_{yy} = -\frac{1}{2}\frac{I^{(+1)} + I^{(-1)} - 2I^{(0)}}{I^{(+1)} + I^{(-1)} + I^{(0)}}, \qquad p_{zz} = \frac{I^{(+1)} + I^{(-1)} - 2I^{(0)}}{I^{(+1)} + I^{(-1)} + I^{(0)}}, \tag{3.2}$$

where $p_{xx} = Q_{xx}/I$, $p_{yy} = Q_{yy}/I$, and $p_{zz} = Q_{zz}/I$.

In the case when the beam passes through the unpolarized target that is considered here, the intensities $I^{(+1)} = I^{(-1)}$. As a result, we have the following expression for p_{zz}:

$$p_{zz} = 2\frac{I^{(\pm1)} - I^{(0)}}{I}, \tag{3.3}$$

where I is the intensity of the initially unpolarized beam, $I = I^{(+1)} + I^{(-1)} + I^{(0)} = 2I^{(\pm1)} + I^{(0)}$.

3.1. Tensor polarization of deuterons passing through a thin target

Let us consider particle transmission through the target with thickness z, which is smaller than the deuteron's free path in matter, i.e., $z < 1/N\sigma$, σ is the total scattering cross section of the deuteron by the nucleus. In the first–order perturbation theory, the solution of kinetic equation (2.4) can be represented in the form:

$$\begin{aligned}\hat{\rho}(\vec{k}, z) = \hat{\rho}(\vec{k}, 0) + \frac{\pi \mathrm{i}}{k} N \left[(\hat{f}(\vec{k}, \vec{k}) + \hat{f}^{+}(\vec{k}, \vec{k})), \hat{\rho}(\vec{k}, 0) \right] z \\ + \frac{\pi \mathrm{i}}{k} N \left\{ (\hat{f}(\vec{k}, \vec{k}) - \hat{f}^{+}(\vec{k}, \vec{k})), \hat{\rho}(\vec{k}, 0) \right\} z + N \hat{f}(\vec{k}, \vec{k}_0) \hat{\rho}(0) \hat{f}^{+}(\vec{k}_0, \vec{k}) z,\end{aligned} \tag{3.4}$$

where $\hat{\rho}(\vec{k}, 0)$ is the density matrix of the beam at entering the target, i.e., when $z = 0$. It describes the momentum distribution of the incoming particles with respect to the direction $\vec{k}_0$.

In deriving (3.4), it was also assumed that the beam's initial angular distribution is much smaller than the characteristic angular width of the differential scattering cross section. In this case, the amplitude $\hat{f}(\vec{k},\vec{k}')$ in the term $\int d\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',0)\hat{f}^{+}(\vec{k}',\vec{k})$ can be removed from the integral sign at point $\vec{k}' = \vec{k}_0$. As a result, we have $\int d\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',0)\hat{f}^{+}(\vec{k}',\vec{k}) \simeq \hat{f}(\vec{k},\vec{k}_0)\hat{\rho}(0)\hat{f}^{+}(\vec{k}_0,\vec{k})$, where $\hat{\rho}(0) = \int d\Omega_{\vec{k}'}\hat{\rho}(\vec{k}',0)$ is the spin part of the beam's density matrix $\hat{\rho}(\vec{k},0)$. The term $N\int d\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',0)\hat{f}^{+}(\vec{k}',\vec{k})$ or $N\hat{f}(\vec{k},\vec{k}_0)\hat{\rho}(0)\hat{f}^{+}(\vec{k}_0,\vec{k})$ describes the contribution to the evolution of the density matrix due to single scattering of particles in the direction of vector $\vec{k}$.

Using the solution (3.4) and the explicit form of the spin structure of the amplitude $\hat{f}(\vec{k},\vec{k}')$ [see (2.6)], one can find the dependence of the beam's intensity and polarization characteristics on the direction of particle scattering and on the distance z passed by the deuteron in matter. In a real experiment, the collimator of the detector has a finite angular width, and so the scattered particles are registered within a certain range of finite momenta. For this reason, the characteristics of the beam transmitted through the target should be studied in the range of solid angles $\Delta\Omega$ with respect to the initial direction of beam propagation (see figure 1). If the collimator has axial symmetry, then $\Delta\Omega$ is defined by the angular width $2\vartheta_{\rm det}$ of the detector's collimator (for further calculations, it is also assumed that $\vartheta_{\rm det}$ is much larger than the initial angular distribution of the beam).

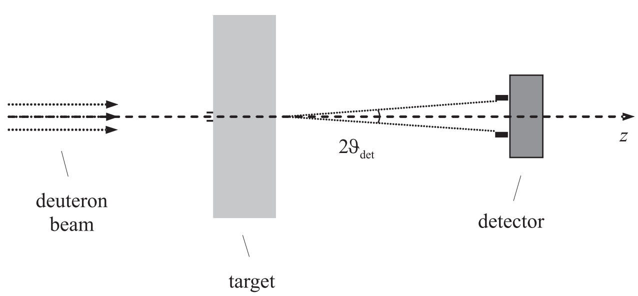

Figure 1. Scheme of detecting the transmitted beam.

We shall further consider the case when the unpolarized beam is incident onto the target.

The number of particles $\mathcal{I}^{(0)}$, $\mathcal{I}^{(\pm1)}$ with quantum numbers $M = 0$ and $M = \pm1$, which are registered by the detector with angular width $\Delta\Omega$ is determined by the diagonal matrix elements $\langle\Psi_{M=0}|\int_{\Delta\Omega} d\Omega_{\vec{k}}\hat{\rho}(\vec{k},z)|\Psi_{M=0}\rangle$ and $\langle\Psi_{M=\pm1}|\int_{\Delta\Omega} d\Omega_{\vec{k}}\hat{\rho}(\vec{k},z)|\Psi_{M=\pm1}\rangle$ of the spin density matrix (the quantization axis is chosen along the direction $\vec{n} = \vec{k}/k$):

$$\mathcal{I}^{(0)}(z) = I_0^{(0)}\left[1 - N\sigma^0_{\rm tot}z + N\int_{\Delta\Omega} d\Omega\,\langle\Psi_{M=0}|\left(\frac{d\hat{\sigma}}{d\Omega}\right)_{\rm sc}|\Psi_{M=0}\rangle z\right],$$

