VOLUME FREE ELECTRON LASERS

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1.Introduction

There has been recently considerable theoretical and experimental interest in the concept of free electron lasers (FELs) [1,2]. It has been shown that free electron lasers can operate due to different radiation processes: "magnetic bremsstrahlung" in the undulator, Smith-Purcel and Cherenkov radiations, radiation in the laser wave. Independing on spontaneous radiation mechanism being principle for a definite FEL scheme, all existing FEL devices use for the feedback forming either two parallel mirrors placed at the ends of the working area or one-dimentional diffraction grating in which transmitted and diffracted (reflected) waves propagate along the electron beam velocity direction (one-dimentional distributed feedback (DFB)) (Fig.1,2).

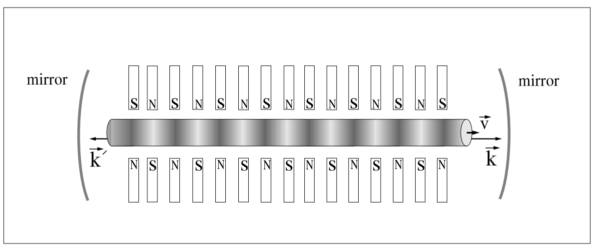

Figure 1: Free electron laser.

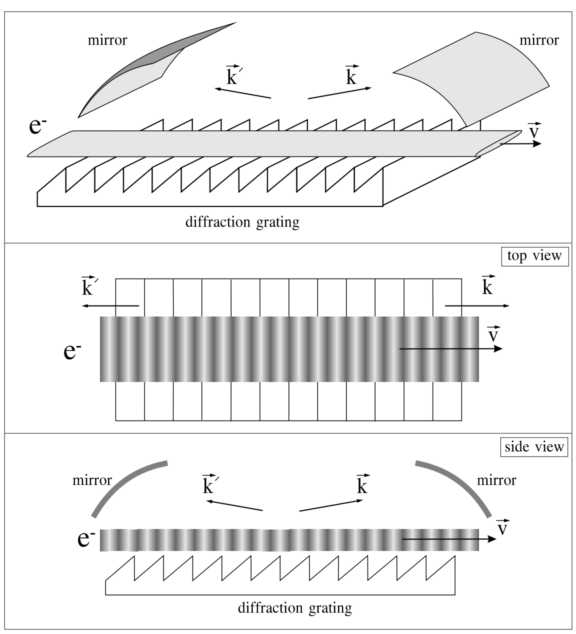

Figure 2: Smith-Purcel FEL (orotron).

According [3], the dispersion equation of the FEL in the collective interaction regime is reduced to that of the conventional travelling wave amplifier [4] and the FEL gain at the conditions of synchronism is proportional to $\rho_o^{1/3}$,where ρ_o is the density of the electron beam.

The volume FEL has been suggested as one of the alternative schemes of FEL which provides possibility to design compact sources in various spectral ranges including ultraviolet and X-ray [5-10].

The main peculiarity of VFEL is the use of one, two or three-dimensional grating as a volume resonator providing three-dimensional distributed feedback (Fig.3-5).

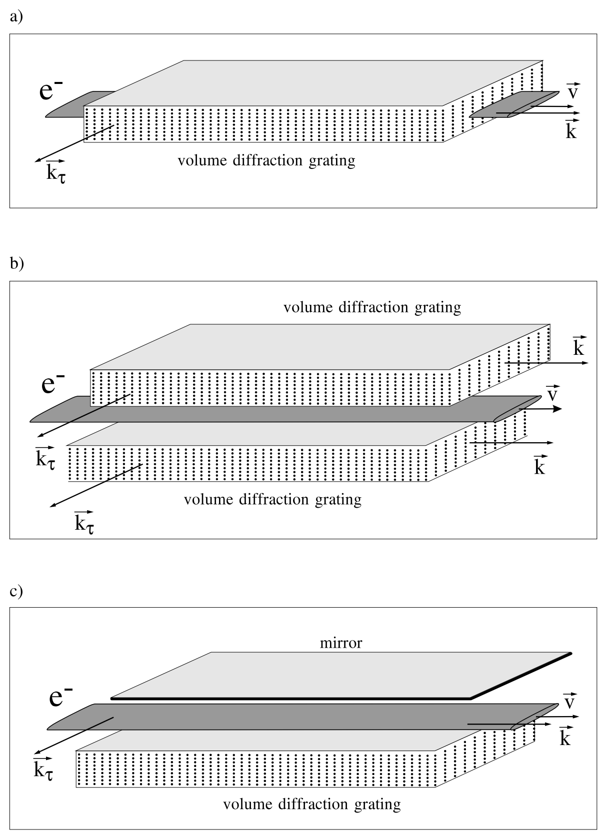

Figure 3: Parametric quasi-Cherenkov VFEL.

It is important to emphasize that even one-dimensional diffraction grating may provide non-one-dimensional (volume) feedback if the diffracted wave moves in nonback direction (the Bragg diffractive angle does not equal $\frac{\pi}{2}$). This results in essential modification of the VFEL gain and lasing processes providing, under specific conditions, more effective radiation process as compared with conventional FELs using one-dimensional distributed feedback. The VFEL gain at the conditions of synchronism is proportional to $\rho_0^{1/S+1}$ where S is the number of diffracted waves. Volume FEL, if realised, could be made with much more compact device structure compared with the conventional FEL and therefore, may be interesting for applications in different wavelength regions: from submillimeter to X-ray [5-9].

