Effect of Electric and Magnetic Fields on Spin Dynamics in the Resonant Electric Dipole Moment Experiment

Alexander J. Silenko

Institute of Nuclear Problems, Belarusian State University,

11 Bobruiskaya Street, Minsk 220080, Belarus[*]

(Dated: June 20, 2012)

Abstract

A buildup of the vertical polarization in the resonant electric dipole moment (EDM) experiment [Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006)] is affected by a horizontal electric field in the particle rest frame oscillating at a resonant frequency. This field is defined by the Lorentz transformation of an oscillating longitudinal electric field and a uniform vertical magnetic one. The effect of a longitudinal electric field is significant, while the contribution from a magnetic field caused by forced coherent longitudinal oscillations of particles is dominant. The effect of electric field on the spin dynamics was not taken into account in previous calculations. This effect is considerable and leads to decreasing the EDM effect for the deuteron and increasing it for the proton. The formula for resonance strengths in the EDM experiment has been derived. The spin dynamics has been calculated.

PACS numbers: 21.10.Ky, 13.40.Em, 14.20.Dh, 27.10.+h

Keywords: electric dipole moment, spin resonances, resonance strengths

[*]Electronic address: silenko@inp.minsk.by

I. INTRODUCTION

Sensitive searches for electric dipole moments (EDMs) of fundamental particles are excellent probes of physics beyond the Standard Model [1]. The Storage Ring EDM Collaboration investigates the possibility to search for a particle EDM by placing a charged particle in a storage ring and observing the precession of its spin [2, 3]. The frozen spin method [2, 3, 4] which consists in cancelling a spin precession in the horizontal plane with a radial electric field can provide a sensitivity of deuteron EDM measurements of 10^{-27} e·cm [3].

The resonance method proposed by Y. Orlov [5] reaches the sensitivity of 10^{-29} e·cm for the deuteron and 10^{-28} e·cm for the proton. This method is based on the idea that the precession caused by the deuteron EDM will accumulate if the beam is forced to undergo a coherent longitudinal oscillation that is in phase with the spin precession in the horizontal plane. This oscillation is caused by an oscillating longitudinal electric field and its frequency should be very close to that of the spin precession. The resonance effect in the storage ring (so-called Orlov ring) is ensured by radio frequency (rf) cavities [5, 6, 7].

The resonance method is free of main systematic error of measurement of the EDM with the frozen spin method caused by the presence of a small vertical electric field (see Refs. [2, 3]). This field leads to the buildup of the vertical polarization (BVP) imitating the EDM effect [2, 3].

The aim of this work is the calculation of effects of electric and magnetic fields on a buildup of the vertical polarization in a resonant EDM experiment [5, 6, 7]. Formulae for resonance strengths are derived. Distinguishing features of the resonant EDM experiment for protons and deuterons are considered and dynamics of components of polarization vector is described.

Throughout the work the system of units $\hbar = c = 1$ is used. In some equations, the speed of light c is explicitly shown.

II. FIELDS AFFECTING THE BUILDUP OF THE VERTICAL POLARIZATION

In the resonance EDM experiment [5, 6, 7], it is planned to stimulate the BVP conditioned by the EDM and to avoid a similar effect caused by the magnetic moment. It is known that the magnetic resonance takes place when a particle placed in a uniform vertical magnetic field is also affected by a horizontal magnetic field oscillating at a frequency close to the frequency of spin rotation (see, e.g., Ref. [8]). If the particle moves, the magnetic resonance can also be stimulated by an oscillating electric field transforming to an oscillating magnetic one in the particle rest frame. The magnetic resonance results in spin flipping for a vertically polarized beam and in the BVP for a horizontally polarized one.

In this work, "spin" means an expectation value of a quantum mechanical spin operator. The polarization vector is defined by $\boldsymbol{P} = \boldsymbol{S}/S$, where $\boldsymbol{S}$ is the spin vector and S is the spin quantum number. The directions of the EDM vector, $\boldsymbol{d}$, and the spin coincide: $\boldsymbol{d} = d\boldsymbol{S}/S$. It is convenient to use the η factor for the EDM which corresponds to the g factor for the magnetic moment and is given by

$$\eta = \frac{2dm}{eS}.$$

Evidently, the magnetic resonance cannot take place when the electric field is longitudinal, because nothing but the oscillating electric field appears in the particle rest frame. Since the frequencies of betatron oscillations are chosen to be far from resonances, these oscillations cannot lead to the resonance effect. However, the resonance takes place if the particle possesses the EDM. The resonance is "electric", because it is conditioned by the electric field in the particle rest frame. In this frame, the electric field possesses the longitudinal component E'_ϕ defined by the oscillating electric field and the radial component E'_ρ caused by the Lorentz transformation of the vertical magnetic field. The latter component has a resonance part because of the modulation of the particle velocity. Only this component has been taken into account in previous calculations [5, 6, 7]. The resonance effect is provided by both components of the electric field in the particle rest frame.

We use the particle rest frame for explaining the origin of the resonance. However, we do not utilize it for calculations and derive all basic equations in a cylindrical coordinate system. The particle spin motion in storage rings is usually specified with respect to the particle trajectory. Main fields are commonly defined relative to the cylindrical coordinate axes. When the ring is either circular or divided into circular sectors by empty spaces, the use of cylindrical coordinates considerably simplifies the analysis of spin effects [9]. The equation of spin motion in the cylindrical coordinate system coincides with that in the frame rotating together with the particle (rotating frame) because the horizontal axes of the cylindrical coordinate system rotate at the instantaneous angular frequency of orbital revolution. The motion of particles in the rotating frame is relatively slow because it can be caused only by oscillations and other deflections of the particles from the ideal trajectory. Therefore, the difference between the spin motion in the cylindrical coordinate system and the particle rest frame can be neglected in many cases.

