Nonlinear Analysis of Quasi-Cherenkov Electron Beam Instability in VFEL (Volume Free Electron Laser)

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(Received 17 January 2005)

Analysis of nonstationary stage of quasi-Cherenkov instability in different VFEL (Volume Free Electron Laser) schemes is carried out. Results of numerical experiments are discussed. Typical bifurcational behavior of the solution at varying physical system parameters is obtained. Common system for modelling of different VFEL with multiwave volume distributed feedback is proposed.

Key words: Volume Free Electron Laser, quasi-Cherenkov instability, numerical modelling, nonlinear integro-differential system, bifurcations

PACS numbers: 41.60.Cr; 35Q60, 35F20, 65M06

1 Introduction

This paper is devoted to the analysis of nonstationary dynamics of electron beam quasi-Cherenkov instability in different realizations of VFEL.

The interest to Free Electron Lasers (FEL) that allows to obtain generation of high intensity and spectral brightness arose more than twenty years ago. Wavelength range for working or projected FEL extends at present time from centimeter to X-ray. VFEL with Volume Distributed Feedback (VDFB) is one of the most promising direction of FEL development. Main useful VFEL feature is effective electromagnetic wave-electron beam interaction in large volume when transverse sizes of electron beam essentially exceed radiation wavelength. This possibility is realized due to Bragg dynamical diffraction which selects modes in large interaction volume. As a result, high output power may be produced in such a system. Next important VFEL property is frequency tuning in wide spectral range. Conception of VFEL based on parametric (quasi-Cherenkov) beam instability was proposed firstly in [1] for generation in X-ray range. Then it was developed in [2]-[8] for other spectral ranges. First lasing of VFEL in millimeter range based on principal ideas referred above was recently obtained by the group from Institute for Nuclear Problems [9]. Experimental work on VFEL goes on [10]-[11]. Investigations on VFEL lead to creation of fundamentally new direction in the domain of high technology. In the future VFEL can be adopted for thermonuclear plasma heating, radiolocation, energy transfer on long distance etc.

Linear regime of VFEL operation was investigated in [1]-[8] and other works. This stage of generation is limited by quite short time period (10^{-8}–10^{-9} s and less) depending on physical parameters. And most part of energy is extracted from electron beam during nonlinear stage. Analysis of this stage requires severe numerical simulations which based on a system of multidimensional first-order nonlinear partial differential equations (PDEs). Boundary conditions can be written at different boundaries and can be PDEs too. Therefore we have to use numerical methods for solving such type of differential problem. Equations to be solved are hyperbolic or of mixed type with nonlinearities holding integrals in right-hand sides. We called them as generalized transfer equations [12]. In our works [13]-[18] we considered mathematical models of different types of VFEL. Details of simulation of quasi-Cherenkov VFEL in millimeter, optical and X-ray region and proposal of common system of PDEs for modelling of different types of VFEL with multiwave VDFB are the goal of the work presented.

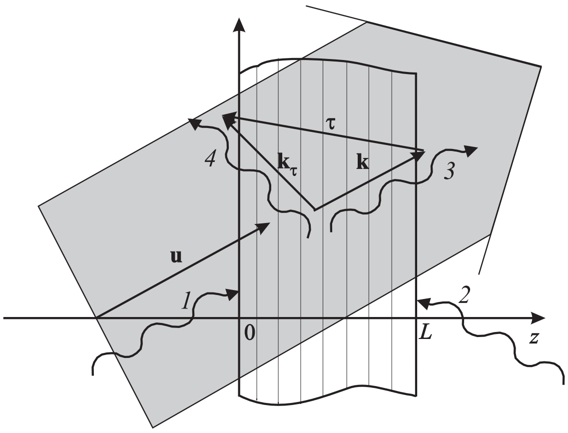

FIG. 1. Simple scheme of quasi-Cherenkov VFEL in Bragg geometry

2 Mathematical model of volume scheme of VFEL

In preceding works [17]-[18] simple scheme of VFEL (see Fig. 1) in millimeter region was considered. The same scheme is suitable for X-ray VFEL. An electron beam with initial electron velocity $\mathbf{u}$ and density n_b passes through the target. This target of length L is a periodic structure. Incident electromagnetic waves 1 or 2 or 1 and 2 simultaneously emerge at it. Two strong waves 3 and 4 excited in the target have the wave vectors $\mathbf{k}$ and $\mathbf{k}_\tau = \mathbf{k} + \boldsymbol{\tau}$, where $\boldsymbol{\tau}$ is the reciprocal lattice vector. When specific conditions