It should be emphasized that a fast destruction of the synchronism condition between a particle and an emitted electromagnetic wave is characteristic for the VFEL scheme with an electron beam passing through a diffraction grating. This leads to the essential

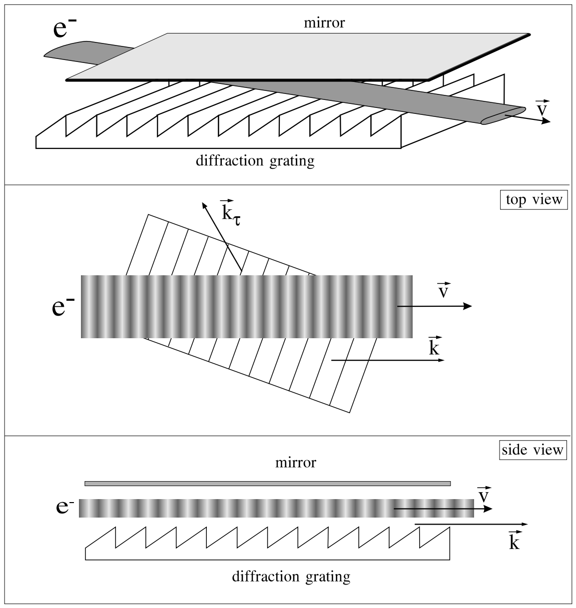

Figure 4: Parametric quasi-Cherenkov VFEL.

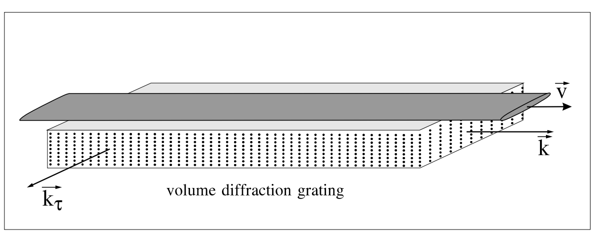

Figure 5: Surface VFEL.

increase of the generation threshold parameters. The reduction of the influence of multiple scattering becomes possible when electron beam moves either in the split of a grating (vacuum VFEL) or over a surface of a grating (surface FEL, SFEL) at a distance $d \leq \lambda\gamma$ (Fig.3-5)(λ is the photon wave length, γ is the Lorentz factor) [10]. The SFEL has been studied in [8].

It is easy to understand that vacuum VFEL turns into SFEL when width of the grating split grows.

Radiation mechanisms being the basis of VFEL and SFEL can be various (Cherenkov, Smith-Purcel and so on). The spontaneous surface parametric radiation (SSPR) [10] may be used for SFEL, for example. We should distinguish the SSPR from Smith-Purcel radiation [11,12].

The difference between these two types of radiation may be shown by analysing the radiation frequency dependence on electron energy. In the case of Smith-Purcel radiation the photon frequency is proportional to:

$$\omega \sim \frac{1}{\frac{1}{\gamma^2} + \theta^2}$$

and, for photons emitted at small angles to the electron velocity, the radiation frequency depends on the electron energy as γ^2. Moreover, this wave propagates in vacuum.
In the case of SSPR, the frequency of photons emitted even at small angles to the electron velocity does not practically depend on the electron energy but is determined by the Bragg condition. This radiation propagates inside the grating and leaves it only through a grating-vacuum boundary. The microscopic nature of both types of radiation is similar: they are stipulated by the medium atoms polarisation caused by an electromagnetic field of a moving charged particle.

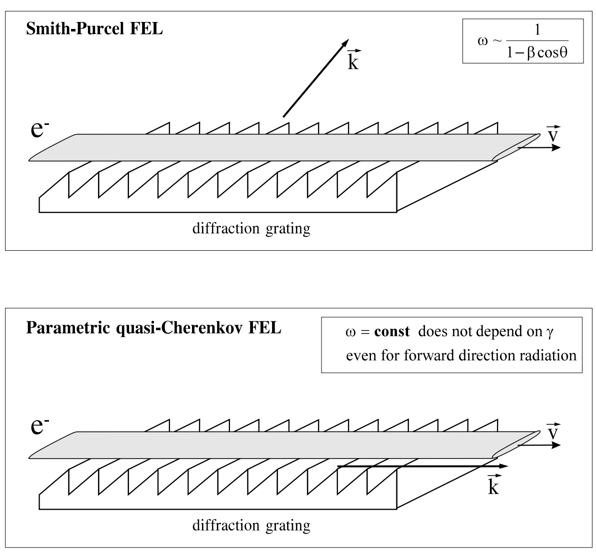

Figure 6: Smith-Purcel FEL and parametric quasi-Cherenkov VFEL comparison.

In the Smith-Purcel FEL (orotron, diffraction radiation generator) (see [13-17]) an electron beam passes over a reflecting diffraction grating, two mirrors (or diffraction grating) are used for one-dimensional feedback forming (Fig.2).

In the VFEL the non-one-dimensional feedback forming by the diffraction grating is used (Fig.3-5).

In the present paper the equations describing the VFEL lasing in case of an electron beam moving either in a split of a diffraction grating or in a vacuum waveguide containing a diffraction grating (vacuum VFEL)(Fig.3,4) have been obtained. The dispersion equation allowing to find the vacuum VFEL gain in one-mode generation regime have been considered.