The general equation of spin motion in the cylindrical coordinates is given by [9]

$$\begin{gathered}\frac{d\boldsymbol{S}}{dt} = \boldsymbol{\omega}_a \times \boldsymbol{S}, \\ \boldsymbol{\omega}_a = -\frac{e}{m}\left\{a\boldsymbol{B} - \frac{a\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) \right. \\ + \left(\frac{1}{\gamma^2-1} - a\right)(\boldsymbol{\beta}\times\boldsymbol{E}) + \frac{1}{\gamma}\left[\boldsymbol{B}_{\|} - \frac{1}{\beta^2}(\boldsymbol{\beta}\times\boldsymbol{E})_{\|}\right] \\ \left. +\frac{\eta}{2}\left(\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right)\right\}, \\ \boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}, \quad a = \frac{g-2}{2}, \end{gathered} \tag{1}$$

where $\boldsymbol{\omega}_a$ is the angular velocity of spin precession relative to axes of the cylindrical coordinate system.

The sign $\|$ means a horizontal projection for any vector. Fields defining perturbations of particle trajectory significantly affect the spin motion, while the explicit dependence of the quantity $\boldsymbol{\omega}_a$ on radial and vertical components of particle momentum conditioning these perturbations can usually be neglected (see Ref. [9]).

To measure the effect, some resonators (rf cavities) should be used. The electric field in a resonator is generated along the central line, and the magnetic field is orthogonally directed [10]. The magnetic field along the central line is equal to zero. If the rf cavities are perfectly placed and longitudinally directed, the magnetic field cannot stimulate any resonance effect. The resonant effect leading to the regular BVP is caused by the terms proportional to η. Therefore, the observed BVP corresponds to the definite value of the EDM. However, both a displacement and an angular deviation of the center line of the rf cavities away from an average particle trajectory lead to a similar behavior of spin imitating the EDM effect. As a result, they create systematic errors in the measurement of the EDM. Most of these errors are not in resonance with the spin precession in the horizontal plane. Therefore, they create background and result in fast oscillations of the vertical component of the polarization vector [5, 6, 7, 11]. Besides this effect, the systematic error can be caused by a radial magnetic field in the particle rest frame oscillating at the resonant frequency. This error will be eliminated by alternately producing two sub-beams with different betatron tunes [5, 7, 11]. In the present work, we calculate only the effects of resonant fields on the BVP in ideal conditions and disregard systematic errors.

Since the velocity oscillates, the vertical magnetic field creates the resonance part of the radial electric field in the particle rest frame. Thus, we need to take into consideration the constant vertical magnetic field and the oscillating longitudinal electric one in the lab frame. This is properly shown by Eq. (1). Resonant terms in the expression for the angular velocity of spin rotation are proportional to the EDM:

$$\boldsymbol{\Omega}_{EDM} = -\frac{e\eta}{2m}\left[\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{B}\right]. \tag{2}$$

Eq. (2) describes the interaction of the EDM with the electric field in the rotating frame in terms of the lab frame fields.

The polarization of the circulating deuteron beam can be best measured when colliding the deuterons with a carbon target and observing reaction products generated due to nuclear interactions [3, 12]. The polarization of the circulating proton beam can be determined by means of elastic proton-proton scattering (see Refs. [12, 13] and references therein).

III. RESONANCE STRENGTHS IN THE EDM EXPERIMENT

The longitudinal electric field acting on the deuterons in the resonator oscillates at the angular frequency ω which should be very close to the angular frequency of spin rotation (g−2 frequency), ω_0, and to the eigenfrequency of free synchrotron oscillations (synchrotron frequency) [7]. The quantity ω_0 is almost equal to the vertical component of $\boldsymbol{\omega}_a$, because other components of this (pseudo)vector are relatively small:

$$\omega_0 = (\omega_a)_z = -\frac{ea}{m}B_0. \tag{3}$$

In Eq. (3), B_0 is the average vertical magnetic field. We suppose the particle charge to be positive and the magnetic field to be upward ($B_0 > 0$). The cyclotron frequency is given by

$$\omega_c = -\frac{eB_0}{\gamma_0 m}, \tag{4}$$

where γ_0 is the average Lorentz factor. The minus sign means that the particle rotates clockwise ($\boldsymbol{\beta}\cdot\boldsymbol{e}_\phi < 0,\ \omega_c < 0$).

Because the oscillating electric field is longitudinally directed,

$$\boldsymbol{E}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E})=\frac{1}{\gamma}\boldsymbol{E}. \tag{5}$$

If we take into account only fields conditioning the electric field in the particle rest frame, the spin motion in the horizontal plane is defined by Eq. (2), where $\boldsymbol{\beta}=-\beta\boldsymbol{e}_\phi$.

The action of resonant electric and quasi-electric ($\boldsymbol{G}\equiv\boldsymbol{\beta}\times\boldsymbol{B}$) fields on the EDM is similar to that of resonant magnetic and quasi-magnetic ($-\boldsymbol{\beta}\times\boldsymbol{E}$) fields on the magnetic moment. Therefore, some previously obtained expressions for resonance strengths (see Refs. [14, 15, 16] and references therein) may be utilized when analyzing EDM-dependent interactions. For the EDM experiment, two regimes with large coherent oscillations are possible: a strongly linear regime (using, for example, a specially designed rf cavity for the linearization of oscillations), or a strongly nonlinear regime with well-stabilized coherent oscillations (see Ref. [7] and references therein). We confine ourselves to the consideration of linear oscillations.

