

Potential for measurement of the tensor polarizabilities of nuclei in storage rings by the frozen spin method

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Abstract

The frozen spin method can be effectively used for a high-precision measurement of the tensor electric and magnetic polarizabilities of the deuteron and other nuclei in storage rings. For the deuteron, this method would provide the determination of the deuteron's polarizabilities with absolute precision of order of 10^{-43} cm³.

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I. INTRODUCTION

Tensor electric and magnetic polarizabilities are important properties of the deuteron and other nuclei defined by spin interactions of nucleons. Their measurement provides a good possibility to examine the theory of spin-dependent nuclear forces. Methods for determining these important electromagnetic properties of the deuteron based on the appearance of interactions quadratic in the spin have been proposed by V. Baryshevsky and co-workers [1, 2, 3]. Additional investigations have been performed in Refs. [4, 5].

Interactions quadratic in the spin and proportional to the tensor electric and magnetic polarizabilities affect spin dynamics. When an electric field in the particle rest frame oscillates at the resonant frequency, an effect similar to the nuclear magnetic resonance takes place. This effect stimulates the buildup of the vertical polarization (BVP) of the deuteron beam [1, 2, 3]. General formulas describing the BVP caused by the tensor electric polarizability of the deuteron in storage rings (the Baryshevsky effect) have been derived in Ref. [4]. The problem of influence of the tensor electric polarizability on spin dynamics in such a deuteron electric-dipole-moment experiment in storage rings has been investigated [4]. It has been proved that doubling the resonant frequency used in this experiment dramatically amplifies the Baryshevsky effect and provides the opportunity to make high-precision measurements of the deuterons tensor electric polarizability [4].

The tensor magnetic polarizability, β_T , produces the spin rotation with two frequencies instead of one, beating with a frequency proportional to β_T , and causes transitions between vector and tensor polarizations [2, 3]. In Ref. [5], the existence of these effects has been confirmed and a detailed calculation of deuteron spin dynamics in storage rings has been carried out. The use of the matrix Hamiltonian derived in Ref. [4] is very helpful for calculating general formulas describing the evolution of the spin. Significant improvement in the precision of possible experiments can be achieved if initial deuteron beams are tensor-polarized [4, 5].

The frozen spin method [6, 7] provides another possibility to measure the tensor polarizabilities of the deuteron and other nuclei. This method remains the spin orientation relatively the momentum direction almost unchanged. In the present work, we also analyze additional advantages ensured by the use of tensor-polarized beams and compute the related spin evolution.

The system of units $\hbar = c = 1$ is used.

II. GENERAL EQUATIONS

The traditional quantum mechanical approach (see Ref. [8]) uses the matrix Hamiltonian equation and the matrix Hamiltonian H for determining an evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = H\Psi, \quad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}. \quad (1)$$

The matrix Hamiltonian H coincides with the Hamilton operator \mathcal{H} expressed in matrix form. This coincidence results from the fact that the Hamilton operator considered in Refs. [4, 5] is independent of coordinates.

Three-component wave function Ψ which is similar to a spinor consists of the amplitudes $C_i(t)$ characterizing states with definite spin projections onto the preferential direction (z axis). Correction to the Hamilton operator caused by the tensor polarizabilities has the form [4]

$$V = -\frac{\alpha_T}{\gamma}(\mathbf{S} \cdot \mathbf{E}')^2 - \frac{\beta_T}{\gamma}(\mathbf{S} \cdot \mathbf{B}')^2, \quad (2)$$

where α_T is the tensor electric polarizability, γ is the Lorentz factor, and \mathbf{E}' and \mathbf{B}' are the electric and magnetic fields in the rest frame of the deuteron.

The use of the rotating frame is very helpful. The particle in this frame is localized and ideally is at rest. Therefore, we can direct the x and y axes in this frame along the radial and longitudinal axes, respectively. This procedure is commonly used (see Ref. [4] and references therein) and results in the simplest forms of spin matrices:

$$S_\rho = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_\phi = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (3)$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The Hamiltonian operator is defined by [4]

$$\mathcal{H} = \mathcal{H}_0 + \mathbf{S} \cdot \boldsymbol{\omega}_a + V, \quad (4)$$

where $\boldsymbol{\omega}_a$ is the angular velocity of the spin precession relatively to the momentum direction (g-2 precession).

In the considered case, the expression of \mathbf{E}' and \mathbf{B}' in terms of the lab fields which are unprimed has the form

$$\mathbf{E}' = \gamma(E_\rho + \beta_\phi B_z) \mathbf{e}_\rho, \quad \mathbf{B}' = \gamma(\beta_\phi E_\rho + B_z) \mathbf{e}_z, \quad (5)$$

where $\beta_\phi = \boldsymbol{\beta} \cdot \mathbf{e}_\phi \equiv \mathbf{v} \cdot \mathbf{e}_\phi / c$.