$$2k_z\tau_z \approx -2\mathbf{k}_\perp \boldsymbol{\tau}_\perp + \tau^2, \qquad (1)$$

(so-called Bragg conditions) are fulfilled for generation of quasi-Cherenkov radiation and if Cherenkov synchronism condition is fulfilled:

$$|\omega - \mathbf{k}\mathbf{u}| \leq \varepsilon\omega, \qquad (2)$$

electron beam emits coherent electromagnetic radiation. Different generation regimes are possible in such VFEL. If electron beam is in synchronism with the wave having positive group velocity, amplification takes place. Such regime is realized in TWT (travelling wave tube). When group velocity is negative the regenerative and oscillation regimes are possible (BWO - backward wave oscillator). There is also a mixed regime when beam-wave synchronism is satisfied for two modes simultaneously (BWO-TWT) [19]. In Bragg geometry (Fig. 1) the transient wave 3 and diffracted wave 4 are directed in opposite directions. In Laue geometry two waves propagate in the same direction.

Let us look for a solution of Maxwell's equations in the next form:

$$\mathbf{E} = \mathbf{e}Ee^{i(\mathbf{k}\mathbf{r}-\omega t)} + \mathbf{e}_\tau E_\tau e^{i(\mathbf{k}_\tau \mathbf{r}-\omega t)}, \qquad (3)$$

where ω is the frequency and $\mathbf{e}$, $\mathbf{e}_\tau$ are the vectors of wave polarizations.

We consider dependence on one spatial coordinate z only. The system of equations for nonstationary quasi-Cherenkov instability has the following form:

$$\frac{\partial E}{\partial t} + \gamma_0 c\frac{\partial E}{\partial z} + \frac{i\omega}{2}(l-\chi_0)E - \frac{i\omega\chi_\tau}{2}E_\tau$$
$$= \frac{4\pi e n_b \Phi u}{k^2}\int_0^{2\pi}\frac{2\pi - p}{8\pi^2}(\exp(-i\Theta(t,z,p)$$
$$+\exp(-i\Theta(t,z,-p)))dp, \qquad (4)$$

$$\frac{\partial E_\tau}{\partial t} + \gamma_1 c\frac{\partial E_\tau}{\partial z} + \frac{i\omega\chi_{-\tau}}{2}E + \frac{i\omega}{2}(l_1 - \chi_0)E_\tau = 0, \qquad (5)$$

$$E(t,0) = E_0, \quad E_\tau(t,L) = E_1, \qquad (6)$$

$$E(0,z) = 0, \quad E_\tau(0,z) = 0, \qquad (7)$$

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k_z - \frac{d\Theta(t,z,p)}{dz}\right)^3 \times$$
$$\mathrm{Re}\left(E(t-z/u,z)\exp(i\Theta(t,z,p))\right), \qquad (8)$$

$$\Theta(t,0,p) = p, \qquad \frac{d\Theta(t,0,p)}{dz} = k_z - \omega/u, \quad (9)$$

where $t > 0$, $z \in [0,L]$, $p \in [-2\pi, 2\pi]$.

In (4)-(9) the following designations are used:

$$k = \omega/c, \quad \gamma_0 = \frac{k_z}{k}, \quad \gamma_1 = \frac{k_{\tau z}}{k},$$

$$l = l_0 + \delta, \quad l_1 = \frac{\chi_\tau \chi_{-\tau}}{l_0},$$

$$\Phi = \sqrt{l_0 + \chi_0 - 1/(\beta\gamma)^2},$$

where χ_0, $\chi_{\pm\tau}$ are Fourier components of the dielectric susceptibility, l_0 is the geometry parameter, δ is the deviation from Cherenkov synchronism, β is the diffraction asymmetry factor, γ is Lorentz-factor of a beam, e, m are charge and mass of an electron respectively.