2. Basic formulas describing vacuum VFEL lasing.

The interaction of an electron beam and an electromagnetic wave propagating along a waveguide in the vacuum VFEL is described by the Maxwell and electron movement equations

$$curl\ curl\ \vec{E}(\vec{r},\omega) - \frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega) = \frac{4\pi i\omega}{c^2}\vec{j}(\vec{r},\omega) \tag{1}$$

$$div\ \varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega) = 4\pi\rho(\vec{r},\omega) \tag{2}$$

$$-\ i\omega\rho(\vec{r},\omega) + div\vec{j}(\vec{r},\omega) = 0 \tag{3}$$

where $\vec{E}(\vec{r},\omega) = \int e^{i\omega t}\vec{E}(\vec{r},t)dt$ is the Fourier transformation of the electric field $\vec{E}(\vec{r},t)$; $\varepsilon(\vec{r},\omega)$ is the dielectric susceptibility of the diffraction grating; $\vec{j}(\vec{r},\omega)$ and $\rho(\vec{r},\omega)$ are the Fourier transformations of the electric current density $\vec{j}(\vec{r},t)$ and electric charge density of the beam $\rho(\vec{r},t)$, respectively.

$$\vec{j}(\vec{r},t) = e\sum_{\alpha}\vec{v}_{\alpha}(t)\delta\left(\vec{r}-\vec{r}_{\alpha}(t)\right) \tag{4}$$

$$\rho(\vec{r},t) = e\sum_{\alpha}\delta\left(\vec{r}-\vec{r}_{\alpha}(t)\right) \tag{5}$$

$\vec{r}_{\alpha}, \vec{v}_{\alpha}(t)$ are electron radius-vector and velocity. The subscript α denotes the particle's number.

Movement equations can be written in the form

$$\frac{d\vec{v}_{\alpha}(t)}{dt} = \frac{e}{m\gamma}\left\{\vec{E}(\vec{r}_{\alpha}(t),t) + \frac{1}{c}\left[\vec{v}_{\alpha}(t)\times\vec{H}((\vec{r}_{\alpha}(t),t)\right] - \frac{\vec{v}_{\alpha}}{c^2}(\vec{v}_{\alpha}\vec{E}(\vec{r}_{\alpha}(t),t)\right\}, \tag{6}$$

where $\vec{E}(\vec{r}_{\alpha}(t),t)$ and $\vec{H}(\vec{r}_{\alpha}(t),t)$ are the electric field and the magnetic field of the electromagnetic wave in the point $\vec{r}_{\alpha}(t)$ at the time moment t, $\gamma = (1-\frac{v_{\alpha}^2}{c^2})^{-\frac{1}{2}}$.

Let us consider a sheet electron beam passed over a diffraction grating placed in a plane waveguide. At first view this generator is similar to Smith-Purcel FEL (orotron or diffraction radiation generator) [3, 13-16]. But, in the volume FEL the radiated wave wavelength λ is of the same order as the diffraction grating period, the wave undergoes Bragg diffraction on the Bragg angle non-equal to $\frac{\pi}{2}$ and the diffraction grating provides the volume distribution feedback. Let the (y,z) coordinate plane be parallel to the waveguide (diffraction grating) surface. In the absence of the electron beam the current $\vec{j}=0$ and the density $\rho=0$. Equations (1, 2) become periodic in y,z directions (they are not periodic in x-direction). In this case the waveguide dielectric susceptibility can be written as:

$$\varepsilon\left(\vec{r},\omega\right) = \varepsilon_0\left(x\right) + \chi\left(\vec{r},\omega\right), \tag{7}$$

where $\varepsilon_0(x) = 1$ in vacuum, $\varepsilon_0(x) = \varepsilon_0$ in the area of the grating disposition, $\chi(\vec{r}, \omega)$ is the space periodic permittivity discribing the diffraction grating.

Let the permittivity $\chi(\vec{r}, \omega)$ be the periodic function of y and z:

$$\chi(\vec{r}, \omega) = \sum_{\vec{\tau} \neq 0} \chi_{\tau}(x) e^{-i\vec{\tau}\vec{\eta}},$$

where $\vec{\eta} = y\vec{e}_2 + z\vec{e}_3$ is the two-dimensional vector, $\vec{e}_{2(3)}$ is the unit vector along $y(z)$ axis and $\vec{\tau} = \tau_y \vec{e}_2 + \tau_z \vec{e}_3$ is the reciprocal lattice vector of the diffraction grating.

In the case $\chi = 0$ the set of equations (1, 2) desribes passing of the electromagnetic waves in the plain waveguide which contains the layer of the matter. The dielectric susceptebility of the matter is ε_0. The waveguide eigenmodes $\left|\vec{Y}_n(x)\right\rangle$and eigenvalues κ_n are well known [18]. They can be used for simplifying the three-dimensional Maxwell equations (1).

First of all, let us rewrite equation (1) as

$$\begin{aligned} -\triangle \vec{E}(\vec{r}, \omega) - \vec{\nabla}\left(\vec{\nabla}\left((\varepsilon_0(x) - 1)\vec{E}(\vec{r}, \omega)\right)\right) - \vec{\nabla}\left(\vec{\nabla}\left(\chi(\vec{r})\vec{E}(\vec{r})\right)\right) - \\ -\frac{\omega^2}{c^2}(\varepsilon_0(x) - 1)\vec{E}(\vec{r}) - \frac{\omega^2}{c^2}\chi(\vec{r})\vec{E}(\vec{r}) - \frac{\omega^2}{c^2}\vec{E}(\vec{r}, \omega) = \\ = \frac{4\pi i\omega}{c^2}\left(\vec{j}(\vec{r}, \omega) + \frac{c^2}{\omega^2}\vec{\nabla}\left(\vec{\nabla}\vec{j}(\vec{r}, \omega)\right)\right). \end{aligned} \quad (8)$$

Equations (8,6) allow us to find the electromagnetic field $\vec{E}(\vec{r}, \omega)$ radiated by electron beam. It is well known that gain and the generation threshold can be find in the linear approximation. In this case the beam current $\vec{j}$ is the linear function of $\vec{E}(\vec{r}, \omega)$: $\vec{j} = \vec{j}_0 + \delta\vec{j}$, where $\vec{j}_0$ is the beam current not perturbated by the radiated field, $\delta\vec{j} \sim \vec{E}(\vec{r}, \omega)$ is the beam current induced by the radiated field. In the linear approximation the set of movement equations (6) may be solved by the following way: the electromagnetic field $\vec{E}(\vec{r}_\alpha(t), \omega)$ in the right side of equations (6) can be represented as $\vec{E}(\vec{r}_{\alpha 0} + \vec{u}t, \omega)$, where $\vec{r}_\alpha(t) \simeq \vec{r}_{\alpha 0} + \vec{u}t$, $\vec{r}_{\alpha 0}$ is the initial coordinate of the electron, $\vec{u}$ is the electron velocity;$\vec{v}_\alpha(t) \simeq \vec{u}$ in the absence of radiated field.