$$\mathcal{I}^{(\pm1)}(z) = I_0^{(\pm1)} \left[1 - N\sigma_{\text{tot}}^{\pm1} z + N \int_{\Delta\Omega} \mathrm{d}\Omega \, \langle \Psi_{M=\pm1} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=\pm1} \rangle \, z \right], \quad (3.5)$$

where $\mathcal{I} \equiv \int_{\Delta\Omega} \mathrm{d}\Omega I(\vec{k}, z)$; $I_0^{(0)}$ and $I_0^{(\pm1)}$ are the partial values of the flow of the deuteron beam at $z = 0$ for the spin states $|\Psi_{M=0}\rangle$ and $|\Psi_{M=\pm1}\rangle$, respectively. For the case of unpolarized beams, considered below, $I_0^{(0)} = \frac{1}{3} I_0$ and $I_0^{(\pm1)} = \frac{1}{3} I_0$, where I_0 is the deuteron flow at the target entrance; $I_0 = \int \mathrm{d}\Omega I(\vec{k}, 0)$. The differential scattering cross section of the deuteron is $(\mathrm{d}\hat{\sigma}/\mathrm{d}\Omega)_{\text{sc}}$.

To find tensor polarization of the registered beam, let us make use of (3.3) and (3.5). As a result, one can obtain the following expression for p_{zz}:

$$\begin{aligned} p_{zz} \simeq & -\frac{2}{3} N \left(\sigma_{\text{tot}}^{\pm1} - \sigma_{\text{tot}}^{0} \right) z \\ & + \frac{2}{3} N \int_{\Delta\Omega} \mathrm{d}\Omega \left\{ \langle \Psi_{M=\pm1} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=\pm1} \rangle \right. \\ & \left. - \langle \Psi_{M=0} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=0} \rangle \right\} z, \end{aligned} \quad (3.6)$$

that is,

$$\begin{aligned} p_{zz} \simeq & -\frac{2}{3} N \left(\sigma_{\text{r}}^{\pm1} - \sigma_{\text{r}}^{0} \right) z \\ & - \frac{2}{3} N \left[\left(\sigma_{\text{el}}^{\pm1} - \sigma_{\text{el}}^{0} \right) - \int_{\Delta\Omega} \mathrm{d}\Omega \left\{ \langle \Psi_{M=\pm1} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=\pm1} \rangle \right. \right. \\ & \left. \left. - \langle \Psi_{M=0} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=0} \rangle \right\} \right] z, \end{aligned} \quad (3.7)$$

where σ_{r} is the inelastic part of the total cross section, which includes all possible nuclear reactions, σ_{el} is the total cross section of elastic scattering.

As is seen, taking account of the scattered particles leads to the presence in the tensor polarization of the term depending on the angular width of the detector's collimator. Particularly, if $\vartheta_{\text{det}} \to 0$, then $\int_{\Delta\Omega} \mathrm{d}\Omega \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} \to 0$. As a consequence [see (3.6)], tensor polarization depends on the difference between the total cross section for the deuterons in the initial spin state with $M = 0$ and that with $M = \pm1$:

$$p_{zz}(\vartheta_{\text{det}} \to 0) \simeq -\frac{2}{3} N \left(\sigma_{\text{tot}}^{\pm1} - \sigma_{\text{tot}}^{0} \right) z. \quad (3.8)$$

If $\vartheta_{\text{det}} \to \pi$, then all the scattered particles are registered. In this case, the second term is proportional to the difference between the total cross sections of elastic scattering:

$$\begin{aligned} \sigma_{\text{el}}^{\pm1} &= \int \mathrm{d}\Omega \langle \Psi_{M=\pm1} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=\pm1} \rangle, \\ \sigma_{\text{el}}^{0} &= \int \mathrm{d}\Omega \langle \Psi_{M=0} | \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega} \right)_{\text{sc}} | \Psi_{M=0} \rangle. \end{aligned} \quad (3.9)$$

As a result, according to (3.7), tensor polarization p_{zz} is described by the following expression:

$$p_{zz}(\vartheta_{\rm det}=\pi) \simeq -\frac{2}{3}N\left(\sigma_{\rm r}^{\pm 1}-\sigma_{\rm r}^{0}\right)z. \qquad (3.10)$$

The differential cross section of elastic scattering for fast particles scattered by the nucleus achieves the largest values in the range of angles $\theta \leq 1/kR$ (R is the radius of action of forces) and decreases rapidly with growing θ ($\theta \gg 1/kR$). For this reason, p_{zz} achieves the value (3.10) already when $\vartheta_{\rm det} \gg 1/kR$.

Thus, in the range of large values of $\vartheta_{\rm det}$, tensor polarization only depends on the difference between the inelastic scattering cross sections for deuteron states with $M=0$ and $M=\pm 1$. It is quite understandable because in this case, there is no elastic-scattering-related losses in the flow of particles registered by the detector: the particles that have passed through the target without being scattered and those that have been elastically scattered equally get into the detector.

3.2. Influence of multiple Coulomb scattering on tensor polarization of deuterons passing through matter

The solution (3.4) applies to such target thicknesses z, for which multiple scattering in matter can be neglected, i.e., for $z \leq 1/N\sigma_{\rm coul}$.

To solve (2.4) for the case $z > 1/N\sigma_{\rm coul}$, let us take into account that in the considered energy range, $\sigma_{\rm nucl} \ll \sigma_{\rm coul}$. As a result, for the zeroth approximation we use the solution of kinetic equation (2.4), where the collision term is only determined by the the Coulomb interaction between the incident particle and the nuclei of matter. The contribution coming to the evolution of the spin density matrix from nuclear scattering and the Coulomb-nuclear interference can be considered as a correction, which is true for the cases when $z \leq 1/N\sigma_{\rm nucl}$.

3.2.1. Solution of the Kinetic Equation. Using the optical theorem (2.8), equation (2.4) can be written as follows:

$$\frac{{\rm d}\hat{\rho}(\vec{k},z)}{{\rm d}z} = -N\sigma_{\rm tot}^{0}\hat{\rho}(\vec{k},z) - \frac{N}{2}\left(\sigma_{\rm tot}^{\pm 1}-\sigma_{\rm tot}^{0}\right)\left\{(\vec{S}\vec{n})^2,\hat{\rho}(\vec{k},z)\right\}$$
$$+\frac{2\pi {\rm i}}{k}N{\rm Re}f_1(0)\left[(\vec{S}\vec{n})^2,\hat{\rho}(\vec{k},z)\right] + N\int {\rm d}\Omega_{\vec{k}'}\hat{f}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',z)\hat{f}^{+}(\vec{k}',\vec{k}) \qquad (3.11)$$

where $\vec{n}$ is the unit vector in the direction of the momentum $\vec{k}$.