Eqs. (4)-(9) is the system of integro-differential equations. In addition to temporal argument there are two independent arguments: spatial coordinate z and initial electron phase p. Amplitudes of electromagnetic fields $E(t,z)$, $E_\tau(t,z)$ are complex-valued. Function $\Theta(t,z,p)$ is the phase of electron in electromagnetic wave. Boundary conditions (6) are written for the case of Bragg geometry.

Equation (8) describes electron beam propagation in VFEL with two-wave VDFB. We model it by averaging over initial phases of electrons. This method is well-known [20] and widely used in simulation of BWT (backward wave tube), TWT, FEL and other electronic devices. Magnetized electron beam with one-dimensional dynamics is considered. Derivation of (8) was proposed in [18].

As was mentioned above we should use numerical methods to solve the system (4)-(9). They were proposed in [18]. We are interested in investigation of dynamics of the whole system, since it is well-known that in laser systems different types of instabilities (bistability, pulsed solutions, chaos) can appear ([21], [22]). So, different types of instabilities and bifurcations leading to transitions between these types of instabilities arising in VFEL were studied on the basis of (4)-(9).

Sequence of bifurcations are generated by variation of control parameters such as current, interaction length L, asymmetry factor β and so on. As an example of simulations carried out, Fig. 2 and Fig. 3 demonstrate saturation and different bifurcations in Bragg geometry in millimeter range with changing current density without and with absorption $Im(\chi_0)$. It is evident, that the current threshold is higher with absorption. Curves form (oscillation period, set of main frequencies) is changed sharply.

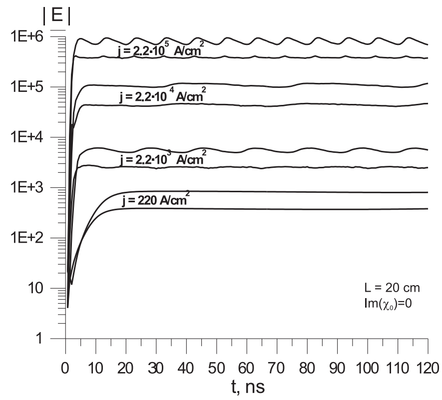

FIG. 2. Numerical solution for different current densities without absorption

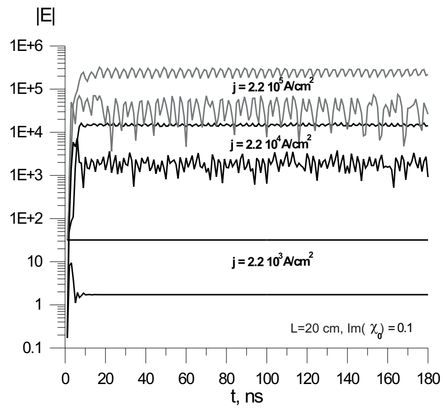

FIG. 3. Numerical solution for different current densities with absorption

The next Fig. 4 demonstrates the presence of different bifurcation regimes for different diffraction geometries and decreasing of the field amplitude with the increase of the parameter l_0. The case of $l_0 \geq 4$ corresponds to BWT.

So, system (4)-(9) can be used for simulation of nonlinear stage of quasi-Cherenkov instability in X-ray VFEL. Such simulation was carried out for Bragg geometry. As an example we chose the