As a result, we can obtain from (6) that

$$\begin{aligned} \delta\vec{v}_\alpha(\omega) = \frac{ie}{\omega m\gamma}\int \frac{d^3k'}{(2\pi)^3} e^{i\vec{k}'\vec{r}_{\alpha 0}}\left\{\frac{\omega}{\omega + \vec{k}'\vec{u}}\vec{E}\left(\vec{k}', \omega + \vec{k}'\vec{u}\right) + \right. \\ \left. + \left(\frac{\vec{k}'}{\omega + \vec{k}'\vec{u}} - \frac{\vec{u}}{c^2}\right)\left(\vec{u}\vec{E}\left(\vec{k}', \omega + \vec{k}'\vec{u}\right)\right)\right\}, \end{aligned} \quad (9)$$

$$\delta\vec{r}_\alpha(\omega) = \frac{i}{\omega}\delta\vec{v}_\alpha(\omega). \quad (10)$$

The beam current induced by the radiated field is

$$\begin{aligned} \delta\vec{j}\left(\vec{k}, \omega\right) = \int e^{-i\vec{k}\vec{r}} e^{i\omega t}\delta j(\vec{r}, t)\, d^3r d\omega = \\ = e\sum_\alpha e^{-i\vec{k}\vec{r}_{\alpha 0}}\left\{\delta\vec{v}_\alpha(\omega - \vec{k}\vec{u}) - i\vec{u}\left(\vec{k}\delta\vec{r}_\alpha(\omega - \vec{k}\vec{u})\right)\right\}. \end{aligned} \quad (11)$$

After substitution of expressions (9, 10, 11) in equation (8) we shall obtain the set of equations for the field $\vec{E}(\vec{r}, \omega)$.

Let us accomplish the Fourier transformation of the field $\vec{E}\,(\vec{r},\omega)$:

$$\vec{E}\,(\vec{r},\omega)=\frac{1}{(2\pi)^2}\int\vec{E}\left(x,\vec{k}_{\|}\right)e^{i\vec{k}_{\|}\vec{\eta}}d^2k_{\|}. \tag{12}$$

To obtain one-dimensional equation for the field $\vec{E}(x,\vec{k}_{\|})$ let us substitute expansion (12) in equation (8).

Let $\chi=0$ (the smooth waveguide) and the electron beam is absent. Then, equation (8) allows us to find eigenfuctions $\vec{Y}_n(x,\vec{k}_{\|})$ and eigenvalues $\kappa_n^2(\vec{k}_{\|})$:

$$\begin{aligned}-\frac{\partial^2}{\partial x^2}\vec{Y}_n(x,\vec{k}_{\|})-\vec{e}_1\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left[(\varepsilon_0(x)-1)\,Y_{nx}(x,\vec{k}_{\|})\right]+i\,(\varepsilon_0(x)-1)\,\vec{k}_{\|}\vec{Y}_n(x,\vec{k}_{\|})\right]-\\-i\vec{k}_{\|}\left[\frac{\partial}{\partial x}\left[(\varepsilon_0(x)-1)\,Y_x(x,\vec{k}_{\|})\right]+i\,(\varepsilon_0(x)-1)\,\vec{k}_{\|}\vec{Y}_n(x,\vec{k}_{\|})\right]-\\-\frac{\omega^2}{c^2}\left(\varepsilon_0\left(x\right)-1\right)\vec{Y}_n=\kappa_n^2\left(k_{\|}\right)\vec{Y}_n.\end{aligned} \tag{13}$$

If the vacuum-matter boundary is sharp, expression (13) gives the well known equation for the waveguide containing the dielectric layer:

a) in vacuum

$$-\frac{\partial^2}{\partial x^2}\vec{Y}_n=\kappa_n^2\vec{Y}_n, \tag{14}$$

b) in medium

$$-\frac{\partial^2}{\partial x^2}\vec{Y}_n-\frac{\omega^2}{c^2}\left(\varepsilon_0-1\right)\vec{Y}_n=\kappa_n^2\vec{Y}_n. \tag{15}$$

Now we can decompose the field $\vec{E}(x,\vec{k}_{\|})$ in terms of the waveguides eigenfunctions $\vec{Y}_n(x,\vec{k}_{\|})$:

$$\vec{E}\left(x,\vec{k}_{\|}\right)=\sum_n c_n\left(\vec{k}_{\|}\right)\left|\vec{Y}_n(x,\vec{k}_{\|})\right\rangle. \tag{16}$$

The field $\vec{E}(\vec{r},\omega)$ may be represented as

$$\vec{E}\,(\vec{r},\omega)=\frac{1}{(2\pi)^2}\sum_n\int c_n(\vec{k}_{\|})\left|\vec{Y}_n(x,\vec{k}_{\|})\right\rangle e^{i\vec{k}_{\|}\vec{\eta}}d^2k_{\|} \tag{17}$$