It follows from the form of the amplitude (2.9) that the elastic part of the total cross section $\sigma_{\rm el}^{0,\pm 1}$ can be presented as a sum of the terms describing the Coulomb scattering cross section, the nuclear cross section, and the contributions to the cross section coming from the interference between the Coulomb and nuclear interactions: $\sigma_{\rm el}^{0,\pm 1} = \sigma_{\rm coul}^{0,\pm 1} + \sigma_{\rm nucl,coul}^{0,\pm 1} + \sigma_{\rm nucl}^{0,\pm 1}$. Let us introduce the following notations: $\sigma_{\rm NC}^{0,\pm 1} = \sigma_{\rm nucl,coul}^{0,\pm 1} + \sigma_{\rm nucl}^{0,\pm 1} + \sigma_{\rm r}^{0,\pm 1}$. Thus, the cross section $\sigma_{\rm tot}^{0,\pm 1}$ can be presented in the form:

$$\sigma_{\rm tot}^{0,\pm 1} = \sigma_{\rm coul}^{0,\pm 1} + \sigma_{\rm NC}^{0,\pm 1}. \qquad (3.12)$$

In view of (3.12) and the representation (2.9) for the scattering amplitude, kinetic equation (3.11) can be written in the form

$$\begin{aligned}\frac{\mathrm{d}\hat{\rho}(\vec{k},z)}{\mathrm{d}z} &= -N\sigma^{0}_{\mathrm{coul}}\hat{\rho}(\vec{k},z) - \frac{N}{2}\left(\sigma^{\pm1}_{\mathrm{coul}} - \sigma^{0}_{\mathrm{coul}}\right)\left\{(\vec{S}\vec{n})^2, \hat{\rho}(\vec{k},z)\right\} \\ &+\frac{2\pi\mathrm{i}}{k}N\mathrm{Re}f_1^{\mathrm{coul}}(0)\left[(\vec{S}\vec{n})^2, \hat{\rho}(\vec{k},z)\right] \\ &+N\int \mathrm{d}\Omega_{\vec{k}'}\hat{f}_{\mathrm{coul}}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',z)\hat{f}^{+}_{\mathrm{coul}}(\vec{k}',\vec{k}) \\ &-N\sigma^{0}_{\mathrm{NC}}\hat{\rho}(\vec{k},z) - \frac{N}{2}\left(\sigma^{\pm1}_{\mathrm{NC}} - \sigma^{0}_{\mathrm{NC}}\right)\left\{(\vec{S}\vec{n})^2, \hat{\rho}(\vec{k},z)\right\} \\ &+\frac{2\pi\mathrm{i}}{k}N\mathrm{Re}f_1^{\mathrm{nucl}}(0)\left[(\vec{S}\vec{n})^2, \hat{\rho}(\vec{k},z)\right] \\ &+N\int \mathrm{d}\Omega_{\vec{k}'}\Big(\hat{f}_{\mathrm{coul}}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',z)\hat{f}^{+}_{\mathrm{nucl,coul}}(\vec{k}',\vec{k}) \\ &+\hat{f}_{\mathrm{nucl,coul}}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',z)\hat{f}^{+}_{\mathrm{coul}}(\vec{k}',\vec{k})\Big) \\ &+N\int \mathrm{d}\Omega_{\vec{k}'}\hat{f}_{\mathrm{nucl,coul}}(\vec{k},\vec{k}')\hat{\rho}(\vec{k}',z)\hat{f}^{+}_{\mathrm{nucl,coul}}(\vec{k}',\vec{k}). \end{aligned} \tag{3.13}$$

As stated above, the solution of (3.13) can be presented as follows:

$$\hat{\rho}(\vec{k},z) = \hat{\rho}^{(0)}(\vec{k},z) + \hat{\rho}^{(1)}(\vec{k},z) + ..., \tag{3.14}$$

where $\hat{\rho}^{(0)}(\vec{k},z)$ is the zeroth approximation, which is the solution of kinetic equation (3.13) in the case when only the Coulomb interaction between the deuteron beam and the nuclei of matter is taken into account; $\hat{\rho}^{(1)}(\vec{k},z)$ is the first-order perturbation theory correction, which includes the contribution of nuclear scattering to the evolution of the beam's polarization characteristics.

The equation for $\hat{\rho}^{(0)}(\vec{k},z)$ has the form:

$$\begin{aligned}\frac{\mathrm{d}\hat{\rho}^{(0)}(\vec{k},z)}{\mathrm{d}z} &= -N\sigma^{0}_{\mathrm{coul}}\hat{\rho}^{(0)}(\vec{k},z) - \frac{N}{2}\left(\sigma^{\pm1}_{\mathrm{coul}} - \sigma^{0}_{\mathrm{coul}}\right)\left\{(\vec{S}\vec{n})^2, \hat{\rho}^{(0)}(\vec{k},z)\right\} \\ &+\frac{2\pi\mathrm{i}}{k}N\mathrm{Re}f_1^{\mathrm{coul}}(0)\left[(\vec{S}\vec{n})^2, \hat{\rho}^{(0)}(\vec{k},z)\right] \\ &+N\int \mathrm{d}\Omega_{\vec{k}'}\hat{f}_{\mathrm{coul}}(\vec{k},\vec{k}')\hat{\rho}^{(0)}(\vec{k}',z)\hat{f}^{+}_{\mathrm{coul}}(\vec{k},\vec{k}'), \end{aligned} \tag{3.15}$$

where the spin structure of $\hat{f}_{\mathrm{coul}}(\vec{k},\vec{k}')$ in the general case has the form (2.6).