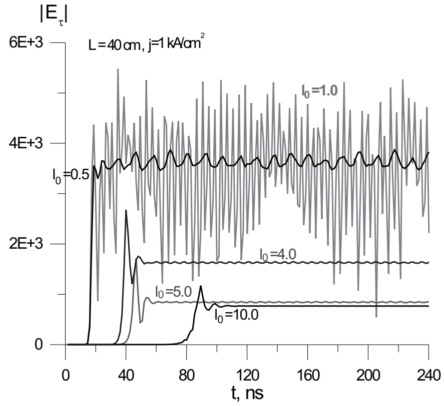

FIG. 4. Numerical solution for different geometry parameter

crystal of LiH with thickness 1 cm and we took the wave length $\lambda = 4.09\mathring{A}$. As one can see in the Fig. 5 the current density threshold is exceeded approximately at current density $j = 300$ MA/cm^2. The moderate bifurcations (transition to oscillation regime is considered as bifurcation also) take place here. However, because of problems with multiple scattering of electrons and deterioration of crystals by electron beam advancing X-ray VFEL cannot be realized yet. X-ray VFEL generation can be realized on electron beams of projected by International collaboration accelerating machines DESY-TESLA.

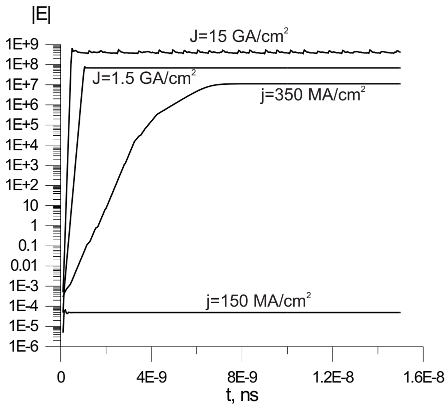

FIG. 5. Numerical solution for X-ray VFEL

3 Mathematical model of surface scheme of VFEL

The surface scheme of quasi-Cherenkov VFEL (where a particle beam moves over a periodic target or at a small angle to this target) was firstly considered in [5]. In [23] the visible qiasi-Cherenkov VFEL was analyzed. In the surface case radiation is formed along to the whole particle trajectory in vacuum and inside a target without multiple scattering. At the same time VDFB is formed by dynamical diffraction from volume optical diffraction grating. Papers [14], [16] were devoted to the detailed analysis of the optical quasi-Cherenkov VFEL. Let us consider the simple scheme of surface VFEL (see Fig. 6).

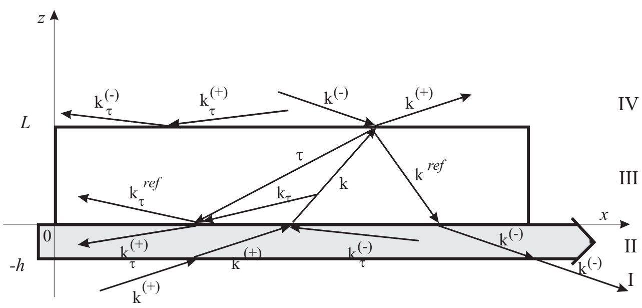

FIG. 6. Surface quasi-Cherenkov VFEL

Domains I and IV are vacuum, domain II is an electron beam, domain III is a spatially periodic target. Here we introduce the following wave vectors: $\mathbf{k} = (\mathbf{k}_\perp, k_z)$, $\mathbf{k}^{ref} = (\mathbf{k}_\perp, -k_z)$, $\mathbf{k}^{(+)} = (\mathbf{k}_\perp, k_{z0})$, $\mathbf{k}^{(-)} = (\mathbf{k}_\perp, -k_{z0})$, $\mathbf{k}_\tau = (\mathbf{k}_\perp + \boldsymbol{\tau}_\perp, k_{\tau z})$, $\mathbf{k}_\tau^{ref} = (\mathbf{k}_\perp + \boldsymbol{\tau}_\perp, -k_{\tau z})$, $\mathbf{k}_\tau^{(+)} = (\mathbf{k}_\perp + \boldsymbol{\tau}_\perp, k_{\tau z0})$, $\mathbf{k}_\tau^{(-)} = (\mathbf{k}_\perp + \boldsymbol{\tau}_\perp, -k_{\tau z0})$, $k_z = (\varepsilon_0 \omega^2/c^2 - k_x^2)^{1/2}$, $k_{z0} = (\omega^2/c^2 - k_x^2)^{1/2}$, $k_{\tau z} = (\varepsilon_0 \omega^2/c^2 - (k_x + \tau_x)^2)^{1/2}$, where $\varepsilon_0 = 1 + \chi_0$ is a dielectric permittivity of the target; $k_{\tau z0} = (\omega^2/c^2 - (k_x + \tau_x)^2)^{1/2}$. The wave amplitudes of corresponding wave vectors are denoted as E, E^{ref}, $E^{(+)}$, $E^{(-)}$, E_τ, E_τ^{ref}, where E and E^{ref} are the radiation wave inside a target and its reflective wave, $E^{(+)}$ and $E^{(-)}$ are that outside the target, E_τ and E_τ^{ref} are the same for the diffracted wave. We can assume without losses of generality that $k_y = t_y = 0$.

