Electrodynamical properties of a "grid" volume resonator for travelling wave tube and backward wave oscillator

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Abstract

The electrodynamical properties of a volume resonator formed by a perodic structure built from the metallic threads inside a rectangular waveguide ("grid" volume resonator) is considered for travelling wave tube and backward wave oscillator operation. Peculiarities of passing of electromagnetic waves with different polarizations through such volume resonator are discussed.

Key words: Volume Free Electron Laser (VFEL), Volume Distributed Feedback (VDFB), diffraction grating, Smith-Purcell radiation, electron beam instability
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1 Introduction

Generation of radiation in millimeter and far-infrared range with nonrelativistic and low-relativistic electron beams gives rise difficulties. Gyrotrons and cyclotron resonance facilities are used as sources in millimeter and sub-millimeter range, but for their operation magnetic field about several tens of kiloGauss ($\omega \sim \frac{eH}{mc}\gamma$) is necessary. Slow-wave devices (TWT, BWT, orotrons)in this range require application of dense and thin ($<$ 0.1 mm) electron beams, because only electrons passing near the slowing structure at the distance $\leq \lambda\beta\gamma/(4\pi)$ can interact with electromagnetic wave effectively. It is difficult to guide thin beams near slowing structure with desired accuracy. And electrical endurance of resonator limits radiation power and density of acceptable electron beam. Conventional waveguide systems are essentially restricted by the requirement for transverse dimensions of resonator, which should not significantly exceed radiation wavelength. Otherwise, generation efficiency decreases abruptly due to excitation of plenty of modes. The most of the above problems can be overpassed in VFEL [1,2,3,4,5]. In VFEL the greater part of electron beam interacts with the electromagnetic wave due to volume distributed interaction. Transverse dimensions of VFEL resonator could significantly exceed radiation wavelength $D \gg \lambda$. In addition, electron beam and radiation power are distributed over the whole volume that is beneficial for electrical endurance of the system. Multi-wave Bragg dynamical diffraction provides mode discrimination in VFEL.

The electrodynamical properties of volume diffraction structures composed from strained dielectric threads was experimentally studied in [6]. In [8] it was shown that nonrelativistic and low-relativistic electron beams passing through such structures can generate in wide frequency range up to terahertz.

In the present paper the electrodynamical properties of a "grid" volume resonator that is formed by a perodic structure built from the metallic threads inside a rectangular waveguide (see Fig.1) is considered.

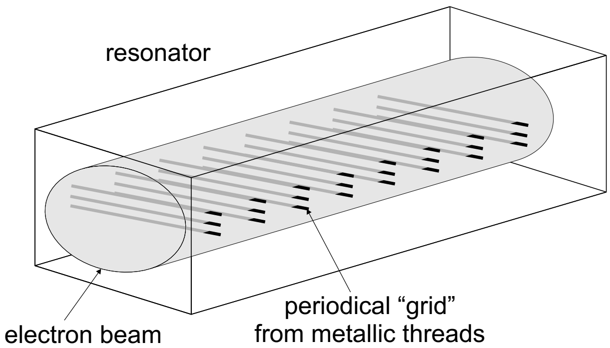

Fig. 1. "Grid" volume resonator

2 Scattering by a set of metallic threads

Let us consider a plane electromagnetic wave $\vec{E} = \Psi\vec{e}$, where $\vec{e}$ is the polarisation vector. Suppose this wave falls onto the cylinder placed into the origin of coordinates and the cylinder axis concides with the axis x (Fig.2) (in further consideration $\vec{e}$ will be omitted). Two orientations of $\vec{e}$ should be considered: $\vec{e}$ is parallel to the cylinder axis x and $\vec{e}$ is perpendicular to the cylinder axis x. For clarity suppose that $\vec{e} \parallel 0x$.