In the small-angle approximation, the terms in (2.6) that are proportional to B, C_1, and C_2 in the Coulomb amplitude, for $\vec{k}' \neq \vec{k}$ lead to depolarization of the registered beam. The magnitude of this depolarization is determined by the quantity $b_{\mathrm{g}}^2\overline{\theta^2}z$ ($b_{\mathrm{g}} = \frac{g-2}{2}\frac{\gamma^2-1}{\gamma} + \frac{\gamma-1}{\gamma}$, g is the gyromagnetic ratio, and $\overline{\theta^2}$ is the mean square angle of Coulomb scattering per unit length) [3, 18]. For the considered path lengths, which are of the order of the nuclear collision length or smaller, the degree of depolarization for deuterons having energies in the range of hundreds of megaelectron-volts and moving in carbon is $\sim 10^{-3}$. As is seen, deuteron depolarization is insignificant and will further be neglected. As a result, in the case of Coulomb interaction, it is sufficient to consider the spin-independent part of scattering amplitude. Then (3.15) reads:

$$\frac{\mathrm{d}\hat{\rho}^{(0)}(\vec{k},z)}{\mathrm{d}z} = -N\sigma^{0}_{\mathrm{coul}}\hat{\rho}^{(0)}(\vec{k},z) + N\int \mathrm{d}\Omega_{\vec{k}'}\left|a(\vec{k},\vec{k}')\right|^2\hat{\rho}^{(0)}(\vec{k}',z), \qquad (3.16)$$

where $a(\vec{k},\vec{k}')$ denotes the spinless part of the amplitude $\hat{f}_{\mathrm{coul}}(\vec{k},\vec{k}')$.

Expression (3.16) is an integro-differential equation. In the limit of small scattering angles, it can be solved by expanding the function $\hat{\rho}^{(0)}(\vec{k}')$ in terms of a small parameter $\vec{q} = \vec{k}' - \vec{k} \ll \vec{k}$ (transmitted momentum) [19]:

$$\hat{\rho}^{(0)}(\vec{k}',z) \approx \hat{\rho}^{(0)}(\vec{k},z) + \frac{\partial\hat{\rho}^{(0)}}{\partial k_x}q_x + \frac{\partial\hat{\rho}^{(0)}}{\partial k_y}q_y + \frac{1}{2}\frac{\partial^2\hat{\rho}^{(0)}}{\partial k_x^2}q_x^2 + \frac{\partial^2\hat{\rho}^{(0)}}{\partial k_x\partial k_y}q_xq_y + \frac{1}{2}\frac{\partial^2\hat{\rho}^{(0)}}{\partial k_y^2}q_y^2 + ..., \qquad (3.17)$$

where $q_x = q\cos\varphi$, $q_y = q\sin\varphi$, and $q \simeq k\theta$ with θ being the scattering angle (the angle between vectors $\vec{k}$ and $\vec{k}'$).

Substitution of the expansion (3.17) into (3.16) and integration over the azimuthal angle φ gives

$$\frac{\mathrm{d}\hat{\rho}^{(0)}(\vec{k},z)}{\mathrm{d}z} = \frac{\overline{\theta^2}}{4}\Delta\hat{\rho}^{(0)}(\vec{k},z) + \frac{\overline{\theta^4}}{64}\Delta\Delta\hat{\rho}^{(0)}(\vec{k},z) + ..., \qquad (3.18)$$

the operator Δ affects the transversal components of vector $\vec{k}$: $\Delta = \frac{\partial^2}{\partial n_x^2} + \frac{\partial^2}{\partial n_y^2}$, where $\vec{n} = \vec{k}/k$, $\overline{\theta^2} = N\int\theta^2\frac{\mathrm{d}\sigma_{\mathrm{coul}}}{\mathrm{d}\Omega}\mathrm{d}\Omega$, and $\overline{\theta^4} = N\int\theta^4\frac{\mathrm{d}\sigma_{\mathrm{coul}}}{\mathrm{d}\Omega}\mathrm{d}\Omega$, etc.

If we confine ourselves to the first term on the right-hand side of (3.18), then integro-differential equation (3.17) can be reduced to a differential equation. The limiting angle in such approximation is obtained from the condition

$$\theta^2_{\max} < \frac{16\Delta\rho^{(0)}(\vec{k},z)}{\Delta\Delta\rho^{(0)}(\vec{k},z)}. \qquad (3.19)$$

As a result, we have the equation for the spin density matrix $\hat{\rho}^{(0)}(\vec{k},z)$, which coincides in form with the equation for the spin density matrix describing multiple scattering of spinless particles:

$$\frac{\mathrm{d}\hat{\rho}^{(0)}(\vec{k},z)}{\mathrm{d}z} = \frac{\overline{\theta^2}}{4}\Delta\hat{\rho}^{(0)}(\vec{k},z). \qquad (3.20)$$

For the initial condition $\hat{\rho}^{(0)}(\vec{k},z=0) = \hat{\rho}_0\delta(n_x)\delta(n_y)$ and infinite media, the solution of this equation can be presented in the form:

$$\hat{\rho}^{(0)}(\vec{k},z) = \hat{\rho}_0 g(\vec{k},z), \qquad (3.21)$$

where $\hat{\rho}_0$ is the spin part of the density matrix [see (2.5)]

$$\hat{\rho}_0 = \frac{1}{3}I_0\hat{\mathrm{I}} + \frac{1}{2}\vec{P}_0\hat{\vec{S}} + \frac{1}{9}Q_{0ik}\hat{Q}_{ik}, \qquad (3.22)$$

and the function

$$g(\vec{k},z) = \frac{1}{\pi\overline{\theta^2}z}\exp\left[-\frac{(\vec{n}-\vec{n}_0)^2}{\overline{\theta^2}z}\right]. \qquad (3.23)$$

The unit vector $\vec{n}=\vec{k}/k$ is counted off with respect to the direction of $\vec{n}_0$ (further, we shall align the z-axis with vector $\vec{n}_0$). In the small-angle approximation, $(\vec{n}-\vec{n}_0)^2=\theta^2$, where θ is the angle between vector $\vec{k}$ and the z-axis; $\overline{\theta^2}$ is the mean square scattering angle per unit path. The mean square scattering angle $(\overline{\theta^2}z)$ at depth z will further be denoted by $\overline{\theta_z^2}$, and its square root — by θ_z.

