Photon Emission from Charged Particles Moving in Undulators Placed in a Photonic Crystal

V.G. Baryshevsky, A.A. Gurinovich

Abstract

The law of photon radiation from electron beams passing through a photonic crystal in the presence of an undulator is considered. The spectral-angular distribution and the time evolution of radiation is found. It is shown that the characteristics of the radiation intensity in time demonstrate oscillations due to the interference between undulator and parametric (quasi-Cherenkov) radiations.

1 Introduction

It was shown in [1, 2] that when a relativistic particle moves in a crystal in which an electromagnetic wave propagates, the radiation spectrum of the particle, oscillating in the wave, changes significantly due to the virtual photon diffraction, occurring in the process of emission in the crystal.

Moreover, even in the X-ray range, in which the refractive index in a crystal is less than unity $n(\omega) < 1$, under diffraction conditions $n(\omega)$ can appear to be greater than unity, and so the induced Vavilov-Cherenkov effect becomes possible.

A relativistic oscillator can also be formed when a charged particle is channeled through a crystal, causing channeling radiation to appear [3–5] , or when a particle moves in a crystal undulator [6–10].

At present, radiation from a relativistic oscillator in a crystal in presence of photon diffraction is called the diffraction radiation of a relativistic oscillator (DRO) or, in the case of channeled particles, the diffracted channeling radiation (DCR). A thorough theoretical consideration of DRO and DCR was given in [11–16].

The spectral-angular distributions of radiation from channeled particles were determined for Laue and Bragg geometries [12, 13] (for more details see also [17]) and for the case of particle motion in a crystal undulator in Laue geometry [18]. In [19, 20] we considered the time evolution of the radiation process in crystals (natural or photonic). It was shown that strong frequency dispersion of waves in crystals can lead to a decrease in group velocity of radiation in crystals (the wave packet can slow down), and the situation can arise when the radiation from the crystal continues even after the particle has escaped from it. In [19, 20] a detailed analysis was carried for the case of parametric (quasi-Cherenkov) radiation.

The present paper studies the law of photon radiation from photonic crystals in the presence of an undulator. It also discusses the law of time evolution of DCR and radiation generated through particle motion inside a crystal undulator. The spectral-angular distribution of radiation when particles travel in the undulator formed in a photonic crystal is found. The formulas are derived that describe the laws of time evolution of radiation from a traveling particle. It is shown that the characteristics of the radiation intensity in time demonstrate oscillations due to the interference between undulator and parametric (quasi-Cherenkov) radiations.

2 Time dependence of the intensity of radiation from a particle moving in a crystal (natural or photonic)

According to [21], the radiation intensity $dI(t)$ in the element of solid angle $d\Omega$ is defined as the amount of energy passing in unit time through the element $dS = r^2 d\Omega$ of the sphere of radius r that is much larger than the size of the radiation source and has the origin of coordinates inside the radiation source.

The radiation intensity $dI(t)$ can be found if we know the expression for the electric (magnetic) component of the electromagnetic wave $\vec{E}(\vec{r},t)$ ($\vec{H}(\vec{r},t)$), produced by the radiation source [21]:

$$dI(t) = \frac{c}{4\pi}|\vec{E}(\vec{r},t)|^2 r^2 d\Omega, \tag{1}$$

where r is the distance from the observation point.

In the case we are considering here, the field $\vec{E}(\vec{r},t)$ appears as a result of interaction of a relativistic particle and a crystal.

The field $\vec{E}(\vec{r},t)$ can be expanded in a Fourier series

$$\vec{E}(\vec{r},t) = \frac{1}{2\pi}\int \vec{E}(\vec{r},\omega) e^{-i\omega t} d\omega. \tag{2}$$

According to the results obtained in [17, 20, 22], at large distance from the crystal, the Fourier component $\vec{E}(\vec{r},\omega)$ can be written as

$$\vec{E}(\vec{r},t) = \frac{e^{ikr}}{r}\frac{i\omega}{c^2}\sum_s e_i^s \int \vec{E}_{\vec{k}}^{(-)s^*}(\vec{r}'\omega)\vec{j}(\vec{r}',\omega) d^3r', \tag{3}$$

where $i = 1, 2, 3$ (x, y, z, respectively); e_i^s is the i-the component of the polarization vector, $s = 1, 2$;

$$\vec{j}(\vec{r},\omega) = \int \vec{j}(\vec{r},t) e^{i\omega t} dt, \tag{4}$$

$\vec{j}(\vec{r},t) = Q\vec{v}(t)\delta(\vec{r} - \vec{r}(t))$ is the current density for a particle of charge Q and $\vec{r}(t)$ is the particle coordinate at time t; $\vec{E}_{\vec{k}}^{(-)s}$ is the solution of Maxwell equations that describes scattering by a crystal of the plane wave having the wave vector $\vec{k} = k\frac{\vec{r}}{r}$ and the wave number $k = \frac{\omega}{c}$. At large distance from the crystal, the solution $\vec{E}_{\vec{k}}^{(-)s}$ has a form of a superposition of a plane and a converging spherical wave [17, 22].

The explicit expression for $\vec{E}^{(-)s}$ that describes the diffraction of the electromagnetic wave in a crystal for the Laue and Bragg cases is given in [17, 22, 23].

Let us discuss the following expression for the the amplitude $A(\omega)$ of the radiated wave in more depth

$$A_{\vec{k}}^{s}(\omega) = \frac{i\omega}{c^2}\int \vec{E}_{\vec{k}}^{(-)s^*}(\vec{r}',\omega)\vec{j}(\vec{r}',\omega)d^3r'. \tag{5}$$

Using (4) and (5), we can write

$$\begin{aligned} A_{\vec{k}}^{s}(\omega) &= \frac{i\omega}{c^2}\int \vec{E}_{\vec{k}}^{(-)s^*}(\vec{r}',\omega)Q\vec{v}(t)\delta(\vec{r}'-\vec{r}(t))e^{i\omega t}dtd^3r' \\ &= \frac{i\omega Q}{c^2}\int \vec{E}_{\vec{k}}^{(-)s^*}(\vec{r}(t),\omega)\vec{v}(t)e^{i\omega t}dt \end{aligned} \tag{6}$$

Let us recall here that $\vec{E}_{\vec{k}}^{(-)s^*} = \vec{E}_{-\vec{k}}^{(+)s}$, where the field $\vec{E}_{-\vec{k}}^{(+)s}$ is the solution of Maxwell equations that describes scattering by a photonic crystal of a plane wave, having the wave vector $(-\vec{k})$, and whose asymptotic form at infinity is a superposition of a plane and a diverging spherical wave. In view of (6), the radiation amplitude is determined by the field $\vec{E}_{\vec{k}}^{(-)s}$ taken at the time moment t and integrated over the time of particle motion.

As it follows from (2) and (3), the expression for the electromagnetic wave emitted by a charged particle traveling through a crystal (natural or photonic) can be written in the form [20]:

$$\vec{E}(\vec{r},t) = \frac{1}{2\pi r}\sum_s \vec{e}^{s}\int A_{\vec{k}}^{s}(\omega)e^{-i\omega(t-r/c)}d\omega, \tag{7}$$

i.e.,

$$\vec{E}(\vec{r},t) = \frac{1}{r}\sum_s \vec{e}^{\ s} A_{\vec{k}}^{s}(t-r/c), \tag{8}$$

where

$$A_{\vec{k}}^{s}(t-r/c) = \frac{1}{2\pi}\int A_{\vec{k}}^{s}(\omega)e^{-i\omega(t-r/c)}d\omega, \quad t > \frac{r}{c}. \tag{9}$$

Hence, the expression for the intensity $\frac{dI(t)}{d\Omega}$ of radiation in the element of solid angle can be written in the form

$$\frac{dI(t)}{d\Omega} = \frac{c}{4\pi}|\sum_s \vec{e}^{\ s} A_{\vec{k}}^{s}(t-r/c)|^2. \tag{10}$$

The intensity of radiation for photons having the polarization $\vec{e}^{\ s}$ is defined as

$$\frac{dI_s(t)}{d\Omega} = \frac{c}{4\pi}|A_{\vec{k}}^{s}(t-r/c)|^2. \tag{11}$$

Equations (10) and (11) describe the intensity of the pulse of radiation generated by a relativistic particle moving along an arbitrary trajectory $\vec{r}(t)$ in a photonic crystal as a function of time.

