Phenomenon of the time-reversal violating magnetic field generation by a static electric field in a medium and vacuum.

Vladimir G.Baryshevsky

Nuclear Problems Research Institute, Bobruiskaya Str. 11,
Minsk 220080 Belarus.
Electronic address: bar@inp.minsk.by
Tel: 00375-172-208481
Fax: 00375-172-265124

P- and T-odd interactions cause mixing of opposite parity levels of atom (molecule) that yields to the appearance of P- and T-odd terms of the atom (molecule) polarizability [1]. This makes possible to observe various optical phenomena, for example, photon polarization plane rotation and circular dichroism in an optically homogeneous medium placed to an electric field, polarization plane rotation (circular dichroism) phenomena for photons moving in an electric (gravitational) field in vacuum [2].

The energy of atom (molecule) in external electromagnetic field includes the term caused by the time reversal violating interactions [1]:

$$\Delta U = -\frac{1}{2}\beta_S^T \overrightarrow{E}\,\overrightarrow{H}, \tag{1}$$

where β_S^T is the scalar T-noninvariant polarizability of atom (molecule), $\overrightarrow{E}$ is the external electric field, $\overrightarrow{H}$ is the external magnetic field.

It's well known [3] that when the external field frequency $\omega \to 0$ the polarizabilities describe the processes of magnetization of medium by a static magnetic field and electric polarization of a medium by a static electric field

The energy of interaction of magnetic moment $\overrightarrow{\mu}$ with magnetic field $\overrightarrow{H}$

$$W_H = -\overrightarrow{\mu}\,\overrightarrow{H} \tag{2}$$

Comparison of (1) and (2) let one to conclude that the action of stationary electric field on an atom (molecule) induces the magnetic moment of atom

$$\overrightarrow{\mu}(\overrightarrow{E}) = \frac{1}{2}\beta_S^T \overrightarrow{E} \tag{3}$$

On the other hand, the energy of interaction of electric dipole moment $\overrightarrow{d}$ with electric field $\overrightarrow{E}$

$$W_E = -\overrightarrow{d}\,\overrightarrow{E}. \tag{4}$$

As it follows from (1) and (4), magnetic field induces the electric dipole moment of atom

$$\overrightarrow{d}(\overrightarrow{H}) = \frac{1}{2}\beta_S^T \overrightarrow{H} \tag{5}$$

As appears from the above, atom (molecule) being placed to static electric field gets the induced magnetic moment which in its part produces magnetic field. And similarly, if atom (molecule) is placed in the area of magnetic field the induced electric dipole moment yields to the appearance of its associated electric field.

Let us consider the simplest possible experiment. Suppose that homogeneous isotropic matter (liquid or gas) is placed to the area occupied by an electric field $\overrightarrow{E}$. From the above it follows that the time reversal violation yields to the appearance of magnetic field $\overrightarrow{H}_T = 4\pi\rho\overrightarrow{\mu}(\overrightarrow{E})$ parallel to $\overrightarrow{E}$ in this area (ρ is the number of atoms (molecules) of matter per cm^3). And vice versa, the electric field $\overrightarrow{E}_T = 4\pi\rho\overrightarrow{d}(\overrightarrow{H})$ appears under matter placement to the area occupied by a magnetic field $\overrightarrow{H}$. Let us estimate the effect value. It is easy to do by β_S^T evaluation. The general case explicit expression for polarizabilities for time dependent fields were derived in [1] (see eqs. (12)-(20) therein).