The scattered wave can be written as [7]

$$\Psi = e^{ikz} + a_0 H_0^{(1)}(k\rho) \quad (1)$$

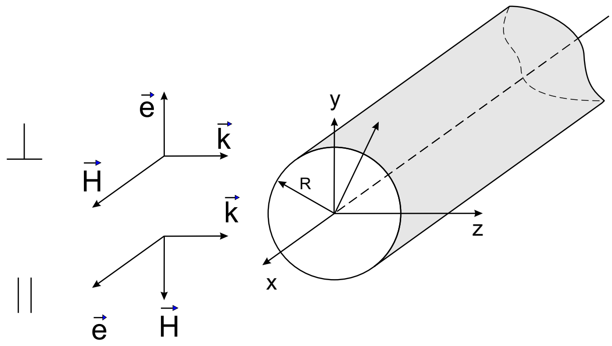

Fig. 2. .

here ρ is the transverse coordinate $\rho = (y, z)$, $H_0^{(1)}$ is the Hankel function

Thus, considering a set of cylinders with $\rho_n = (y_n, z_n)$ one can express the scattered wave as

$$\Psi = e^{ikz} + a_0 \Sigma_n H_0^{(1)}(k\,|\overrightarrow{\rho} - \overrightarrow{\rho}_n|) e^{ikz_n} \tag{2}$$

or using the integral representation for Hankel functions

$$\Psi = e^{ikz} + A_0 \Sigma_n \int\limits_{-\infty}^{\infty} \frac{e^{ik\sqrt{|\overrightarrow{\rho} - \overrightarrow{\rho}_n|^2 - x^2}}}{\sqrt{|\overrightarrow{\rho} - \overrightarrow{\rho}_n|^2 - x^2}} dx e^{ikz_n}, \tag{3}$$

where $A_0 = -\frac{ia_0}{\pi}$, $|\overrightarrow{\rho} - \overrightarrow{\rho}_n|^2 = (y - y_n)^2 + (z - z_n)^2$.

Let us consider the wave passing through the layer of cylinders, which axes are distributed in the plane $x0y$ on the distance d_y. Summation over the coordinates y_n provides the following expression for Ψ:

$$\Psi = e^{ikz} + \frac{2\pi i A_0}{k d_y} e^{ikz}, \tag{4}$$

Thus, after passing m planes (standing out of each other in the distance d_z) the scattered wave can be expressed as:

$$\Psi = \left(\sqrt{\left(1 - \frac{2\pi\ Im A_0}{k\ d_y}\right)^2 + \left(\frac{2\pi\ Re A_0}{k\ d_y}\right)^2} \right)^m e^{ikz} e^{i\varphi m}, \tag{5}$$

where $\varphi = arctg\left(\frac{\frac{2\pi\ Re A_0}{k\ d_y}}{1 - \frac{2\pi\ Im A_0}{k\ d_y}}\right)$, $m = \frac{z}{d_z}$ inside the structure formed by threads. This expression can be easily converted to the form $\Psi = e^{iknz}$, where n is the refraction index defined as

$$n = n' + in'' = \tag{6}$$

$$= \left(1 + \frac{\lambda}{2\pi d_z} Arctg\left(\frac{\frac{\lambda}{d_y} ReA_0}{1 - \frac{\lambda}{d_y} ImA_0}\right)\right) - i\frac{\lambda}{2\pi d_z} \ln\left(\sqrt{\left(\frac{\lambda}{d_y} ReA_0\right)^2 + \left(1 - \frac{\lambda}{d_y} ImA_0\right)^2}\right).$$

here $\lambda = \frac{2\pi}{k}$ is used.

If $ReA_0,\ ImA_0 \ll 1$ then (7) can be expressed as:

$$n = 1 + \frac{2\pi}{d_y d_z k^2} A_0. \tag{7}$$

Radiation frequencies of our interest is $\nu \geq 10$ GHz. In this frequency range skin depth δ is about 1 micron for the most of metals (for example, $\delta_{Cu} = 0.66$ μm, $\delta_{Al} = 0.8$ μm, $\delta_W = 1.16$ μm and so on). Thus, in this frequency range the metallic threads can be considered as perfect conducting.