We shall also point out that for large values of θ, the solution of (3.16) will generally be proportional to $1/\theta^4$ [20, 21]. Thus, approximate equation (3.20) reflects the fact that the large–angle deflections are neglected in a single scattering event. Indeed, for $\theta \gg \theta_z$, the solution (3.21) decreases exponentially, while the subsequent term that corresponds to the solution of the original equation (3.16) diminishes according to the power law θ^{-4}. That is why the angular distribution (3.23) does not describe the characteristics of particles scattered at large angles, $\theta \gg \theta_z$ [20, 21].

The expression for the density matrix in the zeroth approximation (3.21) enables one to obtain the correction $\hat{\rho}^{(1)}(\vec{k},z)$ in the first–order perturbation theory:

$$
\begin{aligned}
\hat{\rho}^{(1)}(\vec{k},z) = N\int_0^z\int G(\vec{k}-\vec{k}'';z-z')\Big(&-\sigma_{\rm NC}^{0}\hat{\rho}^{(0)}(\vec{k}'',z') \\
&-\frac{1}{2}\left(\sigma_{\rm NC}^{\pm1}-\sigma_{\rm NC}^{0}\right)\left\{(\vec{S}\vec{n}'')^2,\hat{\rho}^{(0)}(\vec{k}'',z')\right\} \\
&+\frac{2\pi \mathrm{i}}{k}N\mathrm{Re}f_1^{\rm nucl}(0)\left[(\vec{S}\vec{n}'')^2,\hat{\rho}^{(0)}(\vec{k}'',z')\right]\Big)\,\mathrm{d}\Omega_{\vec{k}''}\mathrm{d}z' \\
&+N\int_0^z\int G(\vec{k}-\vec{k}'';z-z')\left(\hat{f}_{\rm coul}(\vec{k}'',\vec{k}')\hat{\rho}^{(0)}(\vec{k}',z')\hat{f}^{+}_{\rm nucl,coul}(\vec{k}',\vec{k}'')\right. \\
&\left.+\hat{f}_{\rm nucl,coul}(\vec{k}'',\vec{k}')\hat{\rho}^{(0)}(\vec{k}',z')\hat{f}^{+}_{\rm coul}(\vec{k}',\vec{k}'')\right)\mathrm{d}\Omega_{\vec{k}'}\mathrm{d}\Omega_{\vec{k}''}\mathrm{d}z' \\
&+N\int_0^z\int G(\vec{k}-\vec{k}'';z-z') \\
&\times\hat{f}_{\rm nucl,coul}(\vec{k}'',\vec{k}')\hat{\rho}^{(0)}(\vec{k}',z')\hat{f}^{+}_{\rm nucl,coul}(\vec{k}',\vec{k}'')\mathrm{d}\Omega_{\vec{k}'}\mathrm{d}\Omega_{\vec{k}''}\mathrm{d}z', \qquad (3.24)
\end{aligned}
$$

where $G(\vec{k}-\vec{k}'';z-z')$ is the Green function of (3.20): $G(\vec{k}-\vec{k}'';z-z') = \dfrac{1}{\pi\overline{\theta^2}|z-z'|}\exp\left[-\dfrac{(\vec{n}-\vec{n}'')^2}{\overline{\theta^2}|z-z'|}\right]$. In the general case, the amplitudes for Coulomb and nuclear scattering are given by formula (2.6).

In the case of high energies, the characteristic angles for elastic scattering of deuterons by nuclei are $\theta_{\rm n}\sim 1/kR_{\rm d}\ll 1$ ($R_{\rm d}$ is the deuteron radius). As a result, in analysing the polarization of high-energy deuteron beams in transmitted geometry (figure 1) using an axially symmetrical detector, the amplitude of scattering due to nuclear interaction, which appears in the expression for the density matrix (3.24), can be written in the form:

$$
\hat{f}_{\rm nucl,coul}(\vec{k},\vec{k}') = d(\theta)+d_1(\theta)(\vec{S}\vec{n}_0)^2, \qquad (3.25)
$$

where $\vec{n}_0$ is the unit vector directed along the z-axis, which coincides with the direction of motion of the initial beam; $d(\theta)$ and $d_1(\theta)$ are the spin–independent and the spin–dependent parts of the nuclear amplitude (which allows for the distortion caused to the waves incident onto the nuclei by the Coulomb interaction), respectively; θ is the angle between vectors $\vec{k}$ and $\vec{k}'$.

The spin structure of the Coulomb scattering amplitude $\hat{f}_{\rm coul}$ also has the form (3.25). As stated above, however, for this amplitude, the term proportional to the deuteron spin is small and will be dropped further.

3.2.2. Tensor Polarization of Deuterons Passing Through Matter. By substituting the explicit form of the density matrix (2.5) and the scattering amplitude (3.25) into the solution (3.24), one can obtain the characteristics of the deuteron beam, which are of interest to us.

As a result, the number of particles that are registered by the detector with angular width $\Delta\Omega$ can be written in the form:

$$\mathcal{I}(z,\vartheta_{\rm det}) = \left[1-\exp\left(-\frac{\vartheta^2_{\rm det}}{\overline{\theta^2_z}}\right)\right] I_0 + \xi_1(z,\vartheta_{\rm det}) I_0 + \Big(\xi_2(z,\vartheta_{\rm det}) + \xi_3(z,\vartheta_{\rm det})\Big)\left[\frac{2}{3}I_0 + \frac{1}{3}(\mathbf{Q}_0\vec{n}_0)\vec{n}_0\right]. \qquad (3.26)$$

According to (3.26), the intensity of the beam that reaches the detector depends on its possible initial tensor polarization $\mathbf{Q}_0$.