It is convenient to use the quantity

$$\Phi=\phi(t)-\phi(0)=\omega_c t, \tag{6}$$

where ϕ is the azimuth of the particle at the given moment of time. The distance traversed by the beam is equal to

$$L_b=\frac{\beta c}{\omega_c}\Phi=\beta ct.$$

Similarly to the magnetic fields of a rf dipole and a rf solenoid [14, 15, 16], the longitudinal electric field of a rf cavity can be expressed in terms of delta functions:

$$\boldsymbol{E}=E_0\frac{L(\Phi)}{\rho}\sin{(\omega t+\varphi)}\boldsymbol{e}_\phi,\quad L(\Phi)=l\sum_{N=-\infty}^{\infty}\delta(\Phi-\Phi_0-2\pi N),\quad \rho=-\frac{\beta c}{\omega_c}, \tag{7}$$

where E_0,ω and φ are the amplitude, angular frequency and phase of electric field acting on the particle in the resonator, l and Φ_0 define the length and position (azimuth) of the resonator, and ρ is the radius of curvature. Eqs. (6),(7) result in

$$\boldsymbol{E}=\frac{E_0 l}{\rho}\sin{(\nu\Phi+\varphi)}\sum_{N=-\infty}^{\infty}\delta(\Phi-\Phi_0-2\pi N)\boldsymbol{e}_\phi, \tag{8}$$

where $\nu=\omega/\omega_c$.

The equation of particle motion is given by

$$\frac{d\boldsymbol{p}}{dt}=e\boldsymbol{E}. \tag{9}$$

To determine the particle momentum, one needs to integrate this equation. The integral of the delta function is the Heaviside function:

$$\int_{-\infty}^{\Phi}\delta(\phi-\Phi_0-2\pi N)d\phi=H(\Phi-\Phi_0-2\pi N),\quad H(x)=\begin{cases}0, & x<0\\ 1/2, & x=0\\ 1, & x>0\end{cases}.$$

Therefore, the momentum is described by the piecewise continuous function:

$$\boldsymbol{p}=\frac{eE_0 l}{\rho}\sum_{N=-\infty}^{\infty}\sin\left(\nu\Phi_N+\varphi\right)H(\Phi-\Phi_N)\boldsymbol{e}_\phi+\boldsymbol{C}, \tag{10}$$

where $\Phi_N=\Phi_0+2\pi N$ and $\boldsymbol{C}=const.$ The momentum is either minimum or maximum when $\sin\left(\nu\Phi_N+\varphi\right)\approx 0$. Its change defined by the height of a step is maximum when $\sin\left(\nu\Phi_N+\varphi\right)\approx\pm 1$. Therefore, the momentum is modulated by the cosine function $\cos\left(\nu\Phi+\varphi\right)$.

Spin dynamics described by Eqs. (1),(2) is determined by integration of angular velocity of spin motion. The evolution of vertical component of the spin is very slow and resonant fields affecting the spin in the particle rest frame remain almost constant during one revolution.

It would be very inconvenient to use simultaneously the delta and Heaviside functions. It is better to utilize the fact that function (10) can be replaced by another that integration brings the same result. Therefore, an appropriate delta function can be substituted for the sum of the Heaviside functions at every step of function (10). As a result, one obtains a series of delta functions modulated by $\cos\left(\nu\Phi+\varphi\right)$. The quantity

$$\boldsymbol{\pi}=-p_0\boldsymbol{e}_\phi+a\cos\left(\nu\Phi+\varphi\right)\sum_{N=-\infty}^{\infty}\delta(\Phi-\Phi_0-2\pi N)\boldsymbol{e}_\phi \tag{11}$$

may be therefore substituted for the momentum $\boldsymbol{p}$ because the integration of $\boldsymbol{\pi}$ and $\boldsymbol{p}$ leads to the same result. In Eq. (11), $\boldsymbol{p}_0$ is the average momentum and a is the constant that value is defined by the equation of particle motion:

$$\frac{d\boldsymbol{\pi}}{dt}=\omega_c\frac{d\boldsymbol{\pi}}{d\Phi}=e\boldsymbol{E}. \tag{12}$$

Eqs. (7),(11),(12) result in

$$\boldsymbol{\pi}=-\left[p_0+\Delta p_0\cos\left(\nu\Phi+\varphi\right)\right]\boldsymbol{e}_\phi,\quad \Delta p_0=\frac{eE_0L(\Phi)}{\omega\rho}. \tag{13}$$

We can perform calculations to within first-order terms in $\Delta\beta_0$ and neglect beam oscillations at nonresonance frequencies. The normalized velocity is given by

$$\boldsymbol{\beta} = \frac{\boldsymbol{p}}{\sqrt{m^2+\boldsymbol{p}^2}}.$$

In this equation, substitution of $\boldsymbol{\pi}$ for $\boldsymbol{p}$ does not change the result of integration of Eq. (2). Therefore, the normalized velocity can be replaced by the quantity

$$\boldsymbol{\beta}' = \frac{\boldsymbol{\pi}}{\sqrt{m^2+\boldsymbol{\pi}^2}} = -\left[\beta_0 + \Delta\beta_0' \cos\left(\nu\Phi+\varphi\right)\right]\boldsymbol{e}_\phi, \tag{14}$$

where

$$\beta_0 = \frac{p_0}{m\gamma_0}, \quad \gamma_0 = \frac{\sqrt{m^2+p_0^2}}{m}, \quad \Delta\beta_0' = \frac{\Delta p_0}{m\gamma_0^3} = \frac{eE_0L(\Phi)}{m\gamma_0^3\omega\rho}. \tag{15}$$

The primes will be omitted below.