In [14], [16] we proposed detailed mathematical models describing nonstationary quasi-Cherenkov instability surface VFEL for optical regime. Let us write some resulting equations.

First: in vacuum for the wave with amplitude E and wave vector $\mathbf{k}$:

$$\frac{\partial E}{\partial t}+\frac{k_z c^2}{\omega}\frac{\partial E}{\partial z}+\frac{k_x c^2}{\omega}\frac{\partial E}{\partial x}=-\frac{2\pi i}{\omega}\frac{\partial j}{\partial t}-2\pi j \quad (10)$$

where j is a beam current density.

Second: in the target for the wave with amplitude E and wave vector $\mathbf{k}$ that does not satisfy to Bragg diffraction conditions:

$$\frac{\partial E}{\partial t}+\frac{k_z c^2}{\omega\varepsilon_0}\frac{\partial E}{\partial z}+\frac{k_x c^2}{\omega\varepsilon_0}\frac{\partial E}{\partial x}=0. \quad (11)$$

Third: in the target for two waves with amplitude E and E_τ and wave vectors $\mathbf{k}$, $\mathbf{k}_\tau$ respectively, which satisfy to Bragg conditions:

$$\frac{\partial E}{\partial t}+\frac{k_z c^2}{\omega\varepsilon_0}\frac{\partial E}{\partial z}+\frac{k_x c^2}{\omega\varepsilon_0}\frac{\partial E}{\partial x}$$
$$+\frac{i(\mathbf{k}^2-\omega^2/c^2\varepsilon_0)}{2\omega\varepsilon_0}E+\frac{i\omega\chi_\tau}{2\varepsilon_0}E_\tau=0, \quad (12)$$
$$\frac{\partial E_\tau}{\partial t}+\frac{k_{\tau z} c^2}{\omega\varepsilon_0}\frac{\partial E_\tau}{\partial z}+\frac{k_{\tau x} c^2}{\omega\varepsilon_0}\frac{\partial E_\tau}{\partial x}$$
$$+\frac{i\omega\chi_\tau}{2\varepsilon_0}E+\frac{i({\mathbf{k}_\tau}^2-\omega^2/c^2\varepsilon_0)}{2\omega\varepsilon_0}E_\tau=0. \quad (13)$$

Boundary conditions are obtained from the continuity of tangential components of electric and magnetic field. They have the generalized form:

$$A_l^-\frac{\partial E_l}{\partial t}+B_l^-\frac{\partial E_i}{\partial x}+C_l^-E_l+D_lE_3$$
$$+A_k^+\frac{\partial E_k}{\partial t}+B_k^+\frac{\partial E_k}{\partial x}+C_k^+E_k=f_k(x,t,E_k^{(0)},j), \quad (14)$$

where $l=1,2$ and $k=2,1$ for two boundaries with respect to z. $E_k^{(0)}$ is the amplitude of the wave incident to considered boundary from the outside. Waves with amplitudes E_1 and E_3 satisfy to Bragg diffraction conditions.

So, the full scheme of surface VFEL from Fig. 7 is outlined by the system (12)-(13), two equations of type (11) and two equations of type (10) plus three pairs of (14).

For optical VFEL induced by beam-wave interaction, the current was simulated on the basis of kinetic equations for electron distribution functions [14], [16].