It follows from (9) and (10) that time dependence of the intensity $\frac{dI(t)}{d\Omega}$ of radiation produced by a particle passing through a natural or photonic crystal is determined by the radiation amplitude $A_{\vec{k}}^{s}(\omega)$ dependence on frequency. These amplitudes were obtained in [4,6,11,18,22,23] for both X-ray parametric (quasi-Cherenkov) radiation and diffraction X-ray radiation of channeled particles in crystals.

Let us consider expression (6) for the amplitude $A^s_{\vec{k}}(\omega)$. To find this amplitude, we need to know the expression for the field $\vec{E}^{(-)s*}_{\vec{k}}(\vec{r})$ at point $\vec{r}(t)$. The explicit expressions for $\vec{E}^{(-)s*}_{\vec{k}}$ are given in [17, 22, 23]. The general expression for $\vec{E}^{(-)s*}_{\vec{k}}$ in the crystal can be written in the form:

$$\vec{E}^{(-)s}_{\vec{k}}(\vec{r}) = \sum_{\mu=1}^{2} \left[\vec{e}^s \Phi^s_\mu e^{i\vec{k}_{\mu s}\vec{r}} + \vec{e}^s_\tau \Phi^s_{\tau\mu} e^{i\vec{k}_{\mu s\tau}\vec{r}} \right]. \tag{12}$$

The particle coordinate $\vec{r}(t)$ at time moment t can be written as

$$\vec{r}(t) = \vec{r}_0 + \vec{u}t + \delta\vec{r}(t). \tag{13}$$

Here $\vec{r}_0$ is the coordinate of the electron at time $t = 0$, $\vec{u}$ is the velocity of the electron entering the interaction area, and $\delta\vec{r}(t)$ is the change in the particle trajectory caused by the forces acting on the particle in the interaction region.

3 Diffraction radiation from an oscillator when particles move in a magnetostatic undulator inside a photonic crystal

Let us assume for concreteness that a particle is moving in the undulator formed by a periodic magnetostatic field or in the field of an electromagnetic wave propagating along the direction of particle motion that is determined by the initial velocity $\vec{u}$ of the particle. Let us choose the direction of the velocity $\vec{u}$ as the z-axis. The particle oscillation frequency Ω in the laboratory frame can be written as

$$\Omega = \vec{\kappa}\vec{u} - \Omega_0 = \kappa_z u - \Omega_0, \tag{14}$$

where $\kappa_z = \pm\kappa$ is the wave number of the electromagnetic wave, the "+" and "-" signs correspond to the cases when $\vec{u}$ and $\vec{\kappa}$ are parallel and antiparallel, respectively, Ω_0 is the wave frequency, and $\kappa = \frac{\Omega_0}{c} n(\Omega_0)$, where $n(\Omega_0)$ is the refractive index of the medium at the frequency Ω_0. For a static undulator, $\Omega = 0$, $\kappa_z = \kappa = 2\pi/l_0$, where l_0 is the undulator period.

Thus, when a charged particle moves through the region occupied by a variable external field, a moving oscillator having the frequency Ω is formed. It is natural that such an oscillator emits electromagnetic waves, whose frequency ω is determined by the Doppler effect.

$$\omega = \frac{\Omega}{1 - \beta n(\omega)\cos\theta}, \tag{15}$$

where $n(\omega)$ is the refractive index of the medium and θ is the angle between the particle velocity $\vec{u}$ and the direction of motion of the emitted photons.

We shall further assume that the external field (the undulator or the electromagnetic wave) leading to particle oscillations is concentrated in the region occupied by a photonic crystal. For concreteness, we shall further study the features of DRO generated when a particle moves in a magnetostatic undulator placed inside or outside a photonic crystal.

Let us suppose that a particle moves inside a magnetostatic linearly polarized undulator (see Fig.1).

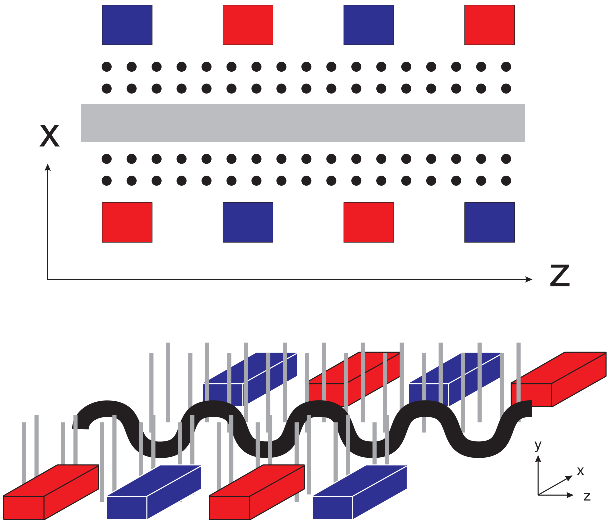

Figure 1: Undulator and photonic crystal

According to [24], the periodic magnetostatic field of the undulator in this case can approximately be written as

$$\vec{B}_\perp(z) = \sqrt{2} B_\perp \vec{n}_y \sin \kappa z, \tag{16}$$

where $\vec{n}_y$ is the unit vector along the y-axis, $B_\perp$ is the root-mean-square amplitude of the field $\kappa = \frac{2\pi}{l_0}$, and l_0 is the undulator period.

As a consequence, the particle undergoes oscillations in the (x, z) plane along both the x (transverse with respect to the z-axis) and the z-axes.

Assuming that the velocity change δv, induced by the field, is small in comparison with the initial velocity $\vec{u}$ of the particle ($\delta v \ll u$), we have for the particle trajectory [24]:

$$x = a \sin \Omega t, \quad z = ut - b \sin 2\Omega t, \tag{17}$$

where $a = \sqrt{2}\frac{u\Omega_\perp}{\gamma\Omega^2}$, $b = \frac{u\Omega_\perp^2}{4\gamma^2\Omega^3}$, and $\Omega_\perp = \frac{eB_\perp}{mc}$. According to (17), the particle velocity can be expressed as:

$$v_x = a\Omega \cos \Omega t, \quad v_z = u - 2b\Omega \cos 2\Omega t. \tag{18}$$

The field $\vec{E}_k^{(-)s*}$ in the region occupied by the crystal (see (6) and (12)) can be presented as

$$\vec{E}_{\vec{k}}^{(-)s*}(\vec{r}) = \sum_{\mu=1}^{2} \left[\vec{e}^s \Phi_\mu^s \exp(-i\vec{k}_{\mu s}\vec{r}) + \vec{e}_\tau^s \Phi_{\tau\mu}^s \exp(-i\vec{k}_{\mu s\tau}\vec{r}) \right]. \tag{19}$$