From the analysis [7] follows that the amplitude A_0 for the perfect conducting cylinder for polarization of the electromagnetic wave parallel to the cylinder axis can be expressed as:

$$A_{0(\|)} = \frac{1}{\pi} \frac{J_0(kR) N_0(kR)}{J_0^2(kR) + N_0^2(kR)} + i\frac{1}{\pi} \frac{J_0^2(kR)}{J_0^2(kR) + N_0^2(kR)} \tag{8}$$

Amplitude A_0 for the perfect conducting cylinder for polarization of the electromagnetic wave perpendicular to the cylinder axis is as follows [7]:

$$A_{0(\perp)} = \frac{1}{\pi} \frac{J_0'(kR) N_0'(kR)}{J_0'^2(kR) + N_0'^2(kR)} + i\frac{1}{\pi} \frac{J_0'^2(kR)}{J_0'^2(kR) + N_0'^2(kR)}, \tag{9}$$

where R is the cylinder (thread) radius, J_0, N_0, J_0' and N_0' are the Bessel and Neumann functions and their derivatives, respectively. Using the asymptotic values for these functions for $kR \ll 1$ one can obtain:

$$\begin{aligned} &J_0(x \to 0) \approx 1,\ N_0(x \to 0) \approx -\tfrac{2}{\pi} \ln \tfrac{2}{1.781 \cdot x}, \\ &J_0'(x \to 0) = -J_1 \approx -\tfrac{x}{2},\ N_0'(x \to 0) = -N_1 \approx -\tfrac{2}{\pi}\tfrac{1}{x}. \end{aligned} \tag{10}$$

Let us consider a particular example. Suppose radiation frequency $\nu = 10$ GHz and the thread radius $R = 0.1$ mm, then

$$ReA_{0(\|)} \approx -0.1087,\ ImA_{0(\|)} \approx 0.0429, \tag{11}$$

$$ReA_{0(\perp)} \approx -0.00011,\ ImA_{0(\perp)} \approx 3.78 \cdot 10^{-8} \tag{12}$$

and

$$n_{\|} = 0.8984 + i \cdot 0.043, \tag{13}$$
$$n_{\perp} = 0.9998 + i \cdot 3.37 \cdot 10^{-9} \tag{14}$$

Such values for n provides to conclude that, in contrast to a solid metal, an electromagnetic wave falling on the described "grid" volume structure is not absorbed on the skin depth, but passes through the "grid" damping in accordance its polarization.

The electromagnetic wave with polarization parallel to the thread axis is strongly absorbed while passing through the structure. Absorption for the wave with polarization perpendicular to the thread axis is weak.

Difference in $n_{\|}$ and $n_{\perp}$ indicates that the system own optical anysotropy (i.e. possesses birefringence and dichroism). To escape this anysotropy we can use alternating the treads position in grid: threads in each layer are orthogonal to the threads in previous and following layer.

And one more important note: When an electron beam passes through volume grid The pulse of transition radiation appears due to large value of the refraction index

The values $ReA_{0(\|)}$ and $ImA_{0(\|)}$ are quite large and for polarization parallel to the thread axis the exact expression (5,7) should be used. Moreover, in all calculations we should carefully check whether the condition $|n-1| \ll 1$ is fullfilled. If no, then we should use more strict description of volume structure and consider rescattering of the wave by different threads.

In this case in contrast to (2) the electromagnetic wave is described by (the wave with polarization along the thread axes is considered):

$$\Psi(\rho) = e^{ikz} + \Sigma_m F_m H_0^{(1)}(k\,|\overrightarrow{\rho} - \overrightarrow{\rho}_m|), \tag{15}$$

where F_m is the effective scattering amplitude defined by

$$F_m = a_0 e^{ikz_m} + a_0 \Sigma_{n\neq m} F_n H_0^{(1)}(k\,|\overrightarrow{\rho} - \overrightarrow{\rho}_n|), \tag{16}$$

Let us consider unregulated set of threads. According to the system of equations (15,16) to find F_m it is necessary to solve the system of algebraic equations.