Eqs. (1)–(15) result in

$$\frac{d\boldsymbol{S}}{d\Phi} = \boldsymbol{F}\times\boldsymbol{S}, \quad \boldsymbol{F} = F_1\boldsymbol{e}_\rho + F_2\boldsymbol{e}_\phi + F_3\boldsymbol{e}_z$$
$$= \frac{e\eta}{2p_0}E_0L(\Phi)\Big[\frac{\omega_0}{a\gamma_0^2\omega}\cos\left(\nu\Phi+\varphi\right)\boldsymbol{e}_\rho + \sin\left(\nu\Phi+\varphi\right)\boldsymbol{e}_\phi\Big] + a\gamma_0\boldsymbol{e}_z, \tag{16}$$

where $F_3 = a\gamma_0$ is the spin precession tune.

It is convenient to use the method of resonant strengths based on the Fourier-series expansion (see Refs. [17, 18, 19]):

$$F_1 - iF_2 = \sum_{K=-\infty}^{\infty}\epsilon_K e^{-i\nu_K\Phi}, \quad \nu_K = \omega_K/\omega_c. \tag{17}$$

In this equation, the resonance strengths ϵ_K are Fourier amplitudes corresponding to spin resonance tunes ν_K and ω_K are harmonic frequencies. In the considered case [14, 17]

$$\nu_K = K\pm\nu, \quad K = 0, \pm1, \pm2, \dots \tag{18}$$

The resonance strengths are defined by

$$\epsilon_K = \frac{1}{2\pi N}\int_{(N)}(F_1 - iF_2)e^{i\nu_K\Phi}d\Phi, \tag{19}$$

where an integration over the infinite number of turns $N\rightarrow\infty$ should be carried out.

Eqs. (7),(16),(18),(19) lead to the following expressions for the resonance strengths:

$$\epsilon_K^+ = \frac{e\eta}{8\pi p_0}E_0 l\left(\frac{\omega_0}{a\gamma_0^2\omega}+1\right)e^{i(K\Phi_0-\varphi)},$$
$$\epsilon_K^- = \frac{e\eta}{8\pi p_0}E_0 l\left(\frac{\omega_0}{a\gamma_0^2\omega}-1\right)e^{i(K\Phi_0+\varphi)}, \tag{20}$$

where ϵ_K^+ and ϵ_K^- correspond to the plus and minus signs in Eq. (18), respectively. Certainly, the dependence of the exponentials from K can be eliminated by appropriate choice of the initial phase ($\Phi_0 = 0$).

Eq. (20) can also be deduced by means of the Fourier expansion of delta functions (see Ref. [16]):

$$\sin{(\nu\Phi+\varphi)}\sum_{N=-\infty}^{\infty}\delta(\Phi-\Phi_0-2\pi N)=\frac{1}{2\pi}\sum_{N=-\infty}^{\infty}\sin{[\pm N(\Phi-\Phi_0)+\nu\Phi+\varphi]},$$
$$\cos{(\nu\Phi+\varphi)}\sum_{N=-\infty}^{\infty}\delta(\Phi-\Phi_0-2\pi N)=\frac{1}{2\pi}\sum_{N=-\infty}^{\infty}\cos{[\pm N(\Phi-\Phi_0)+\nu\Phi+\varphi]}. \qquad (21)$$

Eqs. (16) and (20) define the relationship between the quantities F_1, F_2 and the resonance strengths which agrees with similar relationships derived in Refs. [14, 15, 16] for localized rf magnetic fields. It is of importance because the results obtained in Refs. [14, 15] have been called in question [20, 21, 22].

The main difference between Eq. (20) and the corresponding equations for the resonance strengths deduced in Refs. [14, 15, 16] consists in the noncoincidence of expressions for ϵ_K^+ and ϵ_K^-. The noncoincidence is caused by a more complicated form of the spin precession vector $\boldsymbol{F}$ in the investigated case. Eq. (16) shows that this vector possesses nonzero projections onto two horizontal axes dephased by $\pi/2$.

The substitution of ϵ_K into Eq. (17) results in

$$\begin{aligned}Q=F_1-iF_2&=\sum_{K=-\infty}^{\infty}\left[\epsilon_K^+e^{-i(K+\nu)\Phi}+\epsilon_K^-e^{-i(K-\nu)\Phi}\right]\\&=\frac{e\eta}{8\pi p_0}E_0l\sum_{K=-\infty}^{\infty}\left\{\left(\frac{\omega_0}{a\gamma_0^2\omega}+1\right)e^{-i[K(\Phi-\Phi_0)+\nu\Phi+\varphi]}\right.\\&\qquad\left.+\left(\frac{\omega_0}{a\gamma_0^2\omega}-1\right)e^{-i[K(\Phi-\Phi_0)-\nu\Phi-\varphi]}\right\}.\end{aligned} \qquad (22)$$