As a result, for the radiation amplitude we have

$$A_{\vec{k}}^s(\omega) = \frac{i\omega Q}{c^2} \tag{20}$$
$$\times \int_0^T \left\{ \sum_{\mu=1}^{2} \vec{e}^s \Phi_\mu^s e^{-i\vec{k}_{\mu s}\vec{r}(t)} + \vec{e}_\tau^s \Phi_{\tau\mu}^s e^{-i\vec{k}_{\mu s\tau}\vec{r}(t)} \right\} (\vec{u} - \vec{n}_z 2b\Omega \cos 2\Omega t + \vec{n}_x a\Omega \cos \Omega t)\, e^{i\omega t} dt,$$

where $\vec{r}(t) = \vec{u}t + \vec{n}_x a \sin \Omega t - \vec{n}_z b \sin 2\Omega t$, and $\vec{n}_x$ and $\vec{n}_z$ are the unit vectors along the x and the z axes, respectively.

$$
\begin{aligned}
A^s_{\vec{k}}(\omega) &= \frac{i\omega Q}{c^2}\int_0^T \left\{\sum_{\mu=1}^{2}\vec{e}^{s}\Phi^s_{\mu}e^{-i\vec{k}_{\mu s}\vec{r}(t)} + \vec{e}^{s}_{\tau}\Phi^s_{\tau\mu}e^{-i\vec{k}_{\mu s\tau}\vec{r}(t)}\right\}\vec{u}e^{i\omega t}dt \qquad (21)\\
&- \frac{i\omega Q}{c^2}\int_0^T \left\{\sum_{\mu=1}^{2}\vec{e}^{s}\Phi^s_{\mu}e^{-i\vec{k}_{\mu s}\vec{r}(t)} + \vec{e}^{s}_{\tau}\Phi^s_{\tau\mu}e^{-i\vec{k}_{\mu s\tau}\vec{r}(t)}\right\}\vec{n}_z 2b\,\Omega\cos 2\Omega t\ e^{i\omega t}dt\\
&+ \frac{i\omega Q}{c^2}\int_0^T \left\{\sum_{\mu=1}^{2}\vec{e}^{s}\Phi^s_{\mu}e^{-i\vec{k}_{\mu s}\vec{r}(t)} + \vec{e}^{s}_{\tau}\Phi^s_{\tau\mu}e^{-i\vec{k}_{\mu s\tau}\vec{r}(t)}\right\}\vec{n}_x a\Omega\cos\Omega t\ e^{i\omega t}dt,
\end{aligned}
$$

The exponentials in (20) and (21)

$$e^{-i\vec{k}_{\mu s}\vec{r}(t)} = e^{-i\vec{k}_{\mu s}\vec{u}t}e^{-i\vec{k}_{\mu s}\vec{n}_x a\sin\Omega t}e^{i\vec{k}_{\mu s}\vec{n}_z b\sin 2\Omega t}, \qquad (22)$$

$$e^{-i\vec{k}_{\mu s\tau}\vec{r}(t)} = e^{-i\vec{k}_{\mu s\tau}\vec{u}t}e^{-i\vec{k}_{\mu s\tau}\vec{n}_x a\sin\Omega t}e^{i\vec{k}_{\mu s\tau}\vec{n}_z b\sin 2\Omega t}, \qquad (23)$$

can be conveniently transformed using the relation [26]:

$$e^{\pm id\sin\varphi} = J_0(d) + 2\sum_{n=1}^{\infty}J_{2n}(d)\ \cos 2n\varphi\ \pm\ 2i\sum_{n=0}^{\infty}J_{2n+1}(d)\ \sin(2n+1)\varphi, \qquad (24)$$

or, in some cases

$$e^{\pm id\sin\varphi} = e^{\pm id\cos(\varphi+\pi/2)} = \sum_{m=-\infty}^{\infty}(\pm i)^m J_m(d)\ e^{\pm im(\varphi+\pi/2)}. \qquad (25)$$

In view of (24) and (25), the exponentials in (21) are expressed in terms of the series expansions involving the product of Bessel functions into cos and sin oscillating in time at the frequencies $m\Omega$ ($m = 0, 1, 2, ...$).

As a result, we have quite a cumbersome expression for $A^s_{\vec{k}}(\omega)$

$$
\begin{aligned}
&A^s_{\vec{k}}(\omega) = \frac{i\omega Q}{c^2}\int_0^T \sum_{\mu=1}^{2}\vec{e}^{s}\Phi^s_{\mu}e^{-i\vec{k}^s_{\mu}\vec{u}t} \qquad (26)\\
&\times\left\{J_0(\vec{k}^s_{\mu}\vec{n}_x a) + 2\sum_{n=1}^{\infty}J_{2n}(\vec{k}^s_{\mu}\vec{n}_x a)\cos 2n\Omega t - 2i\sum_{n=0}^{\infty}J_{2n+1}(\vec{k}^s_{\mu}\vec{n}_x a)\sin(2n+1)\Omega t\right\}\\
&\times\left\{J_0(\vec{k}^s_{\mu}\vec{n}_z b) + 2\sum_{n=1}^{\infty}J_{2n}(\vec{k}^s_{\mu}\vec{n}_z b)\cos 4n\Omega t + 2i\sum_{n=0}^{\infty}J_{2n+1}(\vec{k}^s_{\mu}\vec{n}_z b)\sin 2(2n+1)\Omega t\right\}\\
&\times(\vec{u} - \vec{n}_z 2b\Omega\cos 2\Omega t + \vec{n}_x a\Omega\cos\Omega t)\,e^{i\omega t}dt\\
&\quad + \frac{i\omega Q}{c^2}\int_0^T \sum_{\mu=1}^{2}\vec{e}^{s}_{\tau}\Phi^s_{\tau\mu}e^{-i\vec{k}^s_{\mu\tau}\vec{u}t}\\
&\times\left\{J_0(\vec{k}^s_{\mu\tau}\vec{n}_x a) + 2\sum_{n=1}^{\infty}J_{2n}(\vec{k}^s_{\mu\tau}\vec{n}_x a)\cos 2n\Omega t - 2i\sum_{n=0}^{\infty}J_{2n+1}(\vec{k}^s_{\mu\tau}\vec{n}_x a)\sin(2n+1)\Omega t\right\}\\
&\times\left\{J_0(\vec{k}^s_{\mu\tau}\vec{n}_z b) + 2\sum_{n=1}^{\infty}J_{2n}(\vec{k}^s_{\mu\tau}\vec{n}_z b)\cos 4n\Omega t + 2i\sum_{n=0}^{\infty}J_{2n+1}(\vec{k}^s_{\mu\tau}\vec{n}_z b)\sin 2(2n+1)\Omega t\right\}\\
&\times(\vec{u} - \vec{n}_z 2b\Omega\cos 2\Omega t + \vec{n}_x a\Omega\cos\Omega t)\,e^{i\omega t}dt.
\end{aligned}
$$

Using the exponential representation for cos and sin and multiplying the expressions in curly brackets one can present (26) as sums of terms each including product of $J_n(\vec{k}^s_{\mu}\vec{n}_z b)J_{n'}(\vec{k}^s_{\mu}\vec{n}_x a)$ (or $J_n(\vec{k}^s_{\mu\tau}\vec{n}_z b)J_{n'}(\vec{k}^s_{\mu\tau}\vec{n}_x a)$) and exponents.