When the frozen spin method is used, the quantity $\boldsymbol{\omega}_a$ is very small and the fields satisfy the following relation [7, 9]:

$$E_\rho = \frac{a\beta_\phi\gamma^2}{1 - a\beta^2\gamma^2} B_z. \quad (6)$$

For the deuteron, $a \equiv (g - 2)/2 = -0.143$. Therefore,

$$V = -\frac{\gamma B_z^2}{(1 - a\beta^2\gamma^2)^2} [\alpha_T(1 + a)^2\beta^2 S_\rho^2 + \beta_T S_z^2]. \quad (7)$$

The matrix Hamiltonian has the form [4]

$$H = \begin{pmatrix} E_0 + \omega_0 + \mathcal{A} + \mathcal{B} & 0 & \mathcal{A} \\ 0 & E_0 + 2\mathcal{A} & 0 \\ \mathcal{A} & 0 & E_0 - \omega_0 + \mathcal{A} + \mathcal{B} \end{pmatrix}, \quad (8)$$

where E_0 is the zero energy level, $\omega_0 = (\omega_a)_z$,

$$\mathcal{A} = -\alpha_T \frac{(1 + a)^2 \beta^2 \gamma B_z^2}{2(1 - a\beta^2\gamma^2)^2}, \quad \mathcal{B} = -\beta_T \frac{\gamma B_z^2}{(1 - a\beta^2\gamma^2)^2}. \quad (9)$$

Eqs. (8),(9) are basic equations defining dynamics of the deuteron spin in storage rings when the frozen spin method is used.

III. EVOLUTION OF VECTOR POLARIZATION OF DEUTERON BEAM

In Ref. [5], off-diagonal components of Hamiltonian (9) was not taken into account, because their effect on the rotating spin did not satisfy the resonance condition. These

components cannot, however, be neglected in the considered case because the resonant frequency ω_0 can be very small.

The best conditions for a measurement of the tensor polarizabilities of the deuteron and other nuclei can be achieved with the use of tensor-polarized initial beams. In this case, we may confine ourselves to the consideration of a zero projection of the deuteron spin onto the preferential direction. When this direction is defined by the spherical angles θ and ψ [10], the initial polarization is given by

$$\begin{aligned} \mathbf{P}(0) &= 0, \quad P_{\rho\rho}(0) = 1 - 3 \sin^2 \theta \cos^2 \psi, \\ P_{\phi\phi}(0) &= 1 - 3 \sin^2 \theta \sin^2 \psi, \quad P_{zz}(0) = 1 - 3 \cos^2 \theta, \\ P_{\rho\phi}(0) &= -\frac{3}{2} \sin^2 \theta \sin(2\psi), \\ P_{\rho z}(0) &= -\frac{3}{2} \sin(2\theta) \cos \psi, \quad P_{\phi z}(0) = -\frac{3}{2} \sin(2\theta) \sin \psi. \end{aligned} \quad (10)$$

In this case, the general equation describing the evolution of the polarization vector has the form

$$\begin{aligned} P_\rho(t) &= \sin(2\theta) \left\{ \left[\cos(\omega't) \sin \psi \right. \right. \\ &+ \left. \left. \frac{\omega_0}{\omega'} \sin(\omega't) \cos \psi \right] \sin(bt) + \frac{\mathcal{A}}{\omega'} \sin(\omega't) \cos(bt) \sin \psi \right\}, \\ P_\phi(t) &= \sin(2\theta) \left\{ \left[-\cos(\omega't) \cos \psi \right. \right. \\ &+ \left. \left. \frac{\omega_0}{\omega'} \sin(\omega't) \sin \psi \right] \sin(bt) + \frac{\mathcal{A}}{\omega'} \sin(\omega't) \cos(bt) \cos \psi \right\}, \\ P_z(t) &= -\frac{2\mathcal{A}}{\omega'} \sin^2 \theta \sin(\omega't) \left[\cos(\omega't) \sin(2\psi) \right. \\ &+ \left. \frac{\omega_0}{\omega'} \sin(\omega't) \cos(2\psi) \right], \end{aligned} \quad (11)$$

where

$$\omega' = \sqrt{\omega_0^2 + \mathcal{A}^2}, \quad b = \mathcal{B} - \mathcal{A}. \quad (12)$$

When the frozen spin method is used,

$$b = -\frac{\gamma B_z^2}{(1 - a\beta^2\gamma^2)^2} \left[\beta_T - \frac{1}{2} \alpha_T (1 + a)^2 \beta^2 \right]. \quad (13)$$