Since F_1 and F_2 are real, they are given by

$$F_1=\mathrm{Re}(Q), \quad F_2=-\mathrm{Im}(Q). \qquad (23)$$

As a result, Eq. (16) takes the form

$$\frac{d\boldsymbol{S}}{d\Phi} = [\mathrm{Re}(Q)\boldsymbol{e}_\rho - \mathrm{Im}(Q)\boldsymbol{e}_\phi + a\gamma_0\boldsymbol{e}_z] \times \boldsymbol{S}$$

$$= \left(\sum_{K=-\infty}^{\infty}\left\{\mathrm{Re}\left[\epsilon_K^+ e^{-i(K+\nu)\Phi} + \epsilon_K^- e^{-i(K-\nu)\Phi}\right]\boldsymbol{e}_\rho\right.\right.$$

$$\left.\left.-\mathrm{Im}\left[\epsilon_K^+ e^{-i(K+\nu)\Phi} + \epsilon_K^- e^{-i(K-\nu)\Phi}\right]\boldsymbol{e}_\phi\right\} + a\gamma_0\boldsymbol{e}_z\right)\times\boldsymbol{S}. \tag{24}$$

Eqs. (20) and (24) completely define the spin dynamics in the EDM experiment.

The velocity modulation given by Eqs. (14),(15) can be expressed in terms of the resonance strengths. The use of Fourier expansion (21) brings Eq. (14) to the form

$$\boldsymbol{\beta} = -\left\{\beta_0 + \Delta\beta_m \sum_{K=-\infty}^{\infty} \cos\left[\pm K(\Phi - \Phi_0) + \nu\Phi + \varphi\right]\right\}\boldsymbol{e}_\phi, \tag{25}$$

where the amplitude of the velocity modulation is given by

$$\Delta\beta_m = -\frac{e\omega_0}{2\pi a\gamma_0^3 p_0\omega}E_0 l. \tag{26}$$

Eqs. (20),(25),(26) result in

$$\boldsymbol{\beta} = \left\{-\beta_0 + \frac{2}{\eta\gamma_0}\left[\left(1+\frac{a\gamma_0^2\omega}{\omega_0}\right)^{-1}\sum_{K=-\infty}^{\infty}\epsilon_K^+ e^{-i(K+\nu)\Phi}\right.\right.$$

$$\left.\left.+\left(1-\frac{a\gamma_0^2\omega}{\omega_0}\right)^{-1}\sum_{K=-\infty}^{\infty}\epsilon_K^- e^{-i(K-\nu)\Phi}\right]\right\}\boldsymbol{e}_\phi. \tag{27}$$

The relation between the resonance strengths and the amplitude of the velocity modulation has the form

$$\epsilon_K^+ = -\frac{\eta\gamma_0}{4}\left(1+\frac{a\gamma_0^2\omega}{\omega_0}\right)\Delta\beta_m e^{i(K\Phi_0-\varphi)},$$

$$\epsilon_K^- = -\frac{\eta\gamma_0}{4}\left(1-\frac{a\gamma_0^2\omega}{\omega_0}\right)\Delta\beta_m e^{i(K\Phi_0+\varphi)}. \tag{28}$$

IV. RESONANCE EFFECTS OF ELECTRIC AND MAGNETIC FIELDS

The spin rotates counterclockwise (clockwise) if ω_0 is positive (negative). The resonance field should rotate in the same direction and its frequency should be very close to the spin rotation frequency ($\omega_0 \approx \omega_c\nu_R$ and $a\gamma_0 \approx \nu_R$, where the index $K = R$ defines the resonance). The effect of nonresonance harmonics and backward-rotating fields on the spin motion vanishes on the average [8, 17]. Therefore, Eq. (24) contains nonresonant terms which can be excluded. The resulting equation takes the form

$$\frac{d\boldsymbol{S}}{d\Phi} = \boldsymbol{F} \times \boldsymbol{S}, \quad \boldsymbol{F} = \mathrm{Re}\left[\epsilon_R^{\pm} e^{-i(R\pm\nu)\Phi}\right]\boldsymbol{e}_\rho - \mathrm{Im}\left[\epsilon_R^{\pm} e^{-i(R\pm\nu)\Phi}\right]\boldsymbol{e}_\phi + a\gamma_0\boldsymbol{e}_z$$
$$= \frac{e\eta}{8\pi p_0}E_0 l\left(\frac{\omega_0}{a\gamma_0^2\omega} \pm 1\right)\boldsymbol{e}_{\|} + a\gamma_0\boldsymbol{e}_z,$$
$$\boldsymbol{e}_{\|} = \cos(\Psi)\boldsymbol{e}_\rho + \sin(\Psi)\boldsymbol{e}_\phi, \quad \Psi = R(\Phi - \Phi_0) \pm (\nu\Phi + \varphi), \tag{29}$$

where $\boldsymbol{e}_{\|}$ is the unit vector rotating in the horizontal plane with the tune $\nu_R = R \pm \nu$.

The first and second terms in the factor $\left(\frac{\omega_0}{a\gamma_0^2\omega} \pm 1\right)$ are conditioned by the magnetic and electric fields, respectively. The relative importance of electric field can be characterized by the ratio of contributions from the electric and magnetic fields:

$$k = \frac{a\gamma_0^2\omega}{\omega_0}. \tag{30}$$

The amplitude of resonant quasi-electric field is given by

$$G_0 = \frac{E_0}{k\gamma_0}. \tag{31}$$

Eqs. (29)–(31) are valid for any relation between ω_0 and ω.