Let us consider, for example, the terms proportional to $\vec{u}J_0(\vec{k}^s_\mu \vec{n}_z b)J_0(\vec{k}^s_\mu \vec{n}_x a)$ and $\vec{u}J_0(\vec{k}^s_{\mu\tau}\vec{n}_x a)J_0(\vec{k}^s_{\mu\tau}\vec{n}_z b)$ in (26) and integrate them in time t. As a result, these terms will involve the factors $D(\omega)$, having the form

$$D^s_\mu(\omega) = \frac{e^{-i(\vec{k}^s_\mu \vec{u}-\omega)T} - 1}{-i(\vec{k}^s_\mu \vec{u}-\omega)}, \qquad D^s_{\mu\,\tau}(\omega) = \frac{e^{-(i\vec{k}^s_{\mu\,\tau}\vec{u}-\omega)T} - 1}{-i(\vec{k}^s_{\mu\,\tau}\vec{u}-\omega)}. \tag{27}$$

These factors are sharp functions of the differences $(\vec{k}^s_\mu \vec{u}-\omega)$ and $(\vec{k}^s_{\mu\,\tau}\vec{u}-\omega)$ and are proportional to T if

$$\vec{k}^s_\mu \vec{u}-\omega = 0 \quad \text{and} \quad \vec{k}^s_{\mu\,\tau}\vec{u}-\omega. \tag{28}$$

If the conditions under which $|\vec{k}^s_\mu| \simeq |\vec{k}^s_{\mu\,\tau}|$ are fulfilled, then these conditions determine the spectrum of parametric quasi-Cherenkov radiation (see [17,22]). It is to be noted that owing to the transfer of energy to harmonics, the amplitude of quasi-Cherenkov radiation, generated by a particle moving in the undulator, is reduced by a factor of $J_0(a)$ or $J_0(b)$ as compared to the case when the undulator is absent. (Compare with a similar phenomenon in the situation when a particle, moving in the undulator, emits ordinary Cherenkov radiation [27–30]).

When the absolute value of the wave vector of the diffracted photon differs from $|\vec{k}^s_\mu|$, from the equality $\vec{k}^s_{\mu\tau} = \vec{k}^s_\mu + \vec{\tau}$, we can derive the following equation for the radiation frequency

$$\vec{k}^s_\mu \vec{u}-\omega = -\vec{\tau}\vec{u} \tag{29}$$

that defines the spectrum of diffraction radiation (resonance and Smith-Purcell radiations).

Let a particle be incident along the normal to the surface of the photonic crystal. In this case, we can present (29) in the form

$$\omega - k^s_{\mu\,z} u = \tau_z, \tag{30}$$

that is,

$$\omega(1-\beta n\cos\vartheta) = -\tau_z a, \qquad \omega = -\frac{\tau_z}{1-\beta n(\omega)\cos\vartheta}. \tag{31}$$

As is seen, in contrast to the frequency of quasi-Cherenkov radiation, this frequency increases as the particle energy increases.

Let us mention here that in considering the diffraction process with several waves involved, the situation is possible when the photon, emitted in the process of diffraction radiation, will undergo diffraction from a family of planes that are defined by a different reciprocal lattice vector $\vec{\tau}\,'$. This case of radiation in crystals underlies the operation of the first VFEL generator [31].

The exponentials in the remaining terms in (26) include the power indices of the form $\vec{u}(\vec{k}^s_\mu \vec{u}-\omega \pm m\,\Omega)t$ and $\vec{u}(\vec{k}^s_{\mu\,\tau}\vec{u}-\omega \pm m\,\Omega)t$, which lead to the radiation produced by the oscillator that oscillates at the frequency $\pm\, m\,\Omega$. As a result, the spectrum of the diffraction radiation from the oscillator includes the harmonics whose amplitude depends on $\vec{k}^s_\mu \vec{n}_x a$, $\vec{k}^s_\mu \vec{n}_z b$ and $\vec{k}^s_{\mu\,\tau}\vec{n}_x a$, $\vec{k}^s_{\mu\,\tau}\vec{n}_z b$.

If the parameter $k^s_{\mu z} b \ll 1$, i.e., the emitted photons have a wavelength that appreciably exceeds the amplitude of longitudinal oscillations of the particle, then the Bessel function $J_0(k^s_{\mu\,z} b) \approx 1$, while $J_{n\neq 0}(k^s_{\mu\,z} b) \ll 1$, and the contribution to the photon emission process from longitudinal oscillations can be neglected.

Thus, the longitudinal oscillations of particles in undulators that exist along with transverse ones, lead to the formation of additional radiation peaks that are generated through the DRO mechanism at large angles to the velocity of a relativistic particle. The frequency of these peaks is larger than that determined by the fundamental frequency Ω of oscillations.

We should point out that, as stated earlier in this section, owing to the transfer of energy to harmonics, the amplitude of quasi-Cherenkov (parametric) radiation (as well as that of diffraction radiation generated through particle motion in the undulator) is reduced by a factor $J_0(k^s_{\mu z}b)$, $J_0(k^s_{\mu x}a)$, which has an effect on the increment of radiative quasi-Cherenkov (diffraction) instability of a beam moving in a photonic crystal.

Analyzing the time evolution of radiation from a photonic crystal, one can reveal some interesting features of radiation produced by a particle moving in the undulator inside a photonic crystal. As we showed earlier [19, 20], in the case of quasi-Cherenkov radiation, the law of radiation has a form of oscillations, whose frequency is determined by the characteristics of the crystal. When an oscillator moves in a photonic crystal, time oscillations at the frequencies determined by the frequency differences of the harmonics $n\Omega$, where $n = 1, 2, 3, ...$ are added to those mentioned above.

When the radiation intensity is averaged over the time interval being much greater than the oscillation period, these time oscillations disappear, and only the oscillations with a longer period remain, which are determined by the dispersion of the photons diffracted in the crystal.

Now, let us give a more detailed consideration of the time-dependent radiation amplitude $A^s_{\vec{k}}(t)$:

$$A^s_{\vec{k}}(t) = \frac{1}{2\pi}\int A^s_{\vec{k}}(\omega)e^{-i\omega t}d\omega \tag{32}$$

Substituting (26) into (32) and making integration over time as described hereinbefore one can present the amplitude $A^s_{\vec{k}}(t)$ in the form:

$$A^s_{\vec{k}}(t) = \frac{1}{2\pi}\int\sum_{\mu n n'} B^s_\mu(\omega)D^s_\mu(\omega,\, n,\, n')e^{-i\omega t}d\omega \tag{33}$$
$$+\frac{1}{2\pi}\int\sum_{\mu n n'} B^s_{\mu\tau}(\omega)D^s_{\mu\tau}(\omega,\, n,\, n')e^{-i\omega t}d\omega,$$

where $B^s_\mu(\omega) = \Phi^s_\mu(\omega)$ and $B^s_{\mu\tau}(\omega) = \Phi^s_{\mu\tau}(\omega)$ have the meaning of the coefficients of transmission and reflection from the crystal, respectively.

The quantities $D^s_\mu(\omega)$ have a general form

$$D^s_\mu(\omega,\, n,\, n') = \frac{e^{-i(\vec{k}^s_\mu\vec{u}-\omega\pm(n+n')\Omega)T}-1}{-i(\vec{k}^s_\mu\vec{u}-\omega\pm(n+n')\Omega)}, \tag{34}$$

$$D^s_{\mu\,\tau}(\omega,\, n,\, n') = \frac{e^{-i(\vec{k}^s_{\mu\,\tau}\vec{u}-\omega\pm(n+n')\Omega)T}-1}{-i(\vec{k}^s_{\mu\,\tau}\vec{u}-\omega\pm(n+n')\Omega)}. \tag{35}$$

Thus, the amplitude $A^s_{\vec{k}}(t)$ can be written in the form:

$$A^s_{\vec{k}}(t) = \sum_{nn'} \int B^s_\mu(t-t') D^s_\mu(t',n,n')\, dt' + \sum_{nn'} \int B^s_{\mu\tau}(t-t') D^s_{\mu\tau}(t',n,n')\, dt', \tag{36}$$

where $B(t)$ and $D(t)$ are the Fourier transforms of $B(\omega)$ and $D(\omega)$.

Let us compare this expression with that describing the time dependence of the electromagnetic pulse $E_0(t)$ reflected from the crystal

$$E(t) = \frac{1}{2\pi} \int B^s_\mu(\omega) E_0(\omega) e^{-i\omega t} d\omega = \int B^s_\mu(t-t') E_0(t') dt' \tag{37}$$

where $E_0(t') = \frac{1}{2\pi} \int E_0(\omega) e^{-i\omega t'} d\omega$, $B^s_\mu(\omega)$ is the coefficient of reflection (transmission) from the crystal of the wave having the frequency ω.