Let us substitute decomposition (16) into (8) and study the right side of (8) (which is determined by the current $\vec{j}\,(\vec{r},\omega)=\vec{j}_0\,(\vec{r},\omega)+\delta\vec{j}\,(\vec{r},\omega)$) more attentively. The set of equations (8) is a linear system. As a result, we can omit the nonperturbative part of current $\vec{j}_0$ and study (8) containing the induced current $\delta\vec{j}$ only. Decomposition (16) allows us to obtain the following expression for the right side of (8):

$$\begin{aligned}M=\frac{4\pi i\omega}{c^2}\int\left\langle\vec{Y}_n(x,\vec{k}_{\|})\right|e^{-i\vec{k}_{\|}\vec{\eta}}\left\{\delta j\,(\vec{r},\omega)+\frac{c^2}{\omega^2}\vec{\nabla}\left(\vec{\nabla}\delta\vec{j}(\vec{r},\omega)\right)\right\}dxd^2\eta=\\=\frac{4\pi i\omega}{c^2}\frac{1}{(2\pi)^3}\int_l\int\left\langle\vec{Y}_n(x,\vec{k}_{\|})\right|e^{i\vec{k}_{\|}\vec{\eta}}\left\{\delta j(\vec{k},\omega)-\frac{c^2}{\omega^2}\vec{k}\left(\vec{k}\delta\vec{j}(\vec{k},\omega)\right)\right\}e^{i\vec{k}\vec{r}}d^3kdx=\\=\frac{4\pi i\omega}{c^2}\frac{1}{2\pi}\int_l\int\left\langle\vec{Y}_n(x,\vec{k}_{\|})\right|\left\{\delta\vec{j}(\vec{k},\omega)-\frac{c^2}{\omega^2}\vec{k}\left(\vec{k}\delta\vec{j}(\vec{k},\omega)\right)\right\}e^{ik_xx}dxdk_x,\end{aligned} \tag{18}$$

where $\vec{k}=\left(k_x,\vec{k}_{\|}\right)$. It should be mentioned that the electron beam current density $\vec{j}(\vec{r},\omega)$ is not equal to zero only in the vacuum area restricted by the beam transverse size l. This fact results in the appearance of integral $\int\limits_l$ in (18) which means the integration over the area where $\vec{j}(\vec{r},\omega)\neq 0$ (see Fig.3,4). As a result, we have

$$M=\frac{4\pi i\omega}{c^2}\frac{1}{2\pi}\int dk_x\left\langle\vec{Y}_n(k_x,\vec{k}_{\|})\right|\left\{\delta\vec{j}(\vec{k},\omega)-\frac{c^2}{\omega^2}\vec{k}\left(\vec{k}\delta\vec{j}(\vec{k},\omega)\right)\right\}, \tag{19}$$

where

$$\left\langle\vec{Y}_n(k_x,\vec{k}_{\|})\right|=\int\limits_l\left\langle\vec{Y}_n(k_x,\vec{k}_{\|})\right|e^{ik_xx}dx$$

The expression for the current density $\delta\vec{j}(\vec{k},\omega)$ contains the sum $F=\sum\limits_\alpha e^{-i(\vec{k}-\vec{k}')\vec{r}_{\alpha 0}}$. Let us average this sum over distribution of the particles in the beam:

$$\sum_\alpha e^{i\left(\vec{k}-\vec{k}'\right)\vec{r}_{\alpha\,0}}\simeq\Phi\left(k_x-k_x'\right)(2\pi)^3 n_0\delta\left(\vec{k}_{\|}-\vec{k}_{\|}'\right), \tag{20}$$

where $\Phi(k_x-k_x')=\frac{1}{2\pi}\int\limits_a^b e^{-i(k_x-k_x')x}\varphi(x)dx$, $\frac{1}{l}\int\limits_a^b\varphi(x)dx=1$, l is the characteristic transversal size of the beam, function $\varphi(x)$ describes the distribution of the particles along x-direction, n_0 is the electron density of the beam.

Using (20) we can write $\delta j(\vec{k},\omega)$ as

$$\begin{aligned}\delta j(\vec{k},\omega)=\vec{u}\delta\varphi(\vec{k},\omega)=\vec{u}\frac{ie^2n_0}{\omega c^2(\omega-\vec{k}\vec{u})^2m\gamma}\frac{1}{2\pi}\int\limits_l dx'e^{-ik_xx'}\varphi(x')\times\\ \times\left(-ik_x\frac{\partial}{\partial x'}c^2+\frac{c^2-u^2}{u^2}\omega^2\right)\left(\vec{u}\vec{E}(x',\vec{k}_{\|},\omega)\right).\end{aligned} \tag{21}$$

As a result, using (21) we can represent (19) as

$$\begin{aligned}M=\frac{4\pi i\omega}{c^2}\frac{1}{2\pi}\int dk_x\left\langle\vec{Y}_n(k_x,\vec{k}_{\|})\right|\left\{\vec{u}-\frac{c^2}{\omega^2}\vec{k}(\vec{k},\vec{u})\right\}\delta\varphi(\vec{k},\omega)=\\ =\frac{4\pi i\omega}{c^2}\frac{1}{2\pi}\int dk_x\left\langle\vec{E}_n\left(k_x,\vec{k}_{\|}\right)\right|\left\{\vec{u}-\frac{c^2}{\omega^2}\vec{k}\right\}\delta\varphi(\vec{k},\omega)\end{aligned} \tag{22}$$

In the cold beam case (when the condition $k_xu_x\frac{1}{c}\ll L$ to be fulfilled) we can write

$$\begin{aligned}M=\frac{4\pi i\omega}{c^2}\frac{ie^2n_0}{(\omega-\vec{k}_{\|}\vec{u})^2m\gamma\omega c^2}\frac{1}{2\pi}\int\limits_l dx\left\langle\vec{Y}_n(x,\vec{k}_{\|},\omega)\right|\left(\vec{u}-\frac{c^2}{\omega}\vec{k}_{\|}-i\frac{\hat{\partial}}{\partial x}\frac{c^2}{\omega}\vec{e}_1\right)\times\\ \times\left(\frac{\hat{\partial}}{\partial x}\phi(x)\frac{\partial}{\partial x}c^2+\frac{c^2\omega^2}{u^2}\frac{1}{\gamma^2}\right)\left(\vec{u}\vec{E}(x',\vec{k}_{\|},\omega)\right),\end{aligned} \tag{23}$$

where operator $\frac{\hat{\partial}}{\partial x}$ acts on the functions disposed on its left.