Ratio (30) is very different for the proton and deuteron. The gyromagnetic anomaly of the proton is about 12.5 times larger than that of the deuteron. In addition, the deuteron's gyromagnetic anomaly is negative. The resonance at the frequency $\omega \approx \omega_0 = a\gamma_0\omega_c$ is the dominant harmonic for the deuteron. For this harmonic, $R = 0$. In the planned deuteron EDM experiment, the quantity k is equal to $k_d = -0.234$ for deuterons with momentum $p_0 = 1.5$ GeV/c ($\gamma_0 = 1.28$). The electric-field correction is significant because it results in decreasing the EDM effect for the deuteron by 23 percent. Although the contribution from the magnetic field to the BVP is dominant, taking into account this correction is necessary.

For the proton, both $a_p = 1.7928$ and $a_p\gamma_0$ are close to 2 and the angular frequency $\omega = \omega_0$ corresponding to the harmonic $R = 0$ is too high [23]. Therefore, it is useful to modulate the proton velocity at the different angular frequency (see Ref. [23]) $\omega \approx \omega_0 - 2\omega_c$ which conforms to the spin resonance tune $\nu_R = 2 + \nu$ (ν is positive and ω is negative).

A possible choice of proton's kinetic energy is $T = 161$ MeV [23]. This choice gives $\gamma_0 = 1.172$, $\nu = 0.1005$, $\omega/\omega_0 = \nu/(a\gamma_0) = 0.0478$. Ratio (30) is equal to $k_p = 0.118$.

Taking into account the electric-field correction leads to increasing the EDM effect for the proton by 12 percent. Although the contribution from the magnetic field is dominant, this correction is rather important. It should be included in any estimates.

V. RESONANT SPIN DYNAMICS

To calculate the resonant spin dynamics, the theory of magnetic resonance (see Ref. [8]) can be used. The spin is governed by the vertical magnetic field which rotates it in the horizontal plane and the resonant electric and quasi-electric fields. Other fields with spin resonance tunes ν_K that are far from the spin rotation tune $a\gamma_0$ can be disregarded. The resonant spin dynamics is defined by Eq. (29) and the vector $\boldsymbol{e}_{||}$ rotates with the tune $\nu_R \approx a\gamma_0$.

To describe the spin dynamics, it is convenient to use the frame accompanying the spin and rotating at the angular frequency $\omega_R = \omega_c\nu_R$ (see Ref. [8]) relatively to the cylindrical coordinate axes. The vertical component of the angular velocity of spin rotation is much less in this frame than in the cylindrical coordinate system. In the lab frame, the vector $\boldsymbol{e}_{||}$ rotates at an angular frequency which significantly differs from ω because of the rotation of the cylindrical coordinate axes in that frame.

If radial and longitudinal directions in the frame accompanying the spin coincide with those in the cylindrical coordinate system at zero time $t = 0$, they are defined by the unit vectors

$$\begin{aligned} \boldsymbol{e}'_\rho &= \cos{(\nu_R\Phi)}\boldsymbol{e}_\rho + \sin{(\nu_R\Phi)}\boldsymbol{e}_\phi, \\ \boldsymbol{e}'_\phi &= -\sin{(\nu_R\Phi)}\boldsymbol{e}_\rho + \cos{(\nu_R\Phi)}\boldsymbol{e}_\phi. \end{aligned} \qquad (32)$$

All quantities in the frame accompanying the spin are primed. The angular velocity of spin rotation is given by [8]

$$\boldsymbol{\Omega}' = \boldsymbol{\omega}_a - \omega_R\boldsymbol{e}_z = \omega_c\boldsymbol{F} - \omega_R\boldsymbol{e}_z, \qquad (33)$$

where $\boldsymbol{F}$ is defined by Eq. (29). The direction of vector $\boldsymbol{\Omega}'$ is fixed [8] because the unit vector $\boldsymbol{e}_{||}$ transforms to the form

$$\boldsymbol{e}'_{||} = \cos{(\zeta')}\boldsymbol{e}'_\rho + \sin{(\zeta')}\boldsymbol{e}'_\phi, \quad \zeta' = -R\Phi_0 \pm \varphi.$$

The spin rotation tune in the frame accompanying the spin is equal to

$$\nu' = \left|\frac{\boldsymbol{\Omega}'}{\omega_c}\right| = \sqrt{(a\gamma_0 - \nu_R^\pm)^2 + |\epsilon_R^\pm|^2}, \qquad (34)$$

where

$$\nu_R^+ = R + \nu, \quad \nu_R^- = R - \nu.$$

The initial beam polarization is supposed to be horizontal in the resonant EDM experiment. The particle spin dynamics depends on the direction of spin at zero time defined by the azimuth ψ. The azimuth $\psi = 0$ corresponds to the initial radial polarization in the lab frame. The BVP is characterized by the z-component of polarization vector.

Since trajectories of particles in the beam depend on their momenta, corresponding values of ω_0 would vary. However, some experimental techniques will keep the frequency and phase of the forced coherent longitudinal oscillations almost equal to the frequency and phase of the spin rotation. The length of the straight sections is chosen such that the momentum compaction factor

$$\alpha_p = \frac{\Delta p/p}{\Delta\mathcal{L}/\mathcal{L}} = 1,$$

where $\Delta p = p - p_0$, $\Delta\mathcal{L} = \mathcal{L} - \mathcal{L}_0$, $\mathcal{L} \equiv \mathcal{L}(p)$, $\mathcal{L}_0 \equiv \mathcal{L}(p_0)$, and $\mathcal{L}(p)$ is the length of the closed orbit for momentum p. In this case $p/\mathcal{L} = p_0/\mathcal{L}_0$. Since $p = m\gamma(p)\omega_c(p)\rho(p)$ and $\rho/\mathcal{L} = \rho_0/\mathcal{L}_0$, the g$-$2 frequency does not depend on the particle momentum: $a\gamma(p)\omega_c(p) = a\gamma(p_0)\omega_c(p_0)$ and $\omega_c(p) = \omega_c(p_0)$ [7].