As we can see, when radiation is excited by the particle traveling through the crystal, the radiation pulse demonstrates similar pattern of time dependence as that of the reflected from the crystal pulse produced by the incident packet of electromagnetic waves.

This might have been expected, remembering that the emission of electromagnetic waves from a charged particle can be considered as the diffraction in the crystal of the pseudo-photons corresponding to this particle. In this case the coefficient $B(t)$ describes the crystal response function to the δ-type pulse (i.e., if $E_0(t') = \delta(t')$, then the field $E(t) = B(t)$). The explicit expression for the function $B(t)$ was obtained in [19, 20].

Thus, the diffraction of photons emitted from a relativistic particle moving in the undulator inside a photonic crystal leads to complex time oscillations of the radiation amplitude. Consequently, the radiation intensity that is proportional to $|A(t)|^2$. In this case, the oscillations due to the interference between quasi-Cherenkov and undulator radiations occur along with those due to the differences of harmonics $n\Omega$.

Let us note here that the amplitudes $A^s_{\vec{k}}(\omega)$ and $A^s_{\vec{k}}(t)$ of radiation from a particle moving in the undulator in the presence of a photonic crystal are determined by the quantities $D(\omega)$ and $D(t)$ that are analogous to the similar amplitudes of parametric X-ray radiation and X-ray diffraction radiation from a relativistic oscillator in natural crystals. For this reason, we can use the general results obtained for X-ray radiation and derive, in particular, the formulas describing spectral-angular distribution of the radiation energy (the number of emitted quanta) by replacing in the formulas given in [25] the quantities χ_τ, determining the dielectric permittivity of a natural crystal in the X-ray range, by the quantities χ_τ, determining the process of dynamical diffraction in a photonic crystal. In [25], these quantities are given explicitly for a photonic crystal built from metallic threads. According to the analysis in [25], χ_τ in such photonic crystals depends dramatically on the polarization of the wave's electric vector: for the waves whose electric vector $\vec{E}$ is parallel to the thread, $|\chi_{\tau\|}| \ll |\chi_{\tau\perp}|$, where $\chi_{\tau\perp}$ corresponds to the wave whose electric vector is orthogonal to the thread's axis. As a consequence, when emitted at small angles with respect to particle velocity, quasi-Cherenkov radiation has the polarization $\vec{E}$ parallel to the threads. What is more, the angular distribution of radiation becomes anisotropic: the radiation intensity is sharply suppressed if a plane is formed by vectors $\vec{k}$ and $\vec{v}$, orthogonal to the direction of the thread, and reaches its maximum value if this plane is parallel to the thread. This is in sharp contrast to the angular distribution of PXR emitted at small angles with respect to particle velocity, in which case such anisotropy is not observed.

Let us also mention that the intensity of diffraction radiation from a relativistic oscillator demonstrates quite a different time-dependence pattern than the intensity of quasi-Cherenkov radiation, because by selecting the oscillation frequency of the oscillator, one can tune up the center of the pseudo-photon wave packet to such range of frequencies in which the group velocity of the wave packet will reduce appreciably, and the radiation from the crystal will last longer.

4 Radiation from a dynamical undulator

Let us note that above we have given a detailed consideration of particle motion in a magnetostatic undulator, by the particle motion in a dynamical undulator (electromagnetic wave) has some distinct features. The trajectory of the particle moving in a dynamical undulator includes the initial phase of wave oscillation. For example, the particle trajectory that describes one-dimensional oscillations of the particle in the plane transverse to the direction of motion, defined by the velocity $\vec{u}$, can be written as

$$\delta\vec{r}_{\perp}(t) = \vec{a}\cos(\Omega_0 t + \delta), \tag{38}$$

where $\vec{a}$ is the amplitude of particle oscillation in the transverse plane and δ is the initial oscillation phase of the particle. The same phase δ appears in longitudinal oscillations of the particle. Consequently, seeking the radiation intensity in the case when the particle beam entering the undulator is not modulated, we must average the the intensity over the random phase δ, and the interference between quasi-Cherenkov and oscillator radiation (as well as the interference of different harmonics) disappear.

In what follows, we shall consider particle radiation in a dynamical undulator the case of the contribution of longitudinal oscillation of the particle can be neglected.

Let us choose $\vec{u}$ as the z-axis. Then we have for the transverse velocity

$$\vec{v}_{\perp} = -\vec{a}\Omega' \sin(\Omega' t + \delta). \tag{39}$$

The radiation amplitude $A^s_{\vec{k}}(\omega)$ (see (6)) can now be written in the form

$$A^s_{\vec{k}}(\omega) = \frac{i\omega Q}{c^2}\int \vec{E}^{(-)s^*}_{\vec{k}}(\vec{r}(t)\,,\omega)\,(\vec{u} - \vec{a}\,\Omega' \sin(\Omega' t + \delta))\,e^{i\omega t}\,dt, \tag{40}$$

i.e., in this case, the amplitude of photon emission by a charged particle contains the contributions of two types: from parametric quasi-Cherenkov radiation and from the radiation produced by transverse oscillation of a particle in the undulator.

Let us state here that a particle moving in a natural crystal can undergo oscillations due to the fact that it moves in a channeling regime in a straight or a periodically bent channel (crystal undulator) [3–10]. Irrespective of what the mechanism leading to the formation of the undulator may be, the general form of the equality (6) will remain the same. Only specific characteristics of the radiation produced by the particle moving through the crystal will vary.

Thus, the time-dependent amplitude of radiation $A^s_{\vec{k}}(t-\frac{r}{c})$ can be presented in the form ($\tau = t - \frac{r}{c}$):

$$A^s_{\vec{k}}(\tau) = A^s_{\vec{k}\,par}(\tau) + A^s_{\vec{k}\,osc}(\tau), \tag{41}$$

where the amplitude of parametric (quasi-Cherenkov) radiation

$$
\begin{aligned}
A^s_{\vec{k}\,\mathrm{par}}(\tau) &= \frac{1}{2\pi}\int A^s_{\vec{k}\,par}(\omega)e^{-i\omega\tau}d\omega,\\
A^s_{\vec{k}\,\mathrm{par}}(\omega) &= \frac{i\omega\,Q}{c^2}\int \vec{E}^{(-)s*}_{\vec{k}}\,(\vec{r}(t),\omega)\,\vec{u}\,e^{i\omega t}\,dt,
\end{aligned}
\tag{42}
$$

and the amplitude of diffraction radiation from a relativistic oscillator

$$
\begin{aligned}
A^s_{\vec{k}\,\mathrm{osc}}(\tau) &= \frac{1}{2\pi}\int A^s_{\vec{k}\,osc}(\omega)e^{-i\omega\tau}d\omega,\\
A^s_{\vec{k}\,\mathrm{osc}}(\omega) &= \frac{i\omega\,Q}{c^2}\int \vec{E}^{(-)*}_{\vec{k}}\,(\vec{r}(t)\,,\omega)\,(-\vec{u}\,\Omega'\sin(\Omega' t+\delta))\,e^{i\,\omega t}dt.
\end{aligned}
\tag{43}
$$

The time-dependent behavior of parametric quasi-Cherenkov radiation have been studied in the previous section. Let us give now a more detailed consideration of diffraction radiation from an oscillator.