The parameters ξ_1, ξ_2, and ξ_3 in the small-angle approximation are defined as follows:

$$\begin{aligned}
\xi_1(z,\vartheta_{\rm det}) &= -N\sigma^0_{\rm NC}\left[1-\exp\left(-\frac{\vartheta^2_{\rm det}}{\overline{\theta^2_z}}\right)\right] z \\
&+2\pi N z\int_0^\infty P(\chi;\vartheta_{\rm det},\overline{\theta^2_z})\Big(2{\rm Re}[a(\chi)d^*(\chi)] + |d(\chi)|^2\Big)\chi {\rm d}\chi, \\
\xi_2(z,\vartheta_{\rm det}) &= -N(\sigma^{\pm1}_{\rm NC}-\sigma^0_{\rm NC})\left[1-\exp\left(-\frac{\vartheta^2_{\rm det}}{\overline{\theta^2_z}}\right)\right] z \\
&+4\pi N z\int_0^\infty P(\chi;\vartheta_{\rm det},\overline{\theta^2_z})\Big({\rm Re}[a(\chi)d_1^*(\chi)] + {\rm Re}[d(\chi)d_1^*(\chi)]\Big)\chi {\rm d}\chi, \\
\xi_3(z,\vartheta_{\rm det}) &= 2\pi N z\int_0^\infty P(\chi;\vartheta_{\rm det},\overline{\theta^2_z})|d_1(\chi)|^2\chi {\rm d}\chi,
\end{aligned} \qquad (3.27)$$

where the function $P(\chi;\vartheta_{\rm det},\overline{\theta^2_z})$ is defined as

$$P(\chi;\vartheta_{\rm det},\overline{\theta^2_z}) = \frac{2}{\overline{\theta^2_z}}\int_0^{\vartheta_{\rm det}} \exp\left(-\frac{\theta^2+\chi^2}{\overline{\theta^2_z}}\right) {\rm I}_0\left(\frac{2\theta\chi}{\overline{\theta^2_{\rm z}}}\right)\theta {\rm d}\theta, \qquad (3.28)$$

where $\rm I_0(y)$ is the modified Bessel function of the zeroth order.

Let us discuss now what polarization this beam has. Write the expressions for the number of particles that are registered by the detector and have the initial spin projections $M=0$ and $M=\pm1$, respectively:

$$\begin{aligned}
\mathcal{I}^{(0)}(z,\vartheta_{\rm det}) &= \left[1-\exp\left(-\frac{\vartheta^2_{\rm det}}{\overline{\theta^2_z}}\right)\right] I_0^{(0)} + \xi_1 I_0^{(0)}, \\
\mathcal{I}^{(\pm1)}(z,\vartheta_{\rm det}) &= \left[1-\exp\left(-\frac{\vartheta^2_{\rm det}}{\overline{\theta^2_z}}\right)\right] I_0^{(\pm1)} + (\xi_1+\xi_2+\xi_3) I_0^{(\pm1)}
\end{aligned} \qquad (3.29)$$

For the sake of simplicity, let us consider the case when an unpolarized beam is incident onto the target. Using the definition (3.3), find tensor polarization of the beam registered by the detector:

$$p_{zz} \simeq \frac{2}{3\left[1-\exp\left(-\frac{\vartheta_{\text{det}}^2}{\overline{\theta_z^2}}\right)\right]}(\xi_2+\xi_3). \tag{3.30}$$

Here we have retained the terms proportional to the first power of the small quantities ξ_2 and ξ_3.

Using the explicit form of the parameters ξ_2, ξ_3 in (3.27), the expression for tensor polarization of the beam can be written in the form:

$$\begin{aligned} p_{zz}(z,\vartheta_{\text{det}}) = &-\frac{2}{3}N(\sigma_{\text{r}}^{\pm 1}-\sigma_{\text{r}}^{0})z - \frac{2}{3}N\Bigg[(\sigma_{\text{el}}^{\pm 1}-\sigma_{\text{el}}^{0}) \\ &-\frac{4\pi}{\left[1-\exp\left(-\frac{\vartheta_{\text{det}}^2}{\overline{\theta_z^2}}\right)\right]}\int_0^\infty P(\chi,\vartheta_{\text{det}},\overline{\theta_z^2})\,\text{Re}[a(\chi)d_1^*(\chi)]\chi\text{d}\chi \\ &-\frac{2\pi}{\left[1-\exp\left(-\frac{\vartheta_{\text{det}}^2}{\overline{\theta_z^2}}\right)\right]}\int_0^\infty P(\chi,\vartheta_{\text{det}},\overline{\theta_z^2}) \\ &\times\left(2\text{Re}[d(\chi)d_1^*(\chi)]+|d_1(\chi)|^2\right)\chi d\chi\Bigg]z. \end{aligned} \tag{3.31}$$

Equation (3.31) simplifies appreciably for the two limiting values of the detector angle: $\vartheta_{\text{det}} \ll \theta_z$ and $\vartheta_{\text{det}} \gg \theta_z$. As has been stated above, the solution (3.21) does not describe the properties of the beam that is scattered at angles $\theta > \theta_z$. But the integral characteristics of deuterons that are obtained from this solution can be used for $\vartheta_{\text{det}} > \theta_z$. The influence of particles single–scattered due to the Coulomb interaction can be neglected because the major contribution to (3.28) comes from the small values of θ.

When ϑ_{det} is much less than θ_z, the expression for p_{zz} can be written in the form:

$$\begin{aligned} p_{zz}(z,\vartheta_{\text{det}}\ll\theta_z) = &-\frac{2}{3}N(\sigma_{\text{tot}}^{\pm 1}-\sigma_{\text{tot}}^{0})z \\ &+\frac{8}{3}\pi N z\int_0^\infty \exp\left(-\frac{\chi^2}{\overline{\theta_z^2}}\right)\,\text{Re}[a(\chi)d_1^*(\chi)]\chi\text{d}\chi \\ &+\frac{4}{3}\pi N z\int_0^\infty \exp\left(-\frac{\chi^2}{\overline{\theta_z^2}}\right)\left(2\text{Re}[d(\chi)d_1^*(\chi)]+|d_1(\chi)|^2\right)\chi\text{d}\chi. \end{aligned} \tag{3.32}$$

According to (3.32), as a result of multiple scattering, tensor polarization becomes independent of ϑ_{det} for small angles of the detector. Equation (3.32) contains a scattering-related contribution (the second and third terms), which appears in (3.32) as a result of deuteron rescattering from the direction of $\vec{k} \neq \vec{k}_0$ into the direction of vector $\vec{k}_0$.

In the other limiting case, when ϑ_{det} is much greater than θ_z, the terms related to scattering in (3.31) are equal to the scattering cross sections, and so the expression between the square brackets vanishes. Tensor polarization is determined by the difference between the cross sections $\sigma_{\rm r}^{\pm 1}$ and $\sigma_{\rm r}^{0}$ alone, as it occurs in the case of a thin target.