Briefly the calculation technique is as follows. Let us suppose that atom is placed to the arbitrary periodic in time electric and magnetic fields. The energy of interaction of an atom (molecule) with these fields has the routine form

$$W = -\widehat{\overrightarrow{d}}\overrightarrow{E} - \widehat{\overrightarrow{\mu}}\overrightarrow{H} + ... \tag{6}$$

where $\widehat{\overrightarrow{d}}$ is the operator of atom electric dipole moment and $\widehat{\overrightarrow{\mu}}$ is the operator of atom magnetic dipole moment

$$\overrightarrow{E} = \frac{1}{2}\left\{\overrightarrow{E}_0\, e^{-i\omega t} + \overrightarrow{E}_0^*\, e^{i\omega t}\right\},\ \overrightarrow{H} = \frac{1}{2}\left\{\overrightarrow{H}_0\, e^{-i\omega t} + \overrightarrow{H}_0^*\, e^{i\omega t}\right\} \tag{7}$$

The Shrödinger equation describing atom interaction with electromagnetic field is as follows:

$$i\hbar\frac{\partial\psi(\xi,t)}{\partial t} = [H_A(\xi) + W(\xi,t)]\psi(\xi,t), \tag{8}$$

where $H_A(\xi)$ is the atom Hamiltonian taking into account the weak interaction of electrons with nucleus in the center of mass of the system, ξ is the space and spin variable of electron and nucleus, W is the energy of interaction of atom with electromagnetic field of frequency ω

$$\begin{aligned} W &= Ve^{-i\omega t} + V^+e^{i\omega t}, \\ V &= -\frac{1}{2}(\overrightarrow{d}\overrightarrow{E_0} + \overrightarrow{\mu}\overrightarrow{H_0}), V^+ = -\frac{1}{2}(\overrightarrow{d}\overrightarrow{E_0}^* + \overrightarrow{\mu}\overrightarrow{H_0}^*) \end{aligned} \tag{9}$$

Let us perform the transformation $\psi = \exp(-i\frac{H_A}{\hbar}t)\varphi$. Suppose $H_A\psi_n = E_n\psi_n$ ($E_n = E_n^{(0)} - \frac{1}{2}i\Gamma_n$, $E_n^{(0)}$ is the atom level energy, Γ_n is the atom level width), then $\varphi = \sum_n b_n(t)\psi_n$. Therefore it follows from (9)

$$i\hbar\frac{\partial b_n(t)}{\partial t} = \sum_f \{\langle n|V|f\rangle \exp[i(E_n - E_f - \hbar\omega)t/\hbar] + \tag{10}$$

$$+ \langle n|V^+|f\rangle \exp[i(E_n - E_f + \hbar\omega)t/\hbar]\Big\}\, b_f(t),\ \langle\psi_n|\psi_m\rangle \ll 1.$$

Suppose b_{n0} be the ground state amplitude. Let us substitute the amplitude b_f describing the excited atom state into the equation for b_{n0} and study this equation at time $t \gg \tau_f = \hbar/\Gamma_f$ (or $\tau_f = \hbar/\Delta E$); $\Delta E = E_f^{(0)} - E_{n0} - \hbar\omega$; $\Gamma_f \gg |\langle n|V|f\rangle|$ (or $\Delta E \gg |\langle n|V|f\rangle|$). Therefore b_{n0} is defined by equation

$$i\hbar\frac{\partial b_{n0}(t)}{\partial t} = \widehat{U}_{eff}\, b_{n0},\ where$$

$$\widehat{U}_{eff} = -\sum_f \left(\frac{\langle n_0|V|f\rangle\, \langle f|V^+|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|V^+|f\rangle\, \langle f|V|n_0\rangle}{E_f - E_{n0} - \hbar\omega}\right) \tag{11}$$