Since, as the quantities ω_R and ω_0 can slightly differ and one needs to determine a systematical error caused by the nonzero difference $\omega_0 - \omega_R$, it is necessary to use general formulae specifying the spin dynamics. Hereafter the light velocity, c, will be explicitly shown.

The dynamics of the vertical component of polarization vector is the same in the rotating and lab frames. It is given by

$$\begin{gathered} P_z(\Phi) = \frac{\epsilon'}{\nu'} P_0 \left\{ \sin(\psi - \zeta') \sin(\nu'\Phi) + \frac{a\gamma_0 - \nu_R}{\nu'} \cos(\psi - \zeta') \left[1 - \cos(\nu'\Phi)\right] \right\}, \\ \epsilon' = \frac{e\eta}{8\pi c p_0} E_0 l \left(\frac{\omega_0}{a\gamma_0^2\omega} \pm 1 \right), \end{gathered} \quad (35)$$

where ϵ' is the amplitude of the resonance strength ($|\epsilon'| = |\epsilon_R^\pm|$) and P_0 is the initial beam polarization.

Eqs. (29),(33) define the evolution of other components of polarization vector:

$$P_\rho(\Phi) = P_0\left\{\cos(\nu'\Phi)\cos(\nu_R\Phi+\psi) + \frac{\epsilon'^2}{\nu'^2}\cos(\psi-\zeta')\left[1-\cos(\nu'\Phi)\right]\cos(\nu_R\Phi+\zeta') \right.$$
$$\left. -\frac{a\gamma_0-\nu_R}{\nu'}\sin(\nu'\Phi)\sin(\nu_R\Phi+\psi)\right\},$$
$$P_\phi(\Phi) = P_0\left\{\cos(\nu'\Phi)\sin(\nu_R\Phi+\psi) + \frac{\epsilon'^2}{\nu'^2}\cos(\psi-\zeta')\left[1-\cos(\nu'\Phi)\right]\sin(\nu_R\Phi+\zeta') \right.$$
$$\left. +\frac{a\gamma_0-\nu_R}{\nu'}\sin(\nu'\Phi)\cos(\nu_R\Phi+\psi)\right\}. \quad (36)$$

It is important that the substitution $\nu' \to -\nu'$ does not change the values of the components of polarization vector and therefore the quantities ν' and $\nu'\Phi$ can be replaced by Ω'/ω_c and $\Omega' t$, respectively.

When $\nu'\Phi \ll 1$ $(\Omega' t \ll 1)$,

$$P_\rho(\Phi) = P_0\left[\cos(\nu_R\Phi+\psi) - (a\gamma_0-\nu_R)\Phi\sin(\nu_R\Phi+\psi)\right],$$
$$P_\phi(\Phi) = P_0\left[\sin(\nu_R\Phi+\psi) + (a\gamma_0-\nu_R)\Phi\cos(\nu_R\Phi+\psi)\right], \quad (37)$$

$$P_z = P_0\epsilon'\Phi\sin(\psi-\zeta')$$
$$= \frac{e\eta}{8\pi c p_0}P_0E_0 l\left(\frac{\omega_0}{a\gamma_0^2\omega}\pm 1\right)\omega_c t\sin(\psi-\zeta'). \quad (38)$$

For the deuteron EDM experiment, $\omega \approx \omega_0,\ \zeta' = \varphi$, and the plus sign should be chosen in Eq. (38). When the angle $\Upsilon = \psi - \pi/2$ characterizing an initial spin direction about the $\boldsymbol{e}_\phi$ axis is used,

$$\sin(\psi-\zeta') = \cos(\Upsilon-\zeta') = \cos(\Upsilon-\varphi)$$

and Eq. (38) takes the form

$$P_z = \frac{e\eta}{8\pi c p_0}P_0E_0 l\left(\frac{1}{a\gamma_0^2}+1\right)\omega_c t\cos(\Upsilon-\varphi)$$
$$= \frac{e\eta}{8\pi\beta_0 c^2 p_0}P_0E_0 l\left(\frac{1}{a\gamma_0^2}+1\right)\omega_c L_b\cos(\Upsilon-\varphi). \quad (39)$$

Eqs. (26),(39) result in

$$P_z = -\frac{1}{4}\eta P_0\Delta\beta_m\gamma_0\left(1+a\gamma_0^2\right)\omega_c t\cos(\Upsilon-\varphi). \quad (40)$$

The corresponding formula obtained in Refs. [5, 7] is given by

$$P_z = \frac{1}{4}\eta P_0\Delta\beta_m\gamma_0\omega_c t, \quad (41)$$

where the designations accepted in the present work are used. Only the special case of $\omega_0 = \omega,\ \Psi = \varphi$ has been considered in Refs. [5, 7].

Eq. (41) differs from Eq. (40) by the absence of the factor $(1 + a\gamma_0^2)$. It is quite natural because the effect of electric field on the spin dynamics has not been taken into account in Refs. [5, 7]. The formula obtained in Refs. [5, 7] should be added by the above mentioned factor. The signs in Eqs. (40) and (41) are opposite because the quantity ω_c was supposed to be positive in Refs. [5, 7].