Let us consider first the long-wave case ($kd \ll 1$) to obtain the approximate solution; the sum in (16) could be replaced by the integral:

$$F_m(\rho) = a_0 \left\{ e^{ikz} + \frac{1}{\Omega_2} \int\limits_V F_m(\rho') H_0^{(1)}(k \left| \overrightarrow{\rho} - \overrightarrow{\rho}' \right|) d^2\rho' \right\}, \tag{17}$$

where the area V includes all scatterers with the exception of that located in the point ρ (this area V being supplied with the small area ΔV, surrounding the point ρ, gives us the whole area)

If note here that

$$e^{ikz} + \frac{1}{\Omega_2} \int F_m(\rho') H_0^{(1)}(k \left| \overrightarrow{\rho} - \overrightarrow{\rho}' \right|) d^2\rho' = \Psi(\rho), \tag{18}$$

and present the integral over the area V as the difference of integral over the whole area and the integral over the area ΔV we can get

$$F_m(\rho) = a_0 \left\{ \Psi(\rho) - \frac{1}{\Omega_2} \int\limits_{\Delta V} F_m(\rho') H_0^{(1)}(k \left| \overrightarrow{\rho} - \overrightarrow{\rho}' \right|) d^2\rho' \right\}, \tag{19}$$

as the area ΔV is small surrounding of the point ρ and $F_m(\rho')$ does not change significantly within this area, the values $F_m(\rho')$ in the integral over ΔV could be replaced by $F_m(\rho)$. This provides to write

$$F_m(\rho) = a_0 \Psi(\rho) - F_m(\rho) \frac{a_0}{\Omega_2} \int\limits_{\Delta V} H_0^{(1)}(k \left| \overrightarrow{\rho} - \overrightarrow{\rho}' \right|) d^2\rho', \tag{20}$$

Therefore

$$F_m(\rho) = \frac{a_0}{1 + \frac{a_0}{\Omega_2} \int_{\Delta V} H_0^{(1)}(k \left| \overrightarrow{\rho} - \overrightarrow{\rho}' \right|) d^2\rho'} \Psi(\rho), \tag{21}$$

and the equation for Ψ looks as follows

$$\Psi(\rho) = e^{ikz} + \frac{B}{\Omega_2} \int\limits_V H_0^{(1)}(k \left| \overrightarrow{\rho} - \overrightarrow{\rho}' \right|) \Psi(\rho') d^2\rho', \tag{22}$$

or

$$(\Delta + k^2)\Psi(\rho) = \frac{4iB}{\Omega_2} \Psi(\rho) \tag{23}$$

When searching $\Psi(\rho) \sim e^{iq\rho}$ we get

$$-q^2 + k^2 = \frac{4iB}{\Omega_2}, \tag{24}$$

after introducing parameter $b_0 = B/a_0$ and remembering $a_0 = i\pi A_0$ (25) appears

$$q^2 = k^2 + \frac{4\pi A_0 b_0}{\Omega_2}, \tag{25}$$

where $b_0 = \frac{1}{1+\frac{a_0}{\Omega_2}\int_{\Delta V} H_0^{(1)}(k\left|\overrightarrow{\rho}-\overrightarrow{\rho}'\right|)d^2\rho'}$.

To find b_0 the integral should be calculated:

$$\frac{1}{\Omega_2}\int\limits_{\Delta V} H_0^{(1)}(k\left|\overrightarrow{\rho}-\overrightarrow{\rho}'\right|)d^2\rho' = (1 + i\frac{2}{\pi}C) + i\frac{2}{\pi}\ln\frac{kR}{2}, \tag{26}$$

Thus, the refraction index is as follows:

$$n^2 = 1 + \frac{4\pi A_0}{\Omega_2 k^2}\frac{1}{1 + a_0((1 + i\frac{2}{\pi}C) + i\frac{2}{\pi}\ln\frac{kR}{2})}, \tag{27}$$

or rewriting with $a_0 = i\pi A_0$

$$n^2 = 1 + \frac{4\pi A_0}{\Omega_2 k^2}\ \frac{1}{1 + i\pi A_0 - 2CA_0 - 2A_0\ln\frac{kR}{2}}, \tag{28}$$

here $C = 0.5772$ is the Eiler constant.