Substituting V and V^+ into (11) one can obtain

$$\widehat{U}_{eff} = -\frac{1}{2}\widehat{g}_{ik}^{E}E_{0i}E_{0k}^* - \frac{1}{2}\widehat{g}_{ik}^{H}H_{0i}H_{0k}^* - \frac{1}{2}\widehat{g}_{ik}^{EH}E_{0i}H_{0k}^* - \frac{1}{2}\widehat{g}_{ik}^{HE}H_{0i}E_{0k}^*, \tag{12}$$

where the polarizability of atom (molecule) is:

$$\begin{aligned}
\widehat{g}_{ik}^{E} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|d_i|f\rangle\, \langle f|d_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|d_k|f\rangle\, \langle f|d_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega}\right)\\
\widehat{g}_{ik}^{H} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|\mu_i|f\rangle\, \langle f|\mu_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|\mu_k|f\rangle\, \langle f|\mu_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega}\right)\\
\widehat{g}_{ik}^{EH} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|d_i|f\rangle\, \langle f|\mu_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|\mu_k|f\rangle\, \langle f|d_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega}\right)\\
\widehat{g}_{ik}^{HE} &= -\frac{1}{2}\left(\sum_f \frac{\langle n_0|\mu_i|f\rangle\, \langle f|d_k|n_0\rangle}{E_f - E_{n0} + \hbar\omega} + \frac{\langle n_0|d_k|f\rangle\, \langle f|\mu_i|n_0\rangle}{E_f - E_{n0} - \hbar\omega}\right)
\end{aligned}$$

It should be noted that $\widehat{g}_{ik}^{E}$ and $\widehat{g}_{ik}^{H}$ are the P- and T-invariant electric and magnetic polarizability tensors and $\widehat{g}_{ik}^{EH}$ and $\widehat{g}_{ik}^{HE}$ are the P- and T-noninvariant polarizability tensors

Let an atom be placed at the static ($\omega \rightarrow 0$) magnetic and electric fields $\overrightarrow{E}$ and $\overrightarrow{H}$ of the same direction. Then it's perfectly easy to obtain the effective energy of P- and T-odd interaction of an atom with these fields.

$$\widehat{U}^{T,P}_{eff} = -\frac{1}{2}\left(\sum_f \frac{\langle n_0|d_z|f\rangle\ \langle f|\mu_z|n_0\rangle + \langle n_0|\mu_z|f\rangle\ \langle f|d_z|n_0\rangle}{E_f - E_{n_0}}\right) EH \tag{13}$$

Axis z is supposed to be parallel to $\overrightarrow{E}$. Thus from (1)

$$\beta^T_S = \sum_f \frac{\langle n_0|d_z|f\rangle\ \langle f|\mu_z|n_0\rangle + \langle n_0|\mu_z|f\rangle\ \langle f|d_z|n_0\rangle}{E_f - E_{n_0}} \tag{14}$$

Let us estimate the β^T_S order of magnitude. The atom state $|f\rangle$ does not possess the certain parity because of weak T-odd interactions. And over the weakness of V_T the state $|f\rangle$ is mixed with the state of opposite parity of value $\eta_T = \frac{V^T_W}{E_f - E_n}$. According to

$$\beta^T_S \sim \frac{\langle d\rangle\ \langle \mu\rangle}{E_f - E_{n_0}} \eta_T \tag{15}$$

For the heavy atoms the mixing coefficient can attain the value $\eta_T \approx 10^{-14}$. Taking into account that matrix element $\langle\mu\rangle \sim \alpha\langle d\rangle$ (where $\alpha = \frac{1}{137}$ is the fine structure constant) one can obtain $\beta^T_S \sim \eta_T\ \alpha\frac{\langle d\rangle^2}{\Delta} \approx 10^{-16} \cdot \frac{8\cdot 10^{-36}}{10^{-12}} \approx 10^{-40}$. Therefore, the electric field $E = 10^2\ CGSE$ induces magnetic moment $\mu_T \approx 10^{-38}$. Then, the magnetic field in the liquid target can be estimated as follows

$$H = 4\pi\rho\mu_T \approx 10^{23} \cdot 10^{-38} = 10^{-15}\ gauss \tag{16}$$

The magnitude of magnetic field strength can be increased, for example, by tightening of the magnetic field with superconductive shield. In this way the measured field strength can be increased by four orders when one collect the field from the area 1 m^2 to the area 1 cm^2 (Fig.1).