Substituting the decomposition $\vec{E}(x',\vec{k}_{\|},\omega)=\sum_{n}{}' c_{n'}(\vec{k}_{\|})\left|\vec{Y}_n(x,\vec{k}_{\|})\right\rangle$ into (23) and using the ortogonality of the eigenfunctions we obtain from (8, 23)

$$\left(k_{\|}^2-(\frac{\omega^2}{c^2}-\kappa_n^2)\right)c_n(\vec{k}_{\|})-\frac{\omega^2}{c^2}\sum_{\vec{\tau},n'}\chi_{eff}^{nn'}(\vec{k}_{\|},\vec{k}_{\|}+\vec{\tau})c_{n'}(\vec{k}_{\|}+\vec{\tau})= \tag{24}$$

$$=\frac{4\pi i\omega}{c^2}\sum_{n'}\frac{ie^2n_0A_{nn'}}{\left(\omega-\vec{k}_{\|}\vec{u}\right)^2m\gamma\omega}c_{n'}(\vec{k}_{\|}),$$

where

$$A_{nn'}=\frac{1}{2\pi c^2}\int_l dx\left[\left\langle\vec{Y}_n(x,\vec{k}_{\|},\omega)\right|\left\{\vec{u}-\frac{c^2}{\omega}\vec{k}_{\|}-i(\frac{\hat{\partial}}{\partial x}\frac{c^2}{\omega}\vec{e}_1)\right\}\right]\times$$

$$\times\left[\frac{\hat{\partial}}{\partial x}\varphi(x)\frac{\partial}{\partial x}c^2+\frac{c^2\omega^2}{u^2\gamma^2}\right]\left(\vec{u}\left|\vec{Y}_n'(x,\vec{k}_{\|},\omega)\right\rangle\right)\approx\frac{1}{2\pi c^2}\int_a^b dx\left[\left\langle\vec{Y}_n(x)\right|\left(\vec{u}-\frac{c^2}{\omega}\vec{k}_{\|}\right)\right]\times \tag{25}$$

$$\times\left(\frac{\partial\varphi}{\partial x}\frac{\partial}{\partial x}c^2\right)\left(\vec{u}\left|\vec{Y}_n(x)\right\rangle\right),$$

where $\chi_{eff}^{nn'}(\vec{k}_{\|},\vec{k}_{\|}+\tau)$ is the effective permittivity. It contains two terms: the first proportional to $\chi(\vec{r},\omega)$ and the second proportional to $\vec{\nabla}\left(\vec{\nabla}\chi(\vec{r},\omega)\right)$:

$$\chi_{eff}^{nn'}(\vec{k}_{\|},\vec{k}_{\|}+\vec{\tau})=\int dx\left\langle\vec{Y}_n(x,\vec{k}_{\|})\right|\chi_{\tau}(x)\left|\vec{Y}_{n'}(x,\vec{k}_{\|}+\vec{\tau})\right\rangle+ \tag{26}$$

$$\int dx\left\langle\vec{Y}_n(x,\vec{k}_{\|})\right|\frac{c^2}{\omega^2}\hat{\vec{k}}\left(\hat{\vec{k}}\chi_{\tau}(x)\left|\vec{Y}_{n'}(x,\vec{k}_{\|}+\vec{\tau})\right\rangle\right),$$

where $\hat{\vec{k}}=\frac{\partial}{\partial x}\vec{e}_1+i\vec{k}_{\|}$.

Let the condition $\vec{k}_{\|}\gg\frac{2\pi}{d}$ be fullfiled (d is the diametrical size of the waveguide). In this case the second term of (26) is less than the first one.

We can simplify the system (2) solving by assuming a practically important case of a single mode n existance in the waveguide. It is possible when condition $\omega^2/c^2\chi^{nn'}(\vec{k}_{\|},\vec{k}_{\|}+\vec{\tau})\left(\kappa_n^2-\kappa_{n'}^2\right)^{-1}\ll 1$ is fulfilled. In this case all terms with $n'\neq n$ in the sum on n' in (24) can be omitted.

As a result, we obtain the set of equations which is similar to that describing the multiwave dynamical diffraction of electromagnetic waves in a diffraction grating.

In the two wave diffraction case the Bragg condition accomplishes for two waves with wavevectors $\vec{k}_{\|}$ and $\vec{k}_{\|}'$: $\vec{k}_{\|}'\simeq\left(\vec{k}_{\|}+\vec{\tau}\right)$, $\left|\vec{k}_{\|}'\right|\simeq\left|\vec{k}_{\|}\right|$ and the set of equations (24) can be written as

$$\left[k_{\|}^2-\frac{\omega^2}{c^2}\varepsilon_0+\frac{\omega_LA_{nn}}{\gamma c^2\left(\omega-\vec{k}_{\|}\vec{u}\right)^2}\right]c_n(\vec{k}_{\|})-\frac{\omega^2}{c^2}\chi_{eff}^{nn}(\vec{k}_{\|},\vec{k}_{\|}+\vec{\tau})c_n(\vec{k}_{\|}+\vec{\tau})=0, \tag{27}$$

$$-\frac{\omega^2}{c^2}\chi_{eff}^{nn}(\vec{k}_{\|}+\vec{\tau},\vec{k}_{\|})c_n(\vec{k}_{\|})+\left[(\vec{k}_{\|}+\vec{\tau})^2-\frac{\omega^2}{c^2}\varepsilon_0\right]c_n(\vec{k}_{\|}+\vec{\tau})=0,$$

where $\varepsilon_0=1-c^2\kappa_n^2/\omega^2$, ω_L is the Lengmuer frequency of the electron beam ($\omega_L^2=4\pi e^2n_0/m$).