For example, it follows from [20] that in the Laue case, the expression $\vec{E}^{(-)s*}_{\vec{k}}$ appearing in the equation for the radiation amplitude (43) has the following form inside the crystal

$$
\begin{aligned}
\vec{E}^{(-)s*}_{\vec{k}}(\vec{r}) = \vec{e}^{\,s}&\left[-\sum_{\mu=1}^{2}\xi^{0}_{\mu\,s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}(L-z)}\right]e^{-i\vec{k}\vec{r}}\\
&+\vec{e}^{\,s}_{\tau}\beta_1\times\left[\sum_{\mu=1}^{2}\xi^{\tau}_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}(L-z)}\right]e^{-i\vec{k}_\tau\vec{r}},
\end{aligned}
\tag{44}
$$

where

$$
\vec{k}_{\mu s} = \vec{k} + \kappa^*_{\mu s}\vec{N}, \qquad \kappa^*_{\mu s} = \frac{\omega}{c\gamma_0}\varepsilon^*_{\mu s},
$$

$\mu = 1,2$; $\vec{N}$ is the unit vector of a normal to the entrance crystal surface which is directed into the crystal,

$$
\varepsilon_{1(2)s} = \frac{1}{4}\left[(1+\beta_1)\chi_0 - \beta_1\alpha_B\right] \pm \frac{1}{4}\left\{\left[(1-\beta_1)\chi_0+\beta_1\alpha_B\right]^2 + 4\beta_1 C_s^2\chi_{\vec{\tau}}\chi_{-\vec{\tau}}\right\}^{-1/2},
$$

$\alpha_B = (2\vec{k}\vec{\tau}+\tau^2)k^{-2}$ is the off-Bragg parameter ($\alpha_B = 0$ if the exact Bragg condition of diffraction is fulfilled),

$$
\gamma_0 = \vec{n}_\gamma\cdot\vec{N},\quad \vec{n}_\gamma = \frac{\vec{k}}{k},\quad \beta_1 = \frac{\gamma_0}{\gamma_1},\quad \gamma_1 = \vec{n}_{\gamma\tau}\cdot\vec{N},\quad \vec{n}_{\gamma\tau} = \frac{\vec{k}+\vec{\tau}}{|\vec{k}+\vec{\tau}|}.
$$

Note that the trajectory $\delta\,r_\perp(t)$, which depends on the external field, enters into the exponent, and this leads to the appearance of the exponentials of the form $e^{i\,b\cos\varphi}$. For further analysis, it is convenient to use the following equality [26]

$$
e^{i\,b\cos\varphi} = \sum_{m=-\infty}^{\infty} i^m J_m(b)e^{im\varphi}.
\tag{45}
$$

We can write as a result

$$
\begin{aligned}
A^s_{\vec{k}}(\omega) &= \frac{i\omega\, Q}{c^2}\int\limits_0^T (\vec{u}-\vec{a}\Omega'\sin(\Omega' t+\delta))\,\vec{E}^{(-)^*}_{\vec{k}}\,(\vec{r}(t),\,\omega)\,e^{i\omega t}\,dt \\
&= \frac{i\omega\, Q}{c^2}\int\limits_0^T (\vec{u}-\vec{a}\Omega'\sin(\Omega' t+\delta)) \\
&\times \left\{\vec{e}^{\,s}\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}(L-ut)}\right]e^{-i(\vec{k}(\vec{u}t+\vec{a}\cos(\Omega' t+\delta)))}\right. \\
&+ \left.\vec{e}^{\,s}_{\tau}\beta_1\left[\sum_{\mu=1}^{2}\xi^\tau_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}(L-ut)}\right]e^{-i\vec{k}_\tau(\vec{u}t+\vec{a}\cos(\Omega' t+\delta))}\right\}e^{i\omega t}dt. \qquad (46)
\end{aligned}
$$

Make use of the relationships

$$
e^{ib\cos\varphi}=\sum_{m=-\infty}^{+\infty} i^m J_m(b)e^{im\varphi},\qquad e^{-ib\cos\varphi}=\sum_{m=-\infty}^{+\infty}(-i)^m J_m(b)e^{-im\varphi}. \qquad (47)
$$

We have

$$
\begin{aligned}
A^s_{\vec{k}}(\omega) &= \frac{i\omega\, Q}{c^2}\int\limits_0^T (\vec{u}-\vec{a}\Omega'\sin(\Omega' t+\delta)) \qquad (48)\\
&\times \left\{\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})\vec{e}^{\,s}\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}(L-ut)}\right]e^{-i\vec{k}\vec{u}t}e^{-im\Omega' t}e^{im\delta}\right. \\
&+ \left.\sum_{m=-\infty}^{+\infty}(-i)^m\, J_m(\vec{k}_\tau\vec{a})\vec{e}^{\,s}_{\tau}\beta_1\left[\sum_{\mu=1}^{2}\xi^\tau_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}(L-ut)}\right]e^{-i\vec{k}_\tau\vec{u}t}e^{-im\Omega' t}e^{-im\delta}\right\}e^{i\omega t}dt.
\end{aligned}
$$

These expressions include the time-dependent exponential $e^{-i\frac{\omega}{\gamma_0}\varepsilon_{\mu\, s}ut}e^{-i\vec{k}\vec{u}t}e^{-im\Omega' t}$ (in the second term $\vec{k}_\tau\vec{u}$ appears instead of $\vec{k}u$). Let us note here that $\sin(\Omega' t+\delta)=\frac{e^{i(\Omega' t+\delta)}-e^{-i(\Omega' t+\delta)}}{2i}$. As a result, $(\vec{u}-a\Omega'\sin(\Omega' t+\delta))$ is divided into three terms $\left(\vec{u}-a\Omega'\frac{e^{i(\Omega' t+\delta)}-e^{-i(\Omega' t+\delta)}}{2i}\right)$ which are multiplied by a figure bracket $\{\ldots\}$ and $e^{i\omega t}$.

Let also take into account that $\vec{k}+\frac{\omega}{\gamma_0}\varepsilon_{\mu\, s}=\vec{k}_{\mu\, s}$.

Let us first consider radiation in the forward direction. The radiation in the direction of diffraction can be obtained through the replacement

$$
\vec{e}_s\to\beta_1\vec{e}^{\,s}_{\tau},\qquad \vec{k}\to\vec{k}_\tau\qquad\qquad \xi^0_{\mu\, s}\to\xi^\tau_{\mu\, s}.
$$

Now we have for the amplitude in forward direction

$$A^s_{\vec{k}\,fw}(\omega) = \frac{i\omega\,Q}{c^2}\int\limits_0^T\left(\vec{u} - \frac{\vec{a}\Omega'}{2i}e^{i(\Omega' t+\delta)} + \frac{\vec{a}\Omega'}{2i}e^{-i(\Omega' t+\delta)}\right)$$
$$\times\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})\vec{e}^{\,s}\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu\,s}L}e^{-i(\vec{k}_{\mu s}\vec{u}+m\Omega'-\omega)t}\right]e^{-im\delta}dt \tag{49}$$
$$= \frac{i\omega\,Q}{c^2}\left\{\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})(\vec{u}\vec{e}^{\,s})\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu\,s}L}\,\frac{e^{i\Delta^m_{\mu\,s}(\omega)T}-1}{i\Delta^m_{\mu\,s}(\omega)}\right]e^{-im\delta}\right.$$
$$-\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})\left(\frac{(\vec{a}\vec{e}^{\,s})\Omega'}{2i}\right)\int\limits_0^T\left[-\sum_{\mu=1}^{2}\xi^0_{\mu\,s}\,e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu s}L}e^{-i(\vec{k}_{\mu\,s}\vec{u}+(m-1)\Omega'-\omega)t}\right]e^{-i(m-1)\delta}dt$$
$$\left.+\frac{\vec{a}\vec{e}^{\,s}\Omega'}{2i}\sum_{m=-\infty}^{+\infty}(-i)^m J_m(\vec{k}\vec{a})\left[-\sum_{\mu=1}^{2}\xi^0_{\mu\,s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu\,s}L}e^{-i(\vec{k}_{\mu\,s}\vec{u}+(m+1)\omega'-\omega t)}\right]e^{-i(m+1)\delta}\,dt\right\}$$