Let us consider the regular structure (the volume "grid") built from treads. Scattering by a thread is described by:

$$\Psi(\rho) = e^{ikz} + a_0 H_0^{(1)}(k\left|\overrightarrow{\rho}-\overrightarrow{\rho}'\right|)e^{ikz_n} \tag{29}$$

When $z\rho \to z_n\rho_n$ the wavefunction Ψ can be expressed:

$$\Psi(\rho) = e^{ikz_n} + a_0\{1 + i\frac{2}{\pi}C + i\frac{2}{\pi}\ln\frac{k|\vec{\rho}-\vec{\rho}_n|}{2}\}e^{ikz_n} \tag{30}$$

Introducing $\varphi = 1 + a_0(1 + i\frac{2}{\pi}C)$ that gathers non-divergent terms, we can rewrite this expression as follows

$$\Psi(\rho) = e^{ikz_n}\varphi\{1 + \frac{a_0}{1 + a_0(1 + i\frac{2}{\pi}C)} i\frac{2}{\pi}\ln\frac{k|\vec{\rho} - \vec{\rho}_n|}{2}\}e^{ikz_n} \tag{31}$$

Using the similar reasoning for many scatterers (considering $z \to z_n$) we can obtain for the wavefunction:

$$\begin{aligned} \Psi(\rho) &= e^{ikz_n} + (\ldots + \ldots_{non-divergent\ terms}\ldots + \ldots) + F_n i\tfrac{2}{\pi}\ln\tfrac{k|\vec{\rho}-\vec{\rho}_n|}{2}e^{ikz_n} = \\ &= e^{ikz_n}\varphi_n\{1 + \tfrac{F_n}{\varphi_n} i\tfrac{2}{\pi}\ln\tfrac{k|\vec{\rho}-\vec{\rho}_n|}{2}\}, \end{aligned} \tag{32}$$

where $\frac{F_n}{\varphi_n} = \frac{a_0}{1+a_0(1+i\frac{2}{\pi}C)}$.

The solution in a volume "grid" (an artificial crystal) could be presented in the form:

$$\Psi(\vec{\rho}) = \chi(\vec{\rho})e^{i\vec{k}'\vec{\rho}},\ \vec{\rho} = (y, z) \tag{33}$$

The equation for the wavefunction

$$(\Delta + k^2)\Psi(\vec{\rho}) = 4iF\sum_m e^{i\vec{k}'\vec{\rho}}\delta(\overrightarrow{\rho} - \overrightarrow{\rho}_m), \tag{34}$$

where F is an amplitude, provides to get the equation for $\chi(\vec{\rho})$:

$$\Delta\chi(\vec{\rho}) + 2i\vec{k}'\vec{\nabla}\chi(\vec{\rho}) - (k'^2 - k^2)\chi(\vec{\rho}) = 4iF\sum_m e^{i\vec{k}'\vec{\rho}}\delta(\overrightarrow{\rho} - \overrightarrow{\rho}_m), \tag{35}$$

where $\chi(\vec{\rho})$ can be presented as a sum over the reciprocal lattice vectors $\vec{\tau}$

$$\chi(\vec{\rho}) = \sum_\tau c_\tau e^{i\vec{\tau}\vec{\rho}}, \tag{36}$$

Therefore, the wavefunction can be expressed

$$\Psi(\vec{\rho}) = -\frac{4iF}{\Omega_2}\sum_\tau \frac{e^{i(\vec{k}'+\vec{\tau})\vec{\rho}}}{(\vec{k}' + \vec{\tau})^2 - k^2}. \tag{37}$$