VI. CONTRIBUTIONS FROM ELECTRIC AND MAGNETIC FIELDS IN THE EXPERIMENT BASED ON THE FROZEN SPIN METHOD

It is of interest to compare ratio (30) with the corresponding ratio for the EDM experiment based on the frozen spin method.

The radial electric field in this experiment is adjusted to [2]

$$E = \frac{a\beta\gamma^2}{1 - a\beta^2\gamma^2} B.$$

The ratio of contributions from the electric and magnetic fields to the BVP caused by the EDM is

$$\kappa = \frac{E}{|\boldsymbol{\beta} \times \boldsymbol{B}|} = \frac{a\gamma^2}{1 - a\beta^2\gamma^2}. \qquad (42)$$

For the deuteron experiment [3], $a = a_d = -0.14299,\ \beta = 0.35,\ \gamma = 1.068$, and $\kappa = 0.17$. Therefore, the effect of the electric field on the BVP cannot be neglected in the deuteron EDM experiment based on the frozen spin method. The contribution from the electric field is negligible in a similar muon experiment ($a_\mu = 1.1659 \times 10^{-3}$) and predominant in a proton one ($a_p = 1.7928$).

VII. SUMMARY

The BVP in the resonant EDM experiment is affected by the electric and magnetic fields. The effect of the resonant electric field is significant, while the contribution from the magnetic field proportional to the oscillating part of particle velocity is dominant. The effective fields defining the resonant effect have been expressed in terms of the resonance strengths. The spin dynamics in the resonant deuteron EDM experiment has been calculated in the general case. In previous works, only a special case has been considered and the effect of the electric field on the spin dynamics has not been taken into account. The electric-field correction is important because it leads to decreasing the EDM effect by 23 percent for the deuteron experiment and increasing it by 12 percent for the proton one.

Acknowledgements

The author would like to thank Y.K. Semertzidis for helpful discussions and acknowledge a financial support by BRFFR (grant No. Φ06P-074).

[1] E. Accomando, R. Arnowitt, B. Dutta, Phys. Rev. D **61**, 115003 (2000); K. S. Babu, B. Dutta, R. N. Mohapatra, Phys. Rev. Lett. **85**, 5064 (2000); O. Lebedev, K. A. Olive, M. Pospelov, A. Ritz, Phys. Rev. D **70**, 016003 (2004).

[2] F. J. M. Farley, K. Jungmann, J. P. Miller, W. M. Morse, Y. F. Orlov, B. L. Roberts, Y. K. Semertzidis, A. Silenko, and E. J. Stephenson, Phys. Rev. Lett. **93**, 052001 (2004).

[3] EDM Collaboration (M. Aoki *et al.*), *Deuteron EDM Proposal: Search for a Permanent Deuteron Electric Dipole Moment at the* 10^{-27} *e·cm Level*, http://www.bnl.gov/edm/deuteron_proposal_040816.pdf.

[4] D. F. Nelson, A. A. Schupp, R. W. Pidd, and H. R. Crane, Phys. Rev. Lett. **2**, 492 (1959).

[5] Y. F. Orlov, EDM in Storage Rings Internal Note No. 69, 2004 (unpublished); STORI 2005 Conference Proceedings, Schriften des Forschungszentrums Jülich, Matter and Materials, Vol. 30 (2005), pp. 223–226.

[6] Y. K. Semertzidis, STORI 2005 Conference Proceedings, Schriften des Forschungszentrums Jülich, Matter and Materials, Vol. 30 (2005), pp. 70–77.

[7] Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006).

[8] C. P. Slichter, *Principles of Magnetic Resonance: With Examples from Solid State Physics* (Harper and Row, New York, 1963); *Principles of Magnetic Resonance*, 3rd ed. (Springer-Verlag, New York, 1990).

[9] A. J. Silenko, Phys. Rev. ST Accel. Beams **9**, 034003 (2006).

[10] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975), p. 355.

[11] Y. K. Semertzidis, EDM in Storage Rings Internal Notes Nos. 82, 85, and 92, 2005 (unpublished).

[12] D. Chiladze *et al.*, Phys. Rev. ST Accel. Beams **9**, 050101 (2006).

[13] M. Altmeier *et al.*, Eur. Phys. J. A **23**, 351 (2005).

[14] T. Roser, in *Handbook of Accelerator Physics and Engineering*, edited by A.W. Chao and M. Tigner (World Scientific, Singapore, 1999), p. 151.

[15] M. Bai, W. W. MacKay, and T. Roser, Phys. Rev. ST Accel. Beams **8**, 099001 (2005).

[16] S. Y. Lee, Phys. Rev. ST Accel. Beams **9**, 074001 (2006).

[17] E. D. Courant, Bull. Am. Phys. Soc., **7**, 33 (1962); E. D. Courant and R. D. Ruth, BNL Report No. 51270, 1980 (unpublished).

[18] S.Y. Lee, *Spin Dynamics and Snakes in Synchrotrons* (World Scientific, Singapore, 1997).

[19] M. G. Minty and F. Zimmermann, *Measurement and Control of Charged Particle Beams* (Springer-Verlag, Berlin, 2003), Chap. 10.

[20] V. S. Morozov *et al.*, Phys. Rev. ST Accel. Beams **7**, 024002 (2004).

[21] V. S. Morozov *et al.*, Phys. Rev. ST Accel. Beams **8**, 099002 (2005).

[22] M. A. Leonova *et al.*, Phys. Rev. ST Accel. Beams **9**, 051001 (2006).

[23] E. Stephenson, EDM in Storage Rings Internal Note No. 78, 2005 (unpublished).