The set of equations (27) is similar to that for the electromagnetic field amplitudes describing lasing in volume FEL for the case of beam moving in volume diffraction grating [6]. The main discrepancy appeares in the dependence of the equations (27) on $\vec{k}_{\parallel}$. The similar set of equations for volume FEL depends on $\vec{k}$.

As a result, we can conclude that all the main results obtained for VFELs holds true for the vacuum VFEL. First of all, the non-trivial solution of system (27) exists only when the determinant of the system is equal to zero. This allows us to obtain the dispersion equation for $\vec{k}_{\parallel}$ and ω:

$$\left(\omega - \vec{k}_{\parallel}\vec{u}\right)^2 \left[\left(k_{\parallel}^2 c^2 - \omega^2\varepsilon_0\right)\left(\left(\vec{k}_{\parallel} + \vec{\tau}\right) c^2 - \omega^2\varepsilon_0\right) - \omega^4 \chi_{\tau}^{nn}\chi_{-\tau}^{nn}\right] = \\ = -\frac{\omega_L^2}{\gamma} A_{nn}\left(\left(\vec{k}_{\parallel} + \vec{\tau}\right)^2 c^2 - \omega^2\varepsilon_0\right) \tag{28}$$

According to [5-8] the study of the dispersion equation let us find the condition of the appearance of the convection and absolute instability of the beam and, as a result, obtain the gain and the generation threshold.

In the plain waveguide case ($\chi_{\tau}^{nn} = 0$) from (28) we have

$$\left(\omega - \vec{k}_{\parallel}\vec{u}\right)^2 \left(k_{\parallel}^2 c^2 - \omega^2\varepsilon_0\right) = -\frac{\omega_L^2}{\gamma} A_{nn} \tag{29}$$

$$\left(\omega - \vec{k}_{\parallel}\vec{u}\right)^2 \left(k_{\parallel} c - \omega\sqrt{\varepsilon_0}\right) \simeq -\frac{\omega_L^2}{2\omega\sqrt{\varepsilon_0}\gamma} A_{nn} \tag{30}$$

Equations (29), (30) coincide with the equations describing the wave spectrum for Cherenkov instability of the beam in medium. From (30) we can obtain that $k_{\parallel}$ has imaginary part $Imk_{\parallel}$ and

$$Imk_{\parallel} = \frac{\sqrt{\varepsilon_0}}{2c}\left(\frac{\omega_L^2 \left|A_{nn}\right|}{2\omega\varepsilon_0\gamma}\right)^{\frac{1}{3}}, \tag{31}$$

when the Cherenkov condition $1 - \omega/c\sqrt{\varepsilon_0}cos\vartheta = 0$ is fulfilled. As we see, $Imk_{\parallel}$ is proportional to $n_0^{1/3}$ (n_0 is the density of the electron beam). It means that the gain is proportional to $n_0^{1/3}$ as well. This dependence is typical for all types of one-dimensional FEL in the collective regime [3].

Let the waveguide contains a diffraction grating ($\chi_{\tau}^{nn} \neq 0$). The wave spectrum is described by equation (28). When the coefficient $A_{nn} = 0$, equation (28) splites into two equations:

$$\left(k_{\parallel}^2 c^2 - \omega^2\varepsilon_0\right)\left(\left(\vec{k}_{\parallel} + \vec{\tau}\right)^2 c^2 - \omega^2\varepsilon_0\right) - \omega^4\chi_{\tau}^{nn}\chi_{-\tau}^{nn} = D\left(\vec{k}_{\parallel}, \omega\right) = 0 \tag{32}$$

$$\left(\omega - \vec{k}\vec{u}\right)^2 = 0 \tag{33}$$

Equation (32) describes the electromagnetic wave spectrum for the waiveguide containing a diffraction grating. Equation (33) describes the wave spectrum of the electron beam charge density. Let us study the solutions of (28) near the point where the left side of (28) is equal to zero. The solution of (31, 32) in the vicinity of the exact Bragg condition $\left|\vec{k}_{\parallel} + \vec{\tau}\right| \simeq k_{\parallel}$ can be written in the form

$$k_{z0} = k_z^0\left(1 + \delta\right), \quad \omega_0 = k_z^0 u\left(1 + \delta\right), \tag{34}$$

where $\delta \ll 1$ and k_z^0 can be found from exact Bragg conditions

$$k_z^0 = -\frac{2k_y\tau_y + \tau^2}{2\tau_z}, \tag{35}$$

z axis is parallel to the beam velocity $\vec{u}$. From (28,34) we can obtain for δ

$$\delta = \frac{\chi_\tau^{nn}\chi_\tau^{nn} - (\eta+\xi)^2}{2\nu(\eta+\xi)}; \tag{36}$$

$$\nu = \frac{\tau_z}{k_{0z}}, \eta = \frac{k_y^2}{k_z^{02}}, \xi = 1 - \beta^2\varepsilon_0$$

Now we can study equations (27). Let us rewrite (27) as

$$(\omega - k_z u)^2 D(k_z,\omega) = A(k_z,\omega), \tag{37}$$

where

$$A(k_z,\omega) = -\frac{\omega_L^2}{\gamma} A_{nn}\left(\left(\vec{k}_{\|} + \vec{\tau}\right)^2 c^2 - \omega^2\varepsilon_0\right) \tag{38}$$

The solution of (37) can be represented as

$$\omega = \omega_0 + \omega', \quad k_z = k_{zo} + k_z' \tag{39}$$

where $|\omega'| \ll \omega_0$ and $|k_z'| \ll |k_{z0}|$.