As a rule, we can neglect \mathcal{A}^2 as compared with ω_0^2 and use the approximation $bt \ll 1$. In this case,

$$\begin{aligned} P_\rho(t) &= \sin(2\theta) \left[bt \sin(\omega_0 t + \psi) + \frac{\mathcal{A}}{\omega_0} \sin(\omega_0 t) \sin \psi \right], \\ P_\phi(t) &= \sin(2\theta) \left[-bt \cos(\omega_0 t + \psi) + \frac{\mathcal{A}}{\omega_0} \sin(\omega_0 t) \cos \psi \right], \\ P_z(t) &= -\frac{2\mathcal{A}}{\omega_0} \sin^2 \theta \sin(\omega_0 t) \sin(\omega_0 t + 2\psi). \end{aligned} \quad (14)$$

When the initial deuteron beam is vector-polarized and the direction of its polarization is defined by the spherical angles θ and ψ ,

$$\begin{aligned}
P_\rho(0) &= \sin \theta \cos \psi, & P_\phi(0) &= \sin \theta \sin \psi, \\
P_z(0) &= \cos \theta, & P_{\rho\rho}(0) &= \frac{1}{2} (3 \sin^2 \theta \cos^2 \psi - 1), \\
P_{\phi\phi}(0) &= \frac{1}{2} (3 \sin^2 \theta \sin^2 \psi - 1), \\
P_{\rho\phi}(0) &= \frac{3}{4} \sin^2 \theta \sin (2\psi).
\end{aligned}
\tag{15}$$

Such a polarization (with $\theta = \pi/2$) will be used in the planned deuteron electric-dipole-moment (EDM) experiment [11]. The EDM manifests in an appearance of a vertical component of the polarization vector.

The evolution of this component defined by the tensor polarizabilities of the deuteron is given by

$$\begin{aligned}
P_z(t) &= \left[1 - \frac{2\mathcal{A}^2}{\omega'^2} \sin^2(\omega't) \right] \cos \theta \\
&+ \frac{\mathcal{A}}{\omega'} \sin^2 \theta \sin(\omega't) \left[\cos(\omega't) \sin(2\psi) \right. \\
&\quad \left. + \frac{\omega_0}{\omega'} \sin(\omega't) \cos(2\psi) \right].
\end{aligned}
\tag{16}$$

The tensor magnetic polarizability does not influence on P_z .

In the same approximation as before,

$$P_z(t) = \cos \theta + \frac{\mathcal{A}}{\omega_0} \sin^2 \theta \sin(\omega_0 t) \sin(\omega_0 t + 2\psi).
\tag{17}$$

IV. DISCUSSION AND SUMMARY

Experimental conditions needed for the measurement of the tensor polarizabilities and the EDMs of nuclei in storage rings [7, 11] are similar. Eq. (6) shows that the radial electric field should be sufficiently strong in order to eliminate the effect of the vertical magnetic field on the spin. As a result, the frozen spin method provides a weaker magnetic field than other methods. This factor is negative because the evolution of the spin caused by both the tensor polarizabilities and the EDMs strongly depends on B_z . Nevertheless, the Storage Ring EDM collaboration considers the frozen spin method to be capable to detect the deuteron EDM of order of $10^{-29} e \cdot \text{cm}$. Another method for searching for the deuteron EDM in storage rings is the resonance method developed in Ref. [12]. This method is based on a strong vertical magnetic field and an oscillatory resonant longitudinal electric field.

The use of the resonance method for the measurement of the tensor electric polarizability has been proposed in Refs. [1, 2, 3]. This method may provide higher sensitivity than the frozen spin method (see Ref. [4] for comparing the results). However, the realization of the resonance method seems to be more difficult. Evidently, a strong restriction of the spin rotation considerably simplifies a detection of weak spin-dependent effects. In addition, this restriction facilitates adjusting a storage ring lattice which is essentially based on monitoring a behavior of the spin [7, 11, 12].

We can evaluate a precision of measurement of the tensor polarizabilities of the deuteron via its comparison with the expected sensitivity of the deuteron EDM experiment.

Evidently, the tensor electric polarizability can in principle imitate the presence of the EDM. The exact equation of spin motion in storage rings with allowance for EDMs of nuclei has been derived in Ref. [9]. In the considered case, the angular velocity of spin rotation is equal to

$$\boldsymbol{\omega}_a = \omega_0 \mathbf{e}_z + \mathcal{C} \mathbf{e}_\rho, \quad \mathcal{C} = -\frac{e\eta}{2m} \cdot \frac{1+a}{1-a\beta^2\gamma^2} \beta_\phi B_z, \quad (18)$$

where $\eta = 2dm/(eS)$ is the factor similar to g factor for the magnetic moment. d is the EDM.

When the tensor polarizabilities are not taken into account, the spin rotates about the direction

$$\mathbf{e}'_z = \frac{\mathcal{C}}{\omega'} \mathbf{e}_\rho + \frac{\omega_0}{\omega'} \mathbf{e}_z$$

with the angular frequency $\omega' = \sqrt{\omega_0^2 + \mathcal{C}^2}$.