The induced magnetic moment produces magnetic field at the electron (nucleus) of the atom. This field $H^T(E) \sim \mu\ \langle\frac{1}{r^3}\rangle \sim 10^{-38} \cdot 10^{26} = 10^{-12}\ gauss$. Therefore, the frequency of precession of atom magnetic moment μ_A in the magnetic field induced by an external electric field

$$\Omega_E \sim \frac{\mu_A\ \beta\ E\ \langle\frac{1}{r^3}\rangle}{\hbar} = \frac{10^{-20} \cdot 10^{-12}}{10^{-27}} = 10^{-5}\ \sec^{-1} \tag{17}$$

It should be reminded that to measure the electric dipole moment the shift of precession frequency of atom spin in the presence of both magnetic and electric fields is investigated. Then, the T-odd shift of precession frequency of atom spin includes two terms: frequency shift conditioned by interaction of atom electric dipole moment with electric field $\omega_E = \frac{2d_A E}{\hbar}$ and frequency shift $\Omega = \frac{2\mu H^T(E)}{\hbar}$ defined above. This aspect should be considered when

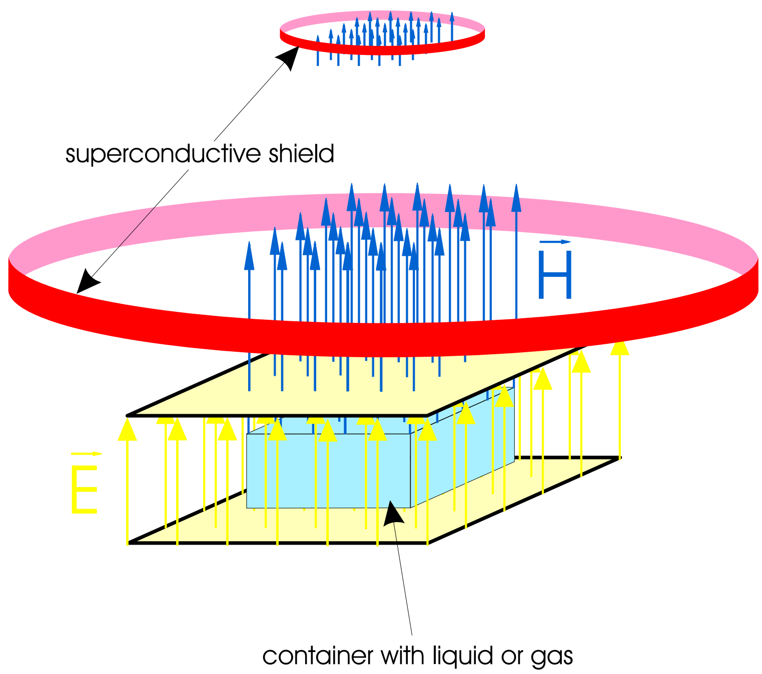

Figure 1:

interpreting the similar experiments. One should take note of the mixing coefficient η_T essential increase when the opposite parity levels are close to each other or even degenerate. Then the effect can grow up as much as several orders $10^5 \div 10^6$ (this occurs, for example, for Dy, TlF, BiS, HgF).

The similar phenomenon of magnetic field induction by electric field can occur in vacuum too.

Due to quantum electrodynamic effect of electron-positron pair creation in strong electric, magnetic or gravitational field, the vacuum is described by the dielectric ε_{ik} and magnetic μ_{ik} permittivity tensors depending on these fields. The theory of ε_{ik} [4] does not take into account the weak interaction of electron and positron with each other. Considering the T- and P-odd weak interaction between electron and positron in the process of pair creation in an electric (magnetic, gravitational) field one can obtain the density of electromagnetic energy of vacuum contains term $\beta_v^T(\vec{E}\vec{H})$ similar (1) (in the case of vacuum polarization by a stationary gravitational field $\beta_g^T(\vec{H}\vec{n_g})$, $\vec{n_g} = \frac{\vec{g}}{g}$, $\vec{g}$ – gravitational acceleration).