where

$$\Delta^m_{\mu\,s}(\omega) = \omega - \vec{k}_{\mu\,s}\vec{u} - m\Omega'$$

Finally,

$$A^s_{\vec{k}\,fw}(\omega) = \frac{i\omega\,Q}{c^2}\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})(\vec{u}\vec{e}^{\,s})$$
$$\times\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu\,s}L}\frac{e^{i\Delta^m_{\mu\,s}(\omega)T}-1}{i\Delta^m_{\mu\,s}(\omega)}\right]e^{-im\delta} \tag{50}$$
$$-\frac{i\omega\,Q}{c^2}\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})\left(\frac{\vec{a}\vec{e}^{\,s}\Omega'}{2i}\right)\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu\,s}L}\,\frac{e^{i\Delta^{m-1}_{\mu\,s}(\omega)T}-1}{i\Delta^{m-1}_{\mu\,s}(\omega)}\right]e^{-i(m-1)\delta}$$
$$+\frac{i\omega\,Q}{c^2}\sum_{m=-\infty}^{\infty}(-i)^m J_m(\vec{k}\vec{a})\left(\frac{\vec{a}\vec{e}^{\,s}\Omega'}{2i}\right)\left[-\sum_{\mu=1}^{2}\xi^0_{\mu s}e^{i\frac{\omega}{\gamma_0}\varepsilon_{\mu\,s}L}\,\frac{e^{i\Delta^{m+1}_{\mu\,s}(\omega)T}-1}{i\Delta^{m+1}_{\mu\,s}(\omega)}\right]e^{-i(m+1)\delta}.$$

The amplitude $A^s_{\vec{k}\,dif}(\omega)$ is obtained by replacing $\vec{e}_s \to \beta_1\vec{e}^{\,s}_{\tau}$ and $\vec{k} \to \vec{k}_\tau$, $\xi^0_{\mu s} \to \xi^\tau_{\mu s}$.

Now let us point out that in the first sum, proportional to $(\vec{u}\,\vec{e}^{\,s})$, the term that includes $m = 0$ describes parametric quasi-Cherenkov radiation in the undulator. The amplitude of this radiation can be described as $A_{par}(\omega)\cdot J_0(\vec{k}\vec{a})$, where $A_{par}(\omega)$ is the amplitude of parametric quasi-Cherenkov radiation of the particle moving with constant velocity $\vec{u}$ [17, 22].

In the second sum, the term including $m = 1$ coincides with the amplitude of ordinary parametric radiation if we replace $(\vec{u}\vec{e}^{\,s})$ by $iJ_1(\vec{k}\vec{a})\frac{1}{2i}(\vec{a}\vec{e}^{\,s})\Omega'$.

In the third sum, the term including $m = -1$ coincides with the amplitude parametric radiation if we replace $(\vec{u}\vec{e}^{\,s})$ by $iJ_{-1}(\vec{k}\vec{a})\frac{1}{2i}(\vec{a}\vec{e}^{\,s})\Omega'$.

Thus, the general expression for the amplitude of parametric radiation can be obtained from that for the ordinary amplitude by replacing $(\vec{u}\vec{e}^{\,s})$ with the following sum:

$$\vec{u}\vec{e}^{\,s} \to \vec{u}\vec{e}^{\,s}J_0(\vec{k}\vec{a}) + \frac{1}{2}J_1(\vec{k}\vec{a})\Omega'(\vec{a}\vec{e}^{\,s}) + \frac{1}{2}J_{-1}(\vec{k}\vec{a})\Omega'(\vec{a}\vec{e}^{\,s})$$

But the sum of the last two terms is zero, since $J_{-1} = -J_1$.

So in the second and third sums, we need to remove the terms with $m = 1$ (in the second) and $m = -1$ (in the third), because their sum equals zero.

Let us note now that in order to obtain the radiation intensity, we need to square the absolute value of the amplitude $A^s_{\vec{k}}$. As a result, we have double sums over $\sum_m \sum_{m'}$, which will initiate oscillations determined by the interference of different harmonics and the interference between quasi-Cherenkov and undulator radiations. The conclusion about the possibility to observe these oscillations depends on whether the phase δ is random or not. The phase δ will be random unless special measures are taken, and averaging over δ will then lead to the absence of the above interference. So it will suffice to study the time dependence of the radiation that is due to a certain harmonic m. The intensity is then equal to the sum of time-dependent intensities of quasi-Cherenkov radiation and diffraction radiation from a relativistic oscillator formed in a dynamical undulator (diffraction radiation of a channeled particle or a particle moving in a crystal undulator).

It is interesting to note that the spectrum of radiation from relativistic protons or nuclei channeled in a crystal lies in the soft spectral range, where the diffraction of the emitted photons can be carried out in a large number of various photonic crystals. This enables one to use such crystals for a thorough study of quasi-Cherenkov (parametric) radiation and diffraction radiation from relativistic oscillators formed by channeled protons or nuclei in both frequency and time domains.

5 Conclusion

The radiation amplitudes have been obtained that determine spectral-angular and time features of radiation from a relativistic particle moving in the undulator placed in (or near) a photonic crystal. Worthy of mention is that for photonic crystals built from metallic threads, the dynamical diffraction of electromagnetic waves has appreciable effects even in the diffraction cases that correspond to large values of $\vec{\tau}$, i.e., for photons with the wavelength $\lambda \ll d$ (d is the lattice period; for a rectangular lattice $\vec{\tau} = (\frac{2\pi n}{d}, \frac{2\pi n_2}{d_2}, \frac{2\pi n_3}{d_3})$). As a result,the effective diffraction reflection in such structures (see Fig.1) is possible, particularly in the terahertz range, even if the lattice period of the photonic crystal is within the centimeter range.

Let an undulator generating radiation in a terahertz range have a period of 10 cm and the length of 1 m, while a photonic crystal have the same length and a period of 1 cm. Then for the reciprocal lattice vector $\vec{\tau}$ with $n = 30$, we have the diffraction reflection in the terahertz range, and so all the phenomena that are due to dynamical diffraction can be observed in this frequency range without the necessity to use a photonic crystal having a period within a sub-millimeter range. An appreciably simpler photonic crystalis quite adequate for such observations. Due to greater values of the increment of radiative instability, volume free electron lasers designed on the basis of this mechanism will have much smaller undulators and photonic crystals than those in FELs without such crystals. Let us point out in this connection that multi-wave cases of diffraction of radiation in photonic crystals have a contribute significantly to the reduction of the resonator size and the generation threshold [34]. This circumstance can also substantially simplify the design and development of a two-stage VFEL for a short-wave range [41].

References

[1] V.G. Baryshevsky, Scattering of light by a flow of electrons through a crystal, *Dokl. Akad. Nauk BSSR* (1971) **15** pp. 306–308.

[2] V.G. Baryshevsky and I.D. Feranchuk, Transition radiation of γ-rays in a crystal *Sov. Phys. JETP* (1972) **34** pp. 502–504.

[3] V.G. Baryshevsky and I.Ya. Dubovskaya, Complex and anomalous Doppler effect for channelized positrons (electrons) *Dokl. Akad. Nauk SSSR* (1976) **231** pp. 1335–1338 (*Sov. Phys. Dokl.* **21** 741).

[4] V.G. Baryshevsky and I.Ya. Dubovskaya, Coherent radiation of the channelling positron (electron), *Phys. Status Sol. (b)*(1977) **82** pp. 403–412.

[5] M. A. Kumakhov, On the theory of electromagnetic radiation of charged particles in a crystal, *Phys. Lett. A* (1976) **57** pp. 17–18.