Ibid, for solving the system of nonlinear first-order differential equations with boundary conditions given also by the nonlinear first-order PDEs we proposed an algorithm of the multicomponent modification of the alternating direction method [24]. It is efficient and unconditionally stable for multidimensional problems in domains with complicated geometry and proved to be effective also when operating with complex arithmetic.

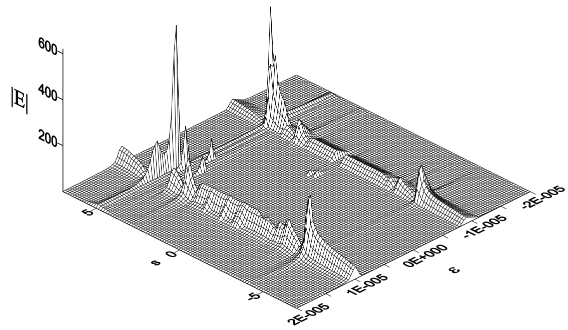

FIG. 7. Optical surface quasi-Cherenkov generation. Dependence of amplification on deviation from Cherenkov synchronism and Bragg condition

Let us demonstrate some examples of optimization of VFEL with two wave distributed feedback. At Fig. 7 dependence of amplification on detuning from the synchronism condition ε (see (2)) and the Bragg condition $s=(l_1-l)\sqrt{-\gamma_1/\gamma_0}/|\chi_\tau|$ (see parameters expansion above) is shown. It is obvious that there is the optimal correlation between these two parameters where the amplification process is developed most effectively.

4 System of PDEs for modelling of VFEL with multiwave distributed feedback

All above mentioned systems and numerical results were produced in the case of two-wave Bragg distributed feedback. Theoretical investigations show the great advantage of multiwave diffraction geometry. System of equations for such a geometry can be derived by the same way as for two-wave geometry. System for modelling of VFEL with multiwave distributed feedback has the following form:

$$\frac{\partial E}{\partial t} + \sum_{i=1}^{N} A_i \frac{\partial E}{\partial x_i} + CE = F(j), \qquad (15)$$

where E is the vector of dimension M of amplitudes of electric field strength inside the target, j is the beam current density.

Matrixes A_i and C are complex-valued. A_i are diagonal. Nonzero lines of C correspond to diffraction components in the system. Initial conditions for (15) can be put equal to zero. Boundary conditions with respect to x_1 are the following:

$$B\frac{\partial E}{\partial t} + \sum_{i=2}^{N} P_i \frac{\partial E}{\partial x_i} + QE = G(j, E^0). \qquad (16)$$

Each of M lines of the system (16) corresponds to one boundary condition. Number of lines M is equal to the number of desired field amplitudes. E^0 is a vector of amplitudes of incident to the system waves.

So, for the first model from Section 1 we have to take $N = 1$, $M = 2$, $E = (E, E_\tau)^T$, $F_2 = 0$, B is a null matrix, Q is an unitary matrix. $G_1 = E_0$ is specified at $x_1 = 0$, $G_2 = E_{\tau 0}$ is specified at $x_1 = L$.

For two-wave surface VFEL (Section 3) we have to take $N = 2$, $M = 6$, $E = (E, E_\tau, E^{ref}, E_\tau^{ref}, E^{(+)}, E^{(-)})^T$. C has two first nonzero lines, corresponding to E and E_τ. F has two last nonzero lines, corresponding to $E^{(+)}$ and $E^{(-)}$. In (16) in each line only ingoing and outgoing from this boundary waves are involved.

For each case we have to write out accurately coefficients of the system obtained and to solve the problem of electron beam simulation.

To solve obtained system of equations we propose to use multicomponent numerical algorithms [12], [14] if the number of spatial variables $N \geq 2$.

5 Conclusions

Mathematical models and proposed numerical algorithms can be used effectively in modelling of operation of different VFEL schemes. They will be useful for experiments on VFEL on the VFEL setup formed in the Institute for Nuclear Problems of Belarusian State University.

Authors thanks prof. Baryshevsky for persistent interest to this work.

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