Let us represent the expression (3.31) for tensor polarization in the form:

$$p_{zz}(z,\vartheta_{\rm det}) = -\frac{2}{3}N(\sigma_{\rm r}^{\pm 1}-\sigma_{\rm r}^{0})z - \frac{2}{3}N(\sigma_{\rm el}^{\pm 1}-\sigma_{\rm el}^{0})z\,{\rm H}(z,\vartheta_{\rm det}), \qquad (3.33)$$

where the function H is defined as

$$\mathrm{H}(z,\vartheta_{\rm det}) = 1 - \frac{1}{(\sigma_{\rm el}^{\pm 1}-\sigma_{\rm el}^{0})}\frac{4\pi}{\left[1-\exp\left(-\frac{\vartheta_{\rm det}^2}{\overline{\theta_z^2}}\right)\right]}\left(\int\limits_0^\infty P(\chi,\vartheta_{\rm det},\overline{\theta_z^2})\,{\rm Re}[a(\chi)d_1^*(\chi)]\chi{\rm d}\chi\right.$$
$$\left. + \int\limits_0^\infty P(\chi,\vartheta_{\rm det},\overline{\theta_z^2})\left({\rm Re}[d(\chi)d_1^*(\chi)] + |d_1(\chi)|^2/2\right)\chi{\rm d}\chi\right). \qquad (3.34)$$

Note that when passing to the limit of a thin target, i.e., $z \to 0$, one should take into account that in this case, the function $P(\chi;\vartheta_{\rm det},\overline{\theta_z^2})$ turns to $P(\chi;\vartheta_{\rm det},0) = \int\limits_0^{\vartheta_{\rm det}}\delta(\vartheta-\chi){\rm d}\vartheta$, where δ is the delta function.

According to (3.33), the dependence of tensor polarization of a beam on the target thickness and the angle $\vartheta_{\rm det}$ is determined by the function $\mathrm{H}(\mathrm{z},\vartheta_{\rm det})$.

Let us recast the equality for the mean square angle of deuteron scattering at depth z in the form [19]:

$$\overline{\theta^2}z = 16\pi Z^2\left(\frac{e^2}{pv}\right)^2\ln(137\,Z^{-1/3})\frac{N_A}{A}\Delta C, \qquad (3.35)$$

where ΔC is the target thickness in g/cm^2, N_A is the Avogadro's number.

The explicit form of the d dependence on θ in the range of small scattering angles for the case of structural particles was derived, for example, in [22]. To estimate the main parameters, let us consider the simplest form of the d and d_1 dependence on θ, namely,

$$d(\theta) = d(0)\exp\left(-k^2R_{\rm d}^2\theta^2/4\right), \qquad d_1(\theta) = d_1(0)\exp\left(-k^2R_{\rm d}^2\theta^2/4\right), \qquad (3.36)$$

where $R_{\rm d}$ is the deuteron radius.

We also used the fact that according to the eikonal-approximation calculations [23], in the range of energies of incident deuterons from 400 to 800 MeV, for the amplitude d we have ${\rm Im}d = 0.75\cdot 10^{-11}$cm, ${\rm Re}d = -0.6\cdot 10^{-12}$cm, and the imaginary part of the amplitude d_1 relates to its real part as ${\rm Im}d_1/{\rm Re}d_1 \sim -10$, with the spinless part of the nuclear amplitude $f_{\rm nucl}$ being much larger than its spin part.

For the considered energies of deuterons scattered by a screened Coulomb potential, the first Born approximation to the amplitude is satisfactory to use as a real part of the Coulomb amplitude $a(\theta)$, and the second term of the Born series can be used as its imaginary part accordingly.

Figure 2 and figure 3 present the results of calculations of the function H for different values of $\vartheta_{\rm det}$ and different deuteron energies when a beam is scattered by a carbon filter.

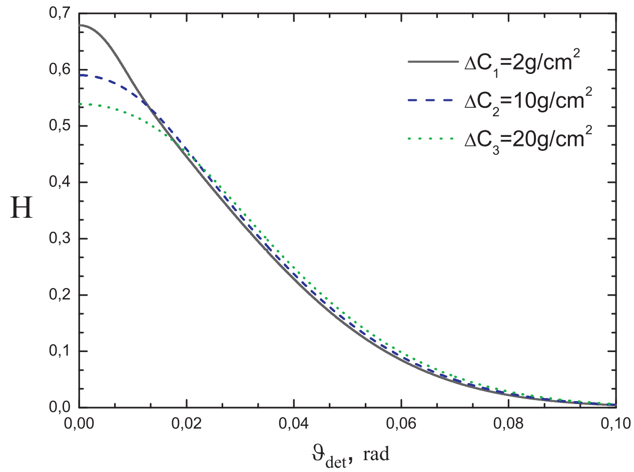

Figure 2. Function H versus the detector angle for deuteron energy of 500 MeV. Solid curve corresponds to the carbon target thickness $\Delta C_1 = 2$ g/cm^2, dashed curve - to $\Delta C_2 = 10$ g/cm^2, dotted curve — to $\Delta C_3 = 20$ g/cm^2.

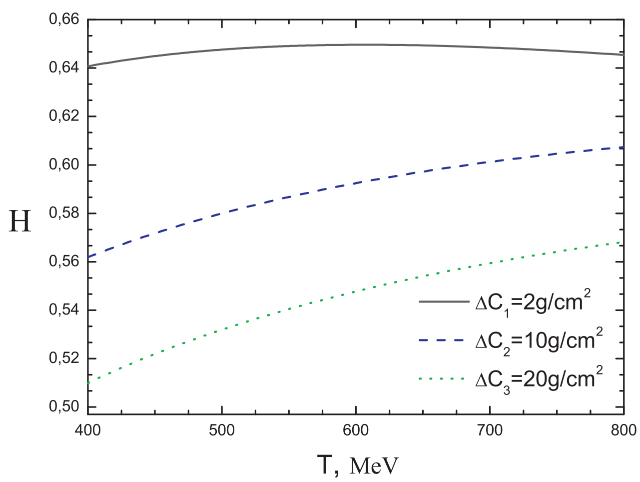

Figure 3. Function H versus the deuteron energy for the detector angle $\vartheta_{\rm det} = 5\cdot 10^{-3}$ rad. Solid curve corresponds to the carbon target thickness $\Delta C_1 = 2$ g/cm^2, dashed curve — to $\Delta C_2 = 10$ g/cm^2, dotted curve — to $\Delta C_3 = 20$ g/cm^2.