Let us write $D(k_z,\omega)$ in the form

$$D(k_z,\omega) = \left(\frac{\partial D}{\partial \omega}\right)_{\omega_0,k_{z0}} \omega' + \left(\frac{\partial D}{\partial k_z}\right)_{\omega_0,k_{z0}} k_z' + \frac{1}{2}\left(\frac{\partial^2 D}{\partial k_z^2}\right)_{\omega_0,k_{z0}} k_z'^2 + \\ + \frac{1}{2}\left(\frac{\partial^2 D}{\partial \omega^2}\right)_{\omega_0,k_{z0}} \omega'^2 + \left(\frac{\partial^2 D}{\partial \omega \partial k_z}\right)_{\omega_0,k_{z0}} \omega' k_z' + \ldots \tag{40}$$

It can be shown that $(\partial D/\partial\omega)_{\omega_0,k_{z0}}$ can not be equal to zero. That is why, we can omit the term proportional to $(\partial^2 D/\partial\omega^2)_{\omega_0,k_{z0}}$. On the other hand, the derivative $(\partial D/\partial k_z)_{\omega_0,k_{z0}}$ can be equal to zero. In this case equation (36) can be written as

$$(\omega' - k_z' u)^2 \left(k_z'^2 - F\omega'\right) = \frac{2A(k_{zo},\omega_0)}{\left(\frac{\partial^2 D}{\partial k_z^2}\right)_{\omega_0,k_{z0}}}, \tag{41}$$

where

$$F = -2\left(\frac{\partial D}{\partial \omega}\right)_{\omega_0,k_{z0}} \left(\frac{\partial^2 D}{\partial k_z^2}\right)^{-1}_{\omega_0,k_{z0}}$$

For $\omega' \to 0$ equation (40) takes form

$$k_z'^4 = \frac{2A}{u}\left(\frac{\partial^2 D}{\partial k_z^2}\right)^{-1}_{\omega_0,k_{z0}} \tag{42}$$

As a result, equation (42) gives

$$Imk_z = \left(\frac{\omega_0 \left|Q\right| \left|\chi_\tau\right| \tau}{u^2 c^2 \sqrt{2\tau_z^2 - \tau^2}} \right)^{1/4} = \left(\frac{\omega_L^2 A_{nn} \left|\chi_\tau\right| \tau}{2\gamma u^2 c^2 \sqrt{2\tau_z^2 - \tau^2}} \right)^{1/4} \tag{43}$$

Let us remind that the Langmuer frequency $\omega_L \sim n_0^{1/2}$. So, accordingly to (43), we have been obtained a very important result: in vacuum VFEL both Imk_z and the gain are proportional to $n_0^{1/4}$ for the definite orientation of the diffraction grating in the waveguide in contrast with the conventional one-dimensional FEL for which the gain is proportional to $n_0^{1/3}$. From (32) and (43) we have:

$$\frac{Imk_z}{Imk_{||}} \approx \left(\frac{\omega_L^2 \left|A_{nn}\right|}{\omega^4 \chi_\tau^3 \gamma} \right)^{-\frac{1}{12}} \gg 1, \tag{44}$$

because $\frac{\omega_L^2 |A_{nn}|}{\omega^4 \chi_\tau^3 \gamma} \ll 1$, $\frac{\omega_L^2}{\omega^2} \ll 1$ and $A_{nn} \ll \omega^2$. As a result, in our case of the volume feedback the gain is larger then that for one-dimensional feedback. For example, the dependence of the threshold current density for the volume and one-dimensional geometries on the length is represented in Fig.7.

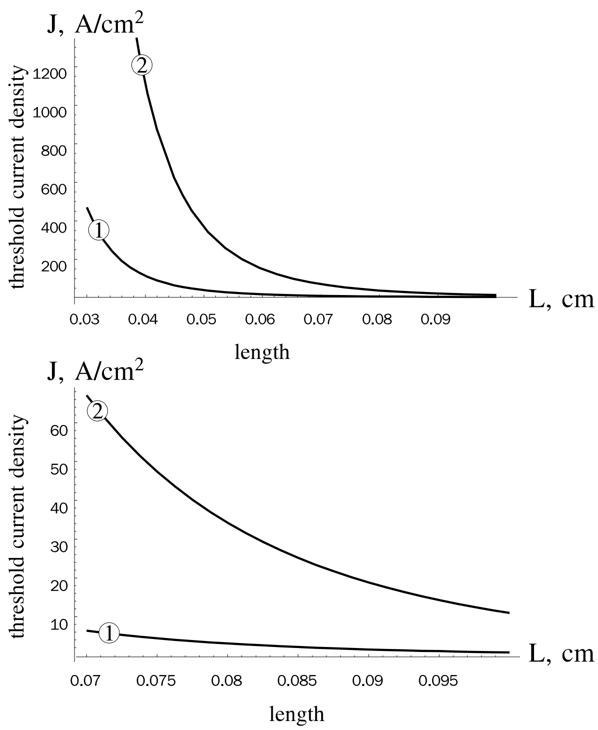

Figure 7: The dependence of the threshold current density for the volume (1) and one-dimensional (2) geometries on the length ($\lambda = 6283\ \mathring{A}$).

The gain becomes higher when the distributed feedback is formed by the multi-wave dynamical diffraction:

$$Imk_z \approx \frac{\omega}{c} \chi_\tau \left(\frac{\omega_L^2 \left|A_{nn}\right|}{\omega^4 \chi_\tau^3 \gamma} \right)^{\frac{1}{S+1}}, \tag{45}$$

where S is the number of diffracted waves.

CONCLUSION

The vacuum VFEL amplification and the generation process develop more intensively then in ordinary FEL using one-dimensional distributed feedback. Such the VFEL, if realised, could be made with much more compact device structure compared with the FEL and therefore, may be interesting for applications in different wavelength ranges: from submillimeter to X-ray. Such the VFEL can be realised on the basis of the existing accelerators.

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