As a result both electric and magnetic fields (directed along the electric field) could exist around an electric charge. But in this case $\oint \vec{B}\,d\vec{S} \neq 0$ ($\vec{B}$ is the magnetic induction) that is impossible in the framework of classic electrodynamics. The existence of such field would means the existence of induced magnetic monopole. If the condition $\oint \vec{B}\,d\vec{S} = 0$ is fulfilled then for the spherically symmetrical case the field appears equal to zero. Surely, the value of this magnetic field is extremely small, but the possibility of its existence is remarkable itself.

The above result can be obtained in the framework of general Lagrangian formalism. Lagrangian density can depend only on the field invariants. Two invariants are known for the quasistatic electromagnetic field: $(\overrightarrow{E}\overrightarrow{H})$ and (E^2-H^2). In conventional T-invariant theory these invariants are included in the Lagrangian L only as (E^2-H^2) and $(\overrightarrow{E}\overrightarrow{H})^2$, i.e. $L=L(E^2-H^2,(\overrightarrow{E}\overrightarrow{H})^2)$ [4]. But while taking into account the T-odd interactions the Lagrangian can include invariant $(\overrightarrow{E}\overrightarrow{H})$ raising to the odd power, i.e.

$$L_T=L_T(E^2-H^2,(\overrightarrow{E}\overrightarrow{H})^2,(\overrightarrow{E}\overrightarrow{H})) \tag{18}$$

Expanding (18) by weak interaction one can obtain

$$L_T=L(E^2-H^2,(\overrightarrow{E}\overrightarrow{H})^2)+\beta_T(\overrightarrow{E}\overrightarrow{H}), \tag{19}$$

where L is the density of Lagrangian in P- and T-invariant electrodynamics, $\beta_T=\beta_T(E^2-H^2,(\overrightarrow{E}\overrightarrow{H})^2)$ is the constant can be found in certain theory. The explicit form of L is cited in [4].

The additions caused by the vacuum polarization can be described by the field dependent dielectric and magnetic permittivity of vacuum. According to [4] the electric induction vector $\overrightarrow{D}$ and magnetic induction vector $\overrightarrow{B}$ are defined as:

$$\overrightarrow{D}=\frac{\partial L}{\partial\overrightarrow{E}},\ \overrightarrow{B}=-\frac{\partial L}{\partial\overrightarrow{H}} \tag{20}$$

Similarly the electric polarization $\overrightarrow{P}$ and magnetization $\overrightarrow{M}$ of vacuum can be found [4]:

$$\overrightarrow{P}=\frac{\partial(L_T-L_0)}{\partial\overrightarrow{E}},\ \overrightarrow{M}=-\frac{\partial(L_T-L_0)}{\partial\overrightarrow{H}}, \tag{21}$$

$$\overrightarrow{D}=\overrightarrow{E}+4\pi\overrightarrow{P},\ \overrightarrow{B}=\overrightarrow{H}+4\pi\overrightarrow{M}. \tag{22}$$

In accordance with the above, the T-noninvariance yields to the appearance of additional P- and T-odd terms to the electric polarization $\overrightarrow{P}$ and magnetization $\overrightarrow{M}$. There are the addition to the vector of electric polarization $\overrightarrow{P}$ proportional to the magnetic field strength $\overrightarrow{H}$ and the addition to the vector of magnetization $\overrightarrow{M}$ proportional to the electric field strength $\overrightarrow{E}$

References

[1] V.G.Baryshevsky Phys.Letters A 177 (1993) p.38-42

[2] V.G.Baryshevsky Phys.Letters A 260 (1999) p.24-30

[3] Landau L., Lifshitz E. Quantum mechanics, 1989, Moscow Science.

[4] Berestetskii V., Lifshitz E., Pitaevskii L. Quantum electrodynamics, 1989, Moscow Science.