[6] V. G. Baryshevsky, A. O. Grubich and I.Ya. Dubovskaya, Generation of γ-quanta by channeled particles in the presence of a variable external field, *Phys. Lett. A* (1980) **77** pp. 61–64.

[7] V. V. Kaplin, S. V. Plotnikov and S. A. Vorobiev, Radiation by charged particles channeled in deformed crystals, *Zh. Tekh. Fiz.* **50** (1980) pp. 1079–1081.

[8] S. Bellucci and V. A. Maisheev, Radiation of relativistic particles for quasiperiodic motion in a transparent medium, *J. Phys.: Condens. Matter* **18** (2006) S2083–S2093.

[9] S. Bellucci and V. A. Maisheev, Calculations of intensity of radiation in crystal undulator, *Nucl. Instrum. Methods B* (2006) **252** pp. 339–346.

[10] V.G. Baryshevsky V.V. Tikhomirov, Crystal undulators: from the prediction to the mature simulations, http://dx.doi.org/10.1016/j.nimb.2013.03.013 *Nucl. Instrum. Methods B*, Available online 21 March 2013

[11] V. G. Baryshevsky and I.Ya. Dubovskaya, Angular distribution of photons from channelled particles, *J. Phys. C*, **16** (1983) pp. 3663–3672.

[12] V.G. Baryshevsky, A.O. Grubich and I.Ya. Dubovskaya, Diffraction of radiation from channelled charged particles *Phys. Stat. Sol. (b)* (1978) **88** pp. 351–358.

[13] V.G. Baryshevsky, A.O. Grubich and I.Ya. Dubovskaya, On photon production by channeled electrons (positrons) *Phys. Stat. Sol. (b)* (1980) **99** pp. 205–213.

[14] Ikeda, T., Matsuda, Y., Nitta, H., Ohtsuki, Y. H. (1996). Parametric X-ray radiation by relativistic channeled particles, *Nucl. Instrum. Methods B* **115**, 1–4, pp. 380–383.

[15] Yabuki, R., Nitta, H., Ikeda, T. and Ohtsuki, Y. H., Theory of diffracted channeling radiation, *Phys. Rev. B* (2001) **63**, 17, pp. 174112 [7 pages].

[16] Korotchenko, K. B., Pivovarov, Yu. L., Tukhfatullin, T. A., Angular distributions of diffracted X-ray radiation from channeled electrons in Si and LiF crystals: Influence of energy levels band structure, *Nucl. Instrum. Methods B* (2008) **266**, 17, pp. 3753–3757.

[17] V.G. Baryshevsky, *High-Energy Nuclear Optics of Polarized Particles*, World Scientific Publishing, Singapore, 2012.

[18] Baryshevsky, V. G. and Dubovskaya, I. Ya. (1991b). Diffraction X-ray radiation from a relativistic oscillator in a crystal, *J. Phys.: Condensed Matter* **3**, 14, (1991) pp. 2421–2430.

[19] S.V. Anishchenko, V.G. Baryshevsky and A.A. Gurinovich, Time dependence of the intensity of parametric X-ray radiation produced by relativistic particles passing through crystals, *Nucl. Instrum. Methods B* (2012) **293** pp. 35–41.

[20] V.G. Baryshevsky and A.A. Gurinovich, Time dependence of the intensity of diffracted radiation produced by a relativistic particle passing through a natural or photonic crystal, Available from: LANL e-print arXiv:1101.4162v1.

[21] L. D. Landau, E. M. Lifshitz and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, in: Landau, L. D. and Lifshitz, E. M. Course of Theoretical Physics Vol. 8, 2nd edn., Butterworth-Heinemann, 1984.

[22] V.G. Baryshevsky, I.D. Feranchuk, A.P. Ulyanenkov, *Parametric X-Ray Radiation in Crystals: Theory, Experiment and Applications*, Series: Springer Tracts in Modern Physics, Vol. 213, 2005.

[23] V.G. Baryshevsky, Parametric X-ray radiation at a small angle near the velocity direction of the relativistic particle, *Nucl. Instrum. Methods B* (1997) **122** pp. 13–18.

[24] T.C. Marshall, *Volume Free Electron Lasers*, Macmillan, New York, 1985.

[25] V. G. Baryshevsky and A. A. Gurinovich, Spontaneous and induced parametric and Smith-Purcell radiation from electrons moving in a photonic crystal built from the metallic threads, *Nucl. Instrum. Methods B*(2006) **252** pp. 92–101.

[26] I.S. Gradshteyn and I.M. Ryzhik *Table of Integrals, Series and Products*, Academic Press Inc., 2nd edition, 1980.

[27] C. Leibowitz and A. Peres, Energy balance of uniformly accelerated charge, *Ann. of Phys.* (NY) (1963) **25** pp. 400–404.

[28] A.Gailitis, Light scattering by a charged particle moving in a medium, I*zv. Vysshikh Uchebn. Zavedenii, Radiofiz.* (1964) **7** pp. 646–651.

[29] V.V. Mysakhanyan, A.I. Nikishov, Changes in Cerenkov radiation caused by an external field, *Sov. Phys. JETP* (1974) **39** pp. 615–619.

[30] V.L. Ginzburg, *Theoretical Physics and Astrophysics*, Pergamon Press, 1979.

[31] V. Baryshevsky, K. Batrakov, A. Gurinovich, I. Ilienko, A. Lobko, V. Moroz, P. Sofronov, V. Stolyarsky, First lasing of a volume FEL (VFEL) at a wavelength range $\lambda \sim 4 - 6\,\mathrm{mm}$, *Nucl. Instrum. Methods A* (2002) **483** pp. 21–23.

[32] V. L. Bratman, N. S. Ginzburg, G. G. Denisov, Use of distributed feedback in a free-electron laser, *Sov. Tech. Phys. Lett.* (1981) **7** pp. 595–567.

[33] V. N Baier, V. M. Katkov and V. M. Strachovenko, *Electromagnetic Processes at High Energies in Oriented Single Crystals* (World Scientific, Singapore, 1998).

[34] V.G. Baryshevsky, Spontaneous and induced radiation by relativistic particles in natural and photonic crystals. Crystal X-ray lasers and volume free electron lasers (VFEL), LANL e-print arXiv:physics/1101.0783.

[35] V.G. Baryshevsky and A.A. Gurinovich, Radiative instability of a relativistic electron beam moving in a photonic crystal, LANL e-print arXiv:physics/1011.2983.

[36] S. V. Anishchenko, V. G. Baryshevsky, and A.A. Gurinovich, Time duration of the parametric x-ray radiation, LANL e-print arXiv::physics/1106.2079v1.

[37] S.V. Anishchenko, V.G. Baryshevsky and A.A. Gurinovich, Time dependence of the intensity of parametric quasi-Cherenkov radiation produced by relativistic particles passing through electromagnetic (photonic) crystals, *J. Nanophoton.* (2012) **6** (1), 061714-1-061714-13.

[38] V.G. Baryshevsky, High power microwave and optical volume free electron lasers (VFELs), LANL e-print arXiv:physics.optics/1211.4769.

[39] V.G. Baryshevsky and A.A Gurinovich, Volume free electron laser - self-phase-locking system, LANL e-print arXiv:physics.acc-ph/1301.4330.

[40] S.V. Anishchenko and V.G. Baryshevsky, Parametric (quasi-Cherenkov) cooperative radiation produced by electron bunches in natural or photonic crystals, LANL e-print arXiv:physics.acc-ph/1304.0960.

[41] V.G. Baryshevsky, A.A. Gurinovich, Generation of radiation in free electron lasers with diffraction gratings (photonic crystal) with the variable spacial period, arXiv:physics/0608068