When the initial polarization of the beam is given by Eq. (15), the polarization vector is equal to

$$\begin{aligned} P_\rho(t) &= \frac{\omega_0 \mathcal{C}}{\omega'^2} [1 - \cos(\omega' t)] \cos \theta \\ &+ \left[1 - \frac{2\omega_0^2}{\omega'^2} \sin^2 \frac{\omega' t}{2} \right] \sin \theta \cos \psi - \frac{\omega_0}{\omega'} \sin(\omega' t) \sin \theta \sin \psi, \\ P_\phi(t) &= \sin(\omega' t) \left(\frac{\omega_0}{\omega'} \sin \theta \cos \psi - \frac{\mathcal{C}}{\omega'} \cos \theta \right) \\ &+ \cos(\omega' t) \sin \theta \sin \psi, \\ P_z(t) &= \left[1 - \frac{2\mathcal{C}^2}{\omega'^2} \sin^2 \frac{\omega' t}{2} \right] \cos \theta \\ &+ \frac{\omega_0 \mathcal{C}}{\omega'^2} [1 - \cos(\omega' t)] \sin \theta \cos \psi + \frac{\mathcal{C}}{\omega'} \sin(\omega' t) \sin \theta \sin \psi. \end{aligned} \quad (19)$$

If we neglect terms of order of \mathcal{C}^2 , the vertical component of the polarization vector takes the form

$$P_z(t) = \cos \theta + \frac{2\mathcal{C}}{\omega_0} \sin \theta \sin \frac{\omega_0 t}{2} \sin \frac{\omega_0 t + 2\psi}{2}. \quad (20)$$

While Eqs. (17) and (20) are similar, the effects of the tensor electric polarizability and the EDM have different angular dependencies and can be properly separated.

For the considered experimental conditions [11], the sensitivity to the EDM of $1 \times 10^{-29} e \cdot \text{cm}$ corresponds to measuring the tensor electric polarizability with the accuracy $\delta\alpha_T \approx 5 \times 10^{-42} \text{ cm}^3$.

There are three independent theoretical predictions for the value of the tensor electric polarizability of the deuteron, namely $\alpha_T = -6.2 \times 10^{-41} \text{ cm}^3$ [13], $-6.8 \times 10^{-41} \text{ cm}^3$ [14], and $3.2 \times 10^{-41} \text{ cm}^3$ [15]. The first two values are very close to each other but they do not agree with the last result. The theoretical estimate for the tensor magnetic polarizability of deuteron is $\beta_T = 1.95 \times 10^{-40} \text{ cm}^3$ [13, 14].

We can therefore conclude that the expected sensitivity of the deuteron EDM experiment allows to measure the tensor electric polarizability with absolute precision $\delta\alpha_T \approx 5 \times 10^{-42} \text{ cm}^3$ which corresponds to the relative precision of order of 10^{-1} . This estimate is made for the vector-polarized initial beam. However, the best sensitivity in the measurement of α_T can be achieved with the use of a tensor-polarized initial beam. When the vector polarization of such a beam is zero, any spin rotation does not occur. In this case, there are no related systematic errors caused by the radial magnetic field and some other reasons. In the general case, such systematic errors are proportional to a residual vector polarization of the beam. This advantage leads to a sufficient increase in experimental accuracy [4, 5]. In this case, our preliminary estimate of experimental accuracy is $\delta\alpha_T \sim 10^{-43} \text{ cm}^3$.

The frozen spin method can also be successively used for the measurement of the tensor magnetic polarizability. Eqs. (11)–(14) show that the preferential direction of initial tensor polarization is defined by $\theta = \pi/2$ and $\theta = \pi/4$ for measuring the tensor electric and magnetic polarizabilities, respectively. In the latter case, the horizontal components of the polarization vector should be measured. Due to a restriction of spin rotation in the horizontal plane, the achievable absolute precision of measurement of the tensor magnetic polarizability of the deuteron is of the same order ($\delta\beta_T \sim 10^{-43} \text{ cm}^3$). A comparison with the theoretical estimate [13, 14] shows that the relative precision of measurement of this quantity can be rather high ($\delta\beta_T/\beta_T \sim 10^{-3}$).

All above derived formulas are applicable to any spin-1 nuclei. Moreover, the evolution of the polarization vector defined by spin tensor effects has to be identical for nuclei with any spin $S \geq 1$ despite difference of spin matrices. This statement follows from the fact that quantum mechanical equations describing spin dynamics should agree with classical spin physics and therefore should not explicitly depend on S .

Thus, the frozen spin method can be effectively used for the high-precision determination of the tensor electric and magnetic polarizabilities of the deuteron and other nuclei.

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