At the limit $\rho \to 0$

$$\Psi - Fi\frac{2}{\pi}\ln\frac{k\rho}{2} = \varphi \tag{38}$$

Substituting the expression (37)

$$-\frac{4iF}{\Omega_2}\sum_{\tau}\frac{e^{i(\vec{k}'+\vec{\tau})\vec{\rho}}}{(\vec{k}'+\vec{\tau})^2-k^2}-Fi\frac{2}{\pi}\ln\frac{k\rho}{2}=\varphi \tag{39}$$

i.e.

$$\frac{4i}{\Omega_2}\sum_{\tau}\frac{e^{i(\vec{k}'+\vec{\tau})\vec{\rho}}}{(\vec{k}'+\vec{\tau})^2-k^2}+i\frac{2}{\pi}\ln\frac{k\rho}{2}=-\frac{\varphi}{F}=\frac{F_n}{\varphi_n}=\frac{a_0}{1+a_0(1+i\frac{2}{\pi}C)} \tag{40}$$

$$k'^2=k^2+\frac{4\pi}{\Omega_2}\frac{A_0}{1+a_0(1+i\frac{2}{\pi}C)}=k^2+\frac{4\pi}{\Omega_2}\frac{A_0}{1+i\pi A_0-2CA_0}, \tag{41}$$

Therefore, the index of refraction is

$$n^2=1+\frac{4\pi A_0}{\Omega_2 k^2}\,\frac{1}{1+i\pi A_0-2CA_0}, \tag{42}$$

Then for the same example ($\nu = 10$ GHz, $R = 0.1$ mm) we obtain:

$$n_{\|}=0.779+i\cdot 1.217\cdot 10^{-19}, \tag{43}$$
$$n_{\perp}=0.99987-i\cdot 1.3464\cdot 10^{-23} \tag{44}$$

(compare this with (12,13)).

Rescattering effects significantly change the index of refraction and its imaginary part appears noticeably reduced.

3 Sketch theory of VFEL lasing using electron beam radiation in a volume "grid" resonator

Maxwell equations and motion equations in this case

$$\begin{aligned}&rot\vec{H}=\tfrac{1}{c}\tfrac{\partial\vec{D}}{\partial t}+\tfrac{4\pi}{c}\vec{j},\ \vec{D}(\vec{r,t})=\textstyle\int_{-\infty}^{\infty}\varepsilon(\vec{r},t-t')\vec{E}(\vec{r},t')dt',\\&rot\vec{E}=-\tfrac{1}{c}\tfrac{\partial\vec{H}}{\partial t},\ div\vec{D}=4\pi\rho,\ \tfrac{\partial\rho}{\partial t}+div\vec{j}=0,\end{aligned} \tag{45}$$

here $\varepsilon(\vec{r},t-t')<0$ at $t<t'$, $D_i(\vec{r},t')=\int\varepsilon(\vec{r},t-t')E_l(\vec{r},t')dt'$ and, therefore, $D_i(\vec{r},\omega)=\varepsilon_{il}(\vec{r},\omega)E_l(\vec{r},\omega)$. Combining the above equation we obtain:

$$
-\Delta\vec{E}+\vec{\nabla}(\vec{\nabla}\vec{E})+\frac{1}{c^2}\frac{\partial^2\vec{D}}{\partial t^2}=-\frac{4\pi}{c^2}\frac{\partial\vec{j}}{\partial t} \tag{46}
$$

after making the Fourier transformation:

$$
\begin{array}{l}
rotrot\vec{E}(\vec{r},\omega)-\frac{\omega^2}{c^2}\varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega=\frac{4\pi i\omega}{c^2}\vec{j}(\vec{r},\omega)\\
div\ \varepsilon(\vec{r},\omega)\vec{E}(\vec{r},\omega)=4\pi\rho(\vec{r},\omega),\\
-i\omega\rho(\vec{r},\omega)+div\vec{j}(\vec{r},\omega)=0,\\
\vec{j}(\vec{r},t)=e\sum_\alpha\vec{v}_\alpha(t)\delta(\vec{r}-\vec{r}_\alpha(t)),\ \rho(\vec{r},t)=e\sum_\alpha\delta(\vec{r}-\vec{r}_\alpha(t)),\\
\frac{d\vec{v}_\alpha}{dt}=\frac{e}{m\gamma}\left\{\vec{E}(\vec{r}_\alpha(t),t)+\frac{1}{c}[\vec{v}_\alpha(t)\times\vec{H}(\vec{r}_\alpha(t),t)]-\frac{\vec{v}_\alpha}{c^2}(\vec{v}_\alpha(t)\vec{E}(\vec{r}_\alpha(t),t))\right\}
\end{array} \tag{47}
$$

where $\gamma=(1-\frac{v_\alpha^2}{c^2})^{-\frac{1}{2}}$.

Applying the method of slow-varying amplitudes the solution for this system can be expressed as

$$
\vec{E}(\vec{r},t)=\vec{e}_1A_1(\vec{r},t)e^{i(\vec{k}_1\vec{r}-\omega t)}+\vec{e}_2A_2(\vec{r},t)e^{i(\vec{k}_2\vec{r}-\omega t)}, \tag{48}
$$

$\vec{k}_2=\vec{k}_1+\vec{\tau}$. Substituting (48) to the exact system of equations and collecting the quick-oscillating terms we obtain the system:

$$
\begin{array}{l}
2i\vec{k}_1\vec{\nabla}A_1(\vec{r},t)-k_1^2A_1(\vec{r},t)+\frac{\omega^2}{c^2}\varepsilon^0(\omega)A_1(\vec{r},t)+i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon^0(\omega)}{\partial\omega}\frac{\partial A_1(\vec{r},t)}{\partial t}+\\
+\frac{\omega^2}{c^2}\varepsilon^{-\tau}(\omega)A_2(\vec{r},t)+i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon^{-\tau}(\omega)}{\partial\omega}\frac{\partial A_2(\vec{r},t)}{\partial t}=J_1,\\
2i\vec{k}_2\vec{\nabla}A_2(\vec{r},t)-k_2^2A_2(\vec{r},t)+\frac{\omega^2}{c^2}\varepsilon^0(\omega)A_2(\vec{r},t)+i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon^0(\omega)}{\partial\omega}\frac{\partial A_2(\vec{r},t)}{\partial t}+\\
+\frac{\omega^2}{c^2}\varepsilon^{\tau}(\omega)A_1(\vec{r},t)+i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon^{\tau}(\omega)}{\partial\omega}\frac{\partial A_1(\vec{r},t)}{\partial t}=J_2,
\end{array} \tag{49}
$$

J_1, J_2 are the currents, their explicit expressions can be found in [9].

The system (49) includes terms describing wave dispersion, if omit these terms we get the system analyzed in the foregoing paper [9].

Consideration of scattering by a diffraction grating in a waveguide requires $\vec{E}(\vec{r},t)$ expansion over the waveguide eigenfunctions $\vec{Y}_{\lambda mn}(\vec{r}_\perp)$ ($\vec{r}_\perp=(x,y)$, the waveguide axes is parallel to the axes z), which meet the equation:

$$
\Delta_\perp\vec{Y}_{\lambda mn}+(\varkappa_{mn}^{\lambda})^2\vec{Y}_{\lambda mn}=0. \tag{50}
$$

Expanding $\vec{E}(\vec{r},t) = \vec{E}(\vec{r}_\perp, z, t)$ over $\vec{Y}_{\lambda mn}(\vec{r}_\perp)$

$$\vec{E}(\vec{r}_\perp, z, t) = \sum_{\lambda mn} C_{\lambda mn}(z,t)\vec{Y}_{\lambda mn}(\vec{r}_\perp) \tag{51}$$

and considering the waveguide with a diffraction grating in vacuum

$$D_i(\vec{r},\omega) = (\delta_{il} + \chi_{il}(\vec{r},\omega))E_l(\vec{r},\omega) = E_i(\vec{r},\omega) + \chi_{il}(\vec{r},\omega)E_l(\vec{r},\omega) \tag{52}$$