The results of calculations given in figure 2 demonstrate that owing to multiple Coulomb scattering, the value of H is less than unity, H < 1, even in a narrow–angle geometry ($\vartheta_{\rm det} \ll \theta_z$). Let us recall that this occurs because of the rescattering in the direction of vector $\vec{k}_0$ of particles that have been scattered in different directions. For $\Delta C = 2$, 10, and 20 g/cm^2, the values of $\theta_z = \sqrt{\overline{\theta^2} z}$ are $\theta_z(\Delta C_1) = 4.5$ mrad, $\theta_z(\Delta C_2) = 10$ mrad, and $\theta_z(\Delta C_3) = 14$ mrad, respectively. As a result, in this limiting case, the values of H for deuterons registered by the detector with angular width $\vartheta_{\rm det} \ll \theta_z$ for target thicknesses $\Delta C = 2$, 10, and 20 g/cm^2 are H$(\Delta C_1) = 0.68$, H$(\Delta C_2) = 0.59$, and H$(\Delta C_3) = 0.54$, respectively. As is seen, these values differ from unity by almost a factor of two, and this difference is the largest for the thickest target.

With growing angle $\vartheta_{\rm det}$, the particles scattered at the angles $\theta \leq \vartheta_{\rm det}$ gain in significance, which leads to a decrease of H. In a wide–angle geometry, which for the fast–particle scattering is defined from the condition $\vartheta_{\rm det} \gg \theta_{\rm eff}$ ($\theta_{\rm eff} = \theta_z$ when $\theta_z > \theta_{\rm n}$, and $\theta_{\rm eff} = \theta_{\rm n}$ when $\theta_z < \theta_{\rm n}$), we have $\mathrm{H} \to 0$. As a result, tensor polarization in this limiting case is determined by the difference between the inelastic parts of the total scattering cross sections for the states with $M = \pm 1$ and $M = 0$.

Now, let us consider the behaviour of the function H for different values of energy at a fixed value of $\vartheta_{\rm det}$ (choose $\vartheta_{\rm det}$ to be $\vartheta_{\rm det} = 5 \cdot 10^{-3}$ rad). According to figure 3, when the deuteron energy grows, the value of H first increases and then starts to diminish.

It should be emphasized that with due account of multiple Coulomb scattering, tensor polarization is a nonlinear function of the target thickness z (3.31). For large values of $\vartheta_{\rm det}$, however, p_{zz} becomes a linear function of z (3.10). With increasing $\overline{\theta_z^2}$, expression (3.32) approaches the expression for tensor polarization (3.10). This means that for large values of z or for low energies of the incident deuterons, the dependence of the tensor polarization on the detector's angle becomes insignificant (it is necessary that the parameter θ_z should be much greater than the characteristic angle $\theta_{\rm n}$ of nuclear scattering). In this case, tensor polarization can be considered linear in z for any values of $\vartheta_{\rm det}$.

4. Conclusion

It has been shown that the magnitude of tensor polarization of the deuteron beam, which arises from the spin dichroism effect, depends appreciably on the angular width $\vartheta_{\rm det}$ of the detector that registers the deuterons transmitted through the target. Even when the angular width of the detector is much less than the mean square angle of multiple Coulomb scattering, the beam's tensor polarization depends noticeably on rescattering. In the case when the angle $\vartheta_{\rm det}$ is much larger than the mean square angle of multiple Coulomb scattering (as well as than the characteristic angle of elastic nuclear scattering), tensor polarization is determined by the difference between the total reaction cross sections for deuteron–nucleus interaction, $\sigma_{\rm r}^{\pm 1} - \sigma_{\rm r}^{0}$. Elastic scattering processes here make no contribution to tensor polarization.

Acknowledgements

The authors are very grateful to A. Rouba for calculating the forward nuclear amplitudes.

References

[1] Baryshevsky V G 1992 *Phys. Lett.* A **171** 431–34
[2] Baryshevsky V G 1993 *J. Phys.* G **19** 273–82
[3] Baryshevsky V G 2012 *High-Energy Nuclear Optics of Polarized Particles* (Singapore: World Scientific)

[4] Baryshevsky V *et al* 2005 First observation of spin dichroism with deuterons up to 20 MeV in a carbon target *Preprint* hep-ex/0501045

[5] Baryshevsky V *et al* 2007 *Proc. of the 17th int. spin physics symp. (Kyoto)* (New York: Melville) p 777

[6] Seyfarth H *et al* 2011 *Phys. Rev. Lett.* **104** 222501

[7] Azhgirey L S *et al* 2008 *Proc. of the XIIth Work. on high energy spin physics (Dubna)* (Dubna: JINR) p 205

[8] Azhgirey L S *et al* 2010 *Phys. Part. Nuclei Lett.* **7** 27–32

[9] Rathmann F and Nikolaev N 2011 *Proc. of the 8th Int. Conf. on Nuclear Physics at Storage Rings (Frascati)* (Italy PoS(STORI11)) p 029

[10] Baryshevsky V G 2008 *J. Phys.* G **35** 035102

[11] Baryshevsky V G and Rouba A 2010 *Phys. Lett.* B **683** 229–34

[12] Baryshevsky V G and Shyrvel A R *Preprint* hep-ph/1101.2408v1

[13] Baryshevsky V G and Shekhtman A G 1996 *Phys. Rev.* C **53** 267–76.

[14] Goldberger M L and Watson K M 1964 *Collision Theory* (New York: John Wiley and Sons)

[15] Ohlsen G G 1972 *Rep. Prog. Phys.* **35** 717–801.

[16] Darden S E 1967 *Am. J. Phys.* **35** 727–38.

[17] Davydov A S 1965 *Quantum Mechanics* (Oxford: Pergamon Press)

[18] Lyuboshitz V L 1980 *Sov. J. Nucl. Phys.* **32** 362–65.

[19] Ter-Mikaelian M L 1972 *High Energy Electromagnetic Processes in Condensed Media* (New York: Wiley Inter-science)

[20] Bethe H A 1953 *Phys. Rev.* **89** 1256–70

[21] Moliere G 1948 *Z. Naturforsch* A **3** 78–96

[22] Czyz W and Maximon L C 1969 *Ann. Phys.* (N.Y.) **52** 59–121.

[23] Rouba A Private Communication