we can write

$$\Delta\vec{E} - \vec{\nabla}(div\vec{E}) - \tfrac{1}{c^2}\tfrac{\partial^2}{\partial t^2}\int \hat{\varepsilon}(t-t')\vec{E}(t')dt' = \tfrac{4\pi}{c^2}\tfrac{\partial \vec{j}}{\partial t}, \tag{53}$$

or

$$\Delta\vec{E} - \tfrac{1}{c^2}\tfrac{\partial^2}{\partial t^2}\int \hat{\varepsilon}(t-t')(\vec{E}(t') - \vec{\nabla}(div\hat{\chi}\vec{E}))dt' = \tfrac{4\pi}{c^2}\tfrac{\partial \vec{j}}{\partial t} + 4\pi\vec{\nabla}\rho, \tag{54}$$

or

$$\Delta\vec{E} - \tfrac{1}{c^2}\tfrac{\partial^2\vec{E}}{\partial t^2} - \tfrac{1}{c^2}\tfrac{\partial^2}{\partial t^2}\int \hat{\chi}(t-t')\vec{E}(t') - \vec{\nabla}(div\hat{\chi}\vec{E}))dt' = \tfrac{4\pi}{c^2}\tfrac{\partial \vec{j}}{\partial t} + 4\pi\vec{\nabla}\rho. \tag{55}$$

After expansion

$$\begin{aligned}
&\tfrac{\partial^2 C_{\lambda mn}(z,t)}{\partial z^2} - \tfrac{1}{c^2}\tfrac{\partial^2 C_{\lambda mn}(z,t)}{\partial t^2} - (\varkappa_{mn}^{\lambda})^2 C_{\lambda mn}(z,t) - \\
&-\tfrac{1}{c^2}\tfrac{\partial^2}{\partial t^2}\int \vec{Y}^*_{\lambda mn}(\vec{r}_\perp)\hat{\chi}(t-t')\textstyle\sum_{\lambda' m' n'} C_{\lambda' m' n'}(z,t')\vec{Y}_{\lambda' m' n'}(\vec{r}_\perp)d^2r_\perp + \\
&+\vec{Y}^*_{\lambda mn}(\vec{r}_\perp)vec\nabla(div\hat{\chi}\textstyle\sum_{\lambda' m' n'} C_{\lambda' m' n'}(z,t'))\vec{Y}_{\lambda' m' n'}(\vec{r}_\perp)d^2r_\perp = \\
&= \tfrac{4\pi}{c^2}\tfrac{\partial}{\partial t}\int \vec{Y}^*_{\lambda mn}(\vec{r}_\perp)\vec{j}_{\lambda mn}d^2(r_\perp) + 4\pi\int \vec{Y}^*_{\lambda mn}(\vec{r}_\perp)\vec{\nabla}\rho d^2(r_\perp)
\end{aligned} \tag{56}$$

Applying the method of slow variable amplitudes we can obtain the system of equations describing the waves excited in the system: $C_{\lambda mn}(z,t) = A_{\lambda mn}(z,t)e^{i(\varkappa_{\lambda mn}z-\omega t)}$ (here $\varkappa_{\lambda mn}$ and ω corresponds to the waveguide without a diffraction grating).

In general case different modes are separated, but grating rotation could mix different modes [4] (similar waves mixing in the vicinity of Bragg condition). To describe this process the equations for the mixing modes should be solved conjointly.

4 Conclusion

In the present paper the electrodynamical properties of a volume resonator that is formed by a periodic structure built from the metallic threads inside a rectangular waveguide is considered. Peculiarities of passing of electromagnetic waves with different polarizations through such volume resonator are discussed. If in the periodic structure built from the metallic threads diffraction conditions are available, then analysis shows that in this system the effect of anomalous transmission for electromagnetic waves could appear similarly to the Bormann effect well-known in the dynamical diffraction theory of